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Brief paper

A nonlinear Luenberger-like observer for nonlinear singular systems[☆]

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ABSTRACT

This paper investigates observer design problem for a large class of nonlinear singular systems with multiple outputs. We first regularize the singular system by injecting the derivative of outputs into the system. Then differential geometric method is applied to transform the regularized system into a simple normal form, for which a Luenberger-like observer is proposed.

1. Introduction

Singular systems widely exist in engineering systems, such as chemical system, biological system, electrical circuit and so on. These systems are governed by mixing differential and algebraic equations, which are the special differences with respect to regular systems, thus the control of such system is a challenging problem (see Campbell, 1980, 1982, Dai, 1989). Due to this reason, many well-defined concepts relative to observation problem for regular (non-singular) systems have to be reconsidered for singular ones.

The solvability, controllability and observability concepts have been studied in Yip and Sincovec (1981) for singular systems with regular matrix pencil. Causal observability has been treated in Hou and Muller (1999a) for linear singular systems. Recently, Bejarano, Floquet, Perruquetti, and Zheng (2013) generalizes the observability for linear singular system to consider the unknown input case, by converting the singular system into a regular one with unknown inputs and algebraic constraints. Moreover, the assumption of regular matrix pencil for singular systems was removed as well in Bejarano et al. (2013). It has been extended in Bejarano, Perruquetti, Floquet, and Zheng (2015) to treat nonlinear singular systems.

Concerning the observer design, a Luenberger-like observer has been proposed in Paraskevopoulos and Koumboulis (1992) for linear singular systems. Darouach and Boutayeb (1995) gave necessary and sufficient condition for the existence of a reduced-order observer for linear singular systems with known inputs. In Hou and Muller (1999b), a generalized observer was studied by involving the derivative of input and output. For linear singular systems with unknown input, a proportional–integral observer was proposed in Koenig and Mammar (2002), and its extension by involving multiple integrations to design unknown input observer for linear singular systems with unknown input was studied in Gao and Ho (2004) and Koenig (2005). For nonlinear singular systems, Kaprielian and Turi (1992) studied an observer in which the system was linearized around the equilibrium point. The same technique was used in Boutayeb and Darouach (1995) to study the reduced-order observer for a class of nonlinear singular systems. Also, many efforts have been made when the system is affected by some disturbances in input, in the model, or in the measurement. In Gao and Ho (2006), a simple linear singular observer was proposed, and necessary and sufficient conditions were given for a special class of linear singular systems with unknown inputs. This result was extended as well to treat nonlinear singular systems under the assumption that the nonlinear term is Lipschitz. Other techniques, such as LMI Darouach and Boutat-Baddas (2008), Lu and Ho (2006) and Lu, Ho, Zheng, and Zheng (2004) and convex optimization Koenig (2006), are proposed as well to design observer for nonlinear singular systems with known (or unknown) inputs. Recently, the technique of regularization by applying the differential geometric method to nonlinear singular systems was introduced in Boutat, Zheng, Boutat-Baddas, and Darouach (2012). But this method works only for nonlinear singular systems with single output. This paper is an extension of that result to treat multiple outputs case. Given a nonlinear singular system, we first regularize it into a nonlinear regular system with the injection of

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the output derivative, then seek a diffeomorphism to transform the regularized system into an observer normal form, based on which a Luenberger-like observer is proposed. Compared to the existing results on nonlinear singular systems whose nonlinear terms need to be Lipschitz (Darouach & Boutat-Baddas, 2008; Gao & Ho, 2006; Shields, 1997), the presented technique works as well when this assumption is not satisfied.

This paper is organized as follows. Section 2 recalls the basic results of Luenberger-like observer for linear singular systems. Section 3 presents the method how to regularize nonlinear singular system into a regular one with output derivatives. Section 4 deduces necessary and sufficient conditions to transform the regularized systems into a simple normal form, for which a Luenberger-like observer has been proposed in Section 5.

