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Brief paper

New unknown input observer and output feedback stabilization for uncertain heat equation ☆

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ABSTRACT

In this paper, we propose a new method, by designing an unknown input type state observer, to stabilize an unstable 1-d heat equation with boundary uncertainty and external disturbance. The state observer is designed in terms of a disturbance estimator. A stabilizing state feedback control is designed for the observer by the backstepping transformation, which is an observer based output feedback stabilizing control for the original system. The well-posedness and stability of the closed-loop system are concluded. The numerical simulations show that the proposed scheme is quite effectively. This is a first result on active disturbance rejection control for a PDE with both boundary uncertainty and external disturbance.

1. Introduction

Disturbance attenuation or rejection is one of the major concerns in modern control theory. Since from the 1970s, there are many methods developed to cope with uncertainty in control systems and most of these methods are generalized to systems described by partial differential equations (PDEs). Among them, internal model principle for special type of disturbances (Rebarber & Weiss, 2003) and adaptive control for unknown parameters (Krstic, 2010) are earlier active disturbance rejection methods in dealing with uncertainty by exploiting estimation/cancellation strategy. Other popular methods include sliding mode control (Guo & Jin, 2013) and robust control method (Christofides, 2001) where the completely unknown uncertainty is passively attenuated.

The idea of estimation/cancellation from internal model principle and adaptive rejection control is later developed in large scale as active disturbance rejection control (ADRC) (Han, 2009) where not only external disturbance but also internal uncertainty are estimated in terms of input and output. The uncertainties dealt with by ADRC are much more complicated. It can be the coupling between unknown internal system dynamics, the external disturbance, and the superadded unknown part of control input, or even if whatever the part that is hardly dealt with by practitioners (Guo & Zhao, 2015). ADRC has been applied to state feedback stabilization for PDEs with external disturbance (Guo & Jin, 2013). The output feedback stabilization for PDEs by ADRC is, however, very complicated. In Guo and Jin (2015), an unknown input observer is first designed for stabilization of 1-d wave equation with external disturbance. However, the observer in Guo and Jin (2015) was designed by variable structure control method, which is very technical and brings many mathematical difficulties. In addition, the extended state observer (ESO) used in ADRC utilizes usually the high gain which is very restrictive from engineering control point of view. So there are several challenges in applying ADRC to PDEs in following typical situations: (a) the total disturbance contains not only external disturbance but also internal uncertainty; (b) output feedback instead of state feedback; (c) the high gain problem in ESO; (d) a finite order derivative of total disturbance is required to be bounded.

In this paper, we meet these challenges by considering unknown type state observer and output feedback stabilization for the following one-dimensional heat equation with boundary unknown nonlinear uncertainty and external disturbance:

$$\begin{cases} w_t(x,t) = w_{xx}(x,t), \ x \in (0,1), \ t > 0, \\ w_x(0,t) = -qw(0,t), t \geq 0, \\ w_x(1,t) = f(w(\cdot,t)) + d(t) + u(t), \ t \geq 0, \\ w(x,0) = w_0(x), \ 0 \leq x \leq 1, \\ y(t) = (w(0,t), w(1,t)), \ t \geq 0, \end{cases} \tag{1}$$

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where $q \in \mathbb{R}$, $y(t)$ is the output (measurement), $u(t)$ is the input (control), $w_0(x)$ is the initial value, $f(\cdot)$ is an unknown nonlinear function that represents the boundary uncertainty, and $d(t)$ is the external disturbance. The "$f(w(\cdot,t)) + d(t)$" is called the "total disturbance" in active disturbance rejection control. When $q > 0$, the uncontrolled system (1) may become unstable. For the sake of simplicity, we drop the obvious time and spatial domains in the rest of the paper.

The model (1) is a general 1-d heat equation with boundary convection. Let k be the thermal conductivity of a solid rod, and let h be the convection heat transfer coefficient which varies with the type of flow, the geometry of the body and flow passage area, the physical properties of the fluid, the average surface and fluid temperatures, and many other parameters. The pure convection boundary condition, physically meaning that the temperature gradient within the solid at the surface is coupled to the convective flux at the solid–fluid interface, is prescribed by

$$-kw_x(\partial,t) = \pm h(w(\partial,t) - w_\infty(t)),$$

where $\partial = 0$ or 1 represents the boundary and $w_\infty(t)$ is the ambient fluid temperature. The special case of zero fluid temperature $w_\infty(t) = 0$, given by

$$-kw_x(\partial,t) = \pm hw(\partial,t),$$

represents convection into a fluid medium at zero temperature, noting that a common practice is to redefine or shift the temperature scale such that the fluid temperature is now zero. When k, h, and $w_\infty(t)$ are not known, the convection boundary condition at $x = 1$ leads to the boundary condition of system (1) at $x = 1$. For more details of physical modeling of heat equation, we refer to Hahn and Özisik (2012).

To illustrate the physical model, we give a sketch of (1) with $q = 0$ in Fig. 1 which depicts flow of heat in a rod that is insulated everywhere except the two ends, where the heat of the right end is controlled by a steam chest with placement of a thermometer and the left end is insulated.

Heat equation with unstable term or source term has been extensively studied by the method of backstepping. Examples can be found in Baccoli, Pisano, and Orlov (2015), Meurer (2012), Krstic (2006), Krstic and Smyshlyaev (2008) and Smyshlyaev and Krstic (2007), to name just a few. The backstepping approach is powerful and is still valid to other distributed parameter systems that are corrupted by disturbance or unknown parameters (Aamo, 2013; Krstic, 2010). There are many other works for the parabolic systems control. For classical output regulation theory for distributed parameter system, we refer to (Aulisa & Gilliam, 2016). Recently, the backstepping-based robust output regulation for boundary controlled parabolic PDEs was discussed in Deutscher (2016). In addition, there exist other methods to cope with disturbance or unknown parameters such as the sliding mode control (Orlov, Pisano & Usai, 2011), unknown input observer based control (Chauvin, 2012), and the internal model principle (Rebarber & Weiss, 2003). Our work, however, is different from the existing ones. The main objective of this paper is to propose a new method to cope with the control-matched disturbance that consists of not only external disturbance but also boundary uncertainty. The approach is inspired by the method of ADRC and is different from the existing results in papers for instance (Aamo, 2013; Chauvin, 2012; Guo & Jin, 2015) where the unknown input observers for distributed parameter systems have been designed.

We proceed as follows. In Section 2, we first present a target system as a preliminary for the design of state observer. An unknown input type infinite-dimensional state observer is proposed in Section 3, where the estimation/cancellation strategy in ADRC is used without invoking high gain. The observer could lead immediately to a total disturbance estimator. A state feedback stabilizing control for the observer is designed in Section 4, which is an observer based feedback control for original system. To do this, the backstepping transformation is applied. Section 5 is devoted to well-posedness and asymptotic stability for the closed-loop system. Numerical simulations are presented in Section 6 to validate the theoretical results, followed by the concluding remarks in Section 7.

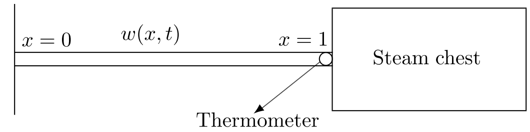

Fig. 1. One-dimensional heated rod.

2. Preliminary: target system for observer

We first consider a stable heat equation:

$$\begin{cases} \hat{z}_t(x,t) = \hat{z}_{xx}(x,t), \\ \hat{z}_x(0,t) = c_0\hat{z}(0,t),\ \hat{z}_x(1,t) = \mathcal{G}(t), \\ \hat{z}(x,0) = \hat{z}_0(x), \end{cases} \tag{2}$$

where $c_0 > 0$ is a constant, $\hat{z}_0(x)$ is the initial value, and $\mathcal{G} \in L^2_{loc}(0,\infty)$ is a given function. System (2) can be written as an evolution equation in $\mathcal{H} := L^2(0,1)$:

$$\frac{d}{dt}\hat{z}(\cdot,t) = A\hat{z}(\cdot,t) + B\mathcal{G}(t), \tag{3}$$

where $B = \delta(x-1)$ with $\delta(\cdot)$ the Dirac distribution, and the operator A is given by

$$\begin{cases} [Af](x) = f''(x),\ \forall f \in D(A), \\ D(A) = \left\{ f \in H^2(0,1) \mid f'(0) = c_0 f(0), f'(1) = 0 \right\}. \end{cases} \tag{4}$$

Lemma 2.1. *For any $\hat{z}_0 \in \mathcal{H}$ and $\mathcal{G} \in L^2_{loc}(0,\infty)$, there exists a unique solution $\hat{z} \in C(0,\infty;\mathcal{H})$ to system (2) such that the following statements hold:*
(i) If we assume further that $\mathcal{G} \in L^\infty(0,\infty)$, then there exists a positive constant L_B, independent of t, such that

$$\sup_{t\in[0,\infty)} \|\hat{z}(\cdot,t)\|_{\mathcal{H}} \le \|\hat{z}_0\|_{\mathcal{H}} + L_B\|\mathcal{G}\|_{L^\infty(0,\infty)} < +\infty; \tag{5}$$

(ii) If $\mathcal{G}(t) \to 0$ as $t \to \infty$, then

$$\|\hat{z}(\cdot,t)\|_{\mathcal{H}} \to 0 \ \text{ as } \ t \to \infty. \tag{6}$$

Proof. Inequality (5) is straightforward by noticing that A generates a C_0-semigroup e^{At} of contractions on $\mathcal{H}$ and B is admissible for e^{At} by invoking Remark 2.6 of Weiss (1989). The convergence (6) is a direct result in Feng and Guo (2014) or Guo and Jin (2013) where the admissibility of B and Remark 2.6 of Weiss (1989) are also used. □

Next, consider the following coupled heat system

$$\begin{cases} \varepsilon_t(x,t) = \varepsilon_{xx}(x,t), \\ \varepsilon_x(0,t) = c_0\varepsilon(0,t),\ \varepsilon_x(1,t) = \tilde{d}_x(1,t), \\ \tilde{d}_t(x,t) = \tilde{d}_{xx}(x,t), \\ \tilde{d}_x(0,t) = c_0\tilde{d}(0,t),\ \tilde{d}(1,t) = 0, \\ \varepsilon(x,0) = \varepsilon_0(x),\ \tilde{d}(x,0) = \tilde{d}_0(x), \end{cases} \tag{7}$$

where $(\varepsilon_0(x), \tilde{d}_0(x))$ is the initial value. System (7) will serve as a target system for the observer design in next section. We consider