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A Lyapunov-based distributed consensus filter for a class of nonlinear stochastic systems☆

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ABSTRACT

This paper considers the distributed state estimation problem for nonlinear stochastic systems over sensor networks. It is assumed that the nonlinear functions are bounded in the pseudo Lipschitz condition. Based on the stochastic Lyapunov stability theory, a distributed consensus filter (DCF) is proposed for both continuous and discrete nonlinear stochastic systems for each node in a sensor network. It will be shown that the estimation errors of the proposed filters are exponentially ultimately bounded in the sense of mean square in terms of linear matrix inequality (LMI). Furthermore, a criterion is presented to optimize the filter gains based on minimizing the upper bound of mean-square error. Numerical examples are used to verify the theoretical results.

1. Introduction

In recent decades, the problem of distributed state estimation (DSE) has received great attention for its successful applications in different areas including environmental monitoring, surveillance, cooperative control of multi-agent systems, target tracking and so on (Olfati-Saber, Fax, & Murray, 2007; Xie, Choi, Kar, & Poor, 2012). The essential principles of DSE algorithms in sensor networks are state estimation at every node and reaching consensus based on the estimated states of each node and its neighboring nodes. Here the word "consensus" means that each sensor node uses distributed filters that can agree on an estimated value with their neighbors. These types of algorithms are called distributed consensus algorithms. In recent decades, many distributed consensus algorithms were introduced and applied (Farina, Ferrari-Trecate, & Scattolini, 2010; Olfati-Saber, 2005, 2007, 2009; Zhu, Chen, Li, Yang, & Guan, 2013).

Distributed consensus algorithms can be classified into four groups: consensus on state estimation, consensus on innovations, consensus on information and H_∞ consensus. The first group has consensus on state estimation, in which estimates are averaged to reach a consensus (Açıkmeşe, Mandić, & Speyer, 2014; Farina et al., 2010; Olfati-Saber, 2007, 2009; Zhu et al., 2013). Some of the first consensus algorithms on state estimation were given in Olfati-Saber (2007, 2009). Actually, by adding a consensus term to the Kalman filter, consensus filters in discrete and continuous forms, which are called Kalman-consensus filters, were introduced in Olfati-Saber (2007). The error covariance of the Kalman-consensus filter was not optimal in a discrete manner, which could cause unacceptable estimation errors. Thus, an optimal consensus filter was proposed in Olfati-Saber (2009) and its stability was investigated. In the second group, a consensus is performed on local innovations (Li & Jia, 2011, 2012; Olfati-Saber, 2005, 2007). In Olfati-Saber (2005), the first consensus filter based on consensus on innovations has been proposed. A modified version of this consensus filter which can be applied in a sensor network with different observation matrices was introduced in Olfati-Saber (2007). In papers Li and Jia (2011, 2012), consensus on innovations was used to design consensus filters for jump Markov systems and discrete-time nonlinear systems with non-Gaussian noise. In the third group, consensus occurs on the inverse of the state estimation covariance matrix or information matrix that was firstly applied in a distributed state estimation problem (Battistelli, Chisci, Morrocchi, & Papi, 2011). Later, a novel consensus filter has been proposed to study the distributed target tracking problem over a sensor network (Battistelli, Chisci, Fantacci, Farina, & Graziano, 2013). This filter was extended to be used in a tracking problem for a maneuvering target (Battistelli, Chisci, Fantacci, Farina, & Graziano, 2015a). More recently, a consensus filter based on the unscented Kalman filter and consensus on information was presented for systems with sensor saturation and state saturation (Li, Wei, & Han, 2014). Finally, the fourth group of consensus filters is H_∞ consensus which was originally introduced in Shen, Wang, and Hung (2010). The main reason

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to apply H_∞ consensus is this fact that the practical systems are often along with parameter uncertainties and disturbances. Consequently, we cannot use ordinary consensus filters. Therefore, the H_∞ consensus has been recently developed (Han, Wei, Song, & Li, 2015; Ugrinovskii, 2014).

Target tracking is one of the fundamental problems in sensor networks in both theory and application (Morbidi & Mariottini, 2013; Olfati-Saber & Jalalkamali, 2012; Ou, Du, & Li, 2012; Ou, Gu, Wang, & Dong, 2015; Zhu et al., 2013). In Olfati-Saber and Jalalkamali (2012), the authors have solved the problem of tracking control for mobile sensors with linear dynamics to estimate the states of a linear target and track that linear target based on a flocking manner. In this work, a distributed Kalman filter is proposed to estimate the states of the target. This filter is developed for tracking a target with linear dynamics in heterogeneous sensor networks (Zhu et al., 2013). In Morbidi and Mariottini (2013), a team of unmanned aerial vehicles are considered as mobile sensors and a distributed estimation and control algorithm is suggested for them to track a target with linear dynamics. In Ou et al. (2012), a distributed controller has been presented to solve the cooperative control problem of mobile sensors with nonlinear dynamics. The proposed controller makes mobile sensors converge to a desired trajectory. In Ou et al. (2015), the problem of finite-time tracking control of multiple nonholonomic mobile robots subject to external disturbances has been solved. An observer is presented to estimate the disturbance and a finite time controller is designed for each robot to track the target with nonholonomic dynamics. In this work, the target's position is assumed to be available.

The targets in the aforementioned target tracking problems have continuous dynamics and their states are considered to be available or have linear dynamics. But, these assumptions actually do not hold in practical environments because many practical targets such as unmanned aerial, ground, or underwater vehicles and satellites have continuous nonlinear dynamics and their states must be estimated by sensor networks. Thus, it is necessary to pay much attention to the design of DSE algorithms for continuous nonlinear systems. Some algorithms have also been proposed for state estimation of targets with nonlinear dynamics (Battistelli, Chisci, Mugnai, Farina, & Graziano, 2015b; Hu & Hu, 2010; Li, Wei, Han, & Liu, 2016). In Hu and Hu (2010), a nonlinear convergent filter has been presented to estimate the target's states in a sensor network. The target's dynamics has been described by a continuous-time linear system whose input is generated by another linear system. In Battistelli et al. (2015b), a hybrid consensus filter is presented with a combination of consensus on information, consensus on measurements and extended Kalman filter algorithm. The stability analysis of the proposed filter is limited to linear systems. In Li et al. (2016), a nonlinear consensus filter has been suggested by employing a consensus approach and unscented Kalman filter. This filter had bounded estimation error in the sense of mean-square and could estimate the states of a target with discrete-time nonlinear dynamics. It should be noted that the proposed algorithms in Battistelli et al. (2015b) and Li et al. (2016) have obtained for state estimation of the targets with discrete-time nonlinear dynamics and cannot be implemented for state estimation of continuous-time nonlinear systems. Furthermore, although Hu and Hu (2010) has proposed a continuous-time consensus filter, the target has a special structure in which most of the targets are not classified. These limitations motivate us for this study.

In this paper, by using the proposed estimator in Xie and Khargonekar (2012) and consensus techniques on state estimation, a DCF for estimating the states of continuous nonlinear systems is proposed. In fact, Xie and Khargonekar (2012) suggested an estimator for estimating states and parameters of a class of continuous nonlinear stochastic system based on Lyapunov stability theory with a suboptimal gain. Additionally, the discrete-time version of proposed DCF is presented for implementation application. Therefore, the main contributions of this paper are as follows:

- A novel DCF is presented to estimate the states of a continuous nonlinear stochastic system. The discrete-time version of the proposed DCF is also introduced.
- The exponentially ultimately boundedness of the estimation errors of proposed DCFs is proved and suboptimal gains are obtained for both continuous and discrete DCF by minimizing the upper bound of the estimation error.

The remainder of this paper is organized as follows. Section 2 introduces a DCF for continuous nonlinear system and analyzes its convergence and optimality. In Section 3, a discrete-time DCF is presented based on the discrete version of Lyapunov theory and a suboptimal gain is obtained. The proposed DCFs performance is studied with numerical examples both for continuous and discrete systems in Section 4. Finally, the conclusions are drawn in Section 5.

Notation and graph theory. $\lambda_{\max}(.)$ and $\lambda_{\min}(.)$ denote the biggest and the smallest eigenvalues, respectively. $\otimes$ represents the Kronecker product. E[.] denotes the expectation operator. $C^{2,1}$ denotes the family of all nonnegative functions $V(x(t), t)$ that are continuously twice differentiable in x and once differentiable in t. I_M denotes an $M \times M$ identity matrix.

The sensor nodes of the network are communicated over an undirected graph $G = (\upsilon, \varepsilon, A)$, where $\upsilon = \{1, 2, \dots, N\}$ is the sensor node set, $\varepsilon \in \upsilon \times \upsilon = \{(i, j) : i, j \in \upsilon\}$ is the communication link set and $A = [a_{ij}] \in R^{N\times N}$ is the adjacent matrix. If nodes i and j are connected, then the node i is the neighbor of node j and $a_{ij} = a_{ji} > 0$. The Laplacian matrix for graph G is defined as $L = D - A$, in which D is a diagonal matrix with the diagonal elements $d_i = \sum_{j\in N_i} a_{ij}$. The eigenvalues of a Laplacian matrix can be ordered as $\lambda_1(L) \le \lambda_2(L) \le \dots \le \lambda_N(L)$ in which the second smallest eigenvalue, $\lambda_2(L)$ is called the algebraic connectivity of the network. $N_i = \{j \in \upsilon : (i, j) \in \varepsilon, j \ne i\}$ denotes the set of neighbors of node i. In this paper, the assumption is that no node is connected with itself, i.e. $a_{ii} = 0 ; 1 \le i \le N$. If there is a link between nodes i and j, the corresponding element in the adjacent matrix will be 1. G is called a connected graph if and only if there is at least one path between every two arbitrary nodes. It is a critical point that an undirected graph is connected if and only if its algebraic connectivity is positive: $\lambda_2(L) > 0$.

2. Continuous-time distributed consensus filter (CDCF)

Consider the continuous-time nonlinear system with the following dynamic equations:

$$\dot{x}(t) = f(x(t)) + Bw(t), \qquad x(t) \in R^M \tag{1}$$

$$y_i(t) = h_i(x(t), t) + D_i v_i(t), \quad i = 1, \dots, N \tag{2}$$

where $w(t)$ and $v_i(t)$ are white noises with covariances $Q(t)$ and $R_i(t)$, respectively. $x(t)$ is the state vector, $y_i(t)$ is the ith sensor measurement vector. $f(.)$ and $h_i(.)$ are nonlinear functions. It is noteworthy that B can be a function of $x(t)$. The problem is to estimate the states of the nonlinear system (1) by providing a novel distributed estimation algorithm in a sensor network. As mentioned in the introduction section, the proposed CDCF is concluded from Xie and Khargonekar (2012). In fact, Xie and Khargonekar (2012) has employed the observers in Tarn and Rasis (1976), Yaz and Azemi (1993), Cho and Rajamani (1997) and introduced an estimator that could estimate the states and unknown parameters of a class of nonlinear stochastic systems. This reference has used the structure in Tarn and Rasis (1976) and sufficient conditions for estimation error boundedness and the optimal filter gain have