2. Recall for linear singular system

First, let us recall some basic results for linear singular systems. Consider a general linear singular system as follows:

$$\begin{cases} \bar{E}\dot{\zeta} = \bar{A}\zeta \\ \bar{y} = \bar{C}\zeta \end{cases} \tag{1}$$

with $\zeta \in \mathbb{R}^n$, $\text{rank}\bar{E} < n$, and it is assumed that the matrix pencil $s\bar{E} - \bar{A}$ is regular. In Darouach and Boutayeb (1995), the following two conditions are proposed:

$$\text{rank}\begin{bmatrix} \bar{E} & \bar{A} \\ 0 & \bar{E} \\ 0 & \bar{C} \end{bmatrix} = n + \text{rank}\bar{E} \tag{2}$$

and

$$\text{rank}\begin{bmatrix} s\bar{E} - \bar{A} \\ \bar{C} \end{bmatrix} = n, \forall s \in \mathbb{C}, \text{Re}(s) \geq 0 \tag{3}$$

which guarantee the existence of a simple Luenberger-like observer

$$\begin{cases} \dot{\xi} = N\xi + Ly \\ \hat{\zeta} = \xi + Ky \end{cases} \tag{4}$$

with properly chosen matrices N, K and L.

The following section will show how to generalize this idea to treat nonlinear singular systems. Similar conditions as those for linear singular systems will be proposed, and we will show that those conditions coincide with the above two conditions (2) and (3) if the linear case is studied.

3. Regularization of nonlinear singular systems

Consider the following class of nonlinear singular systems:

$$\begin{cases} \bar{E}\dot{\zeta} = \bar{f}(\zeta) \\ \bar{y} = \bar{h}(\zeta) \end{cases} \tag{5}$$

where $\zeta \in \Omega_\zeta \in \mathbb{R}^n, \bar{f} : \mathbb{R}^n \to \mathbb{R}^n$ and $\bar{h} = \left[\bar{h}_1, \ldots, \bar{h}_m\right]^T : \mathbb{R}^n \to \mathbb{R}^m$ are smooth, with $\bar{E} \in \mathbb{R}^{n\times n}$ being singular, i.e. $\text{rank}\bar{E} < n$. Without loss of generalities, it is assumed that $\{d\bar{h}_1(\zeta), \ldots, d\bar{h}_m(\zeta)\}$ are linearly independent for all $\zeta \in \Omega_\zeta$ where $d\bar{h}_1$ means the differential of $\bar{h}_1$. It is worth noting that a corresponding regularity assumption is not required for the studied nonlinear system (5).

Assumption 1. For any $\zeta \in \Omega_\zeta$, it is assumed that the following condition

$$\text{rank}\begin{bmatrix} \bar{E} & \frac{\partial \bar{f}(\zeta)}{\partial \zeta} \\ 0 & \bar{E} \\ 0 & \frac{\partial \bar{h}(\zeta)}{\partial \zeta} \end{bmatrix} = n + \text{rank}\bar{E} \tag{6}$$

is satisfied.

Remark 1. Eq. (6) in Assumption 1 can be seen as a generalization of (2) to treat nonlinear singular system. In fact, if we consider the linear case of (5), i.e. $\bar{f}(\zeta) = \bar{A}\zeta$ and $\bar{h}(\zeta) = \bar{C}\zeta$, Eq. (6) becomes exactly (2):

$$\text{rank}\begin{bmatrix} \bar{E} & \frac{\partial \bar{f}(\zeta)}{\partial \zeta} \\ 0 & \bar{E} \\ 0 & \frac{\partial \bar{h}(\zeta)}{\partial \zeta} \end{bmatrix} = \text{rank}\begin{bmatrix} \bar{E} & \bar{A} \\ 0 & \bar{E} \\ 0 & \bar{C} \end{bmatrix} = n + \text{rank}\bar{E}$$

which is the necessary and sufficient condition of observability for linear singular systems.

Note $\text{rank}\bar{E} = q < n$, then there exist two elementary matrices T and S such that

$$S\bar{E}T = \begin{bmatrix} I_q & 0 \\ 0 & 0 \end{bmatrix} \tag{7}$$

thus, by introducing $x = T^{-1}\zeta : \Omega_\zeta \to \Omega_x$, system (5) becomes

$$\begin{cases} S\bar{E}T\dot{x} = S\bar{f}(Tx) \\ \bar{y} = \bar{h}(Tx) \end{cases}$$

which is equivalent to

$$\begin{cases} E\dot{x} = \tilde{f}(x) \\ \bar{y} = \tilde{h}(x) \end{cases} \tag{8}$$

with $E = \begin{bmatrix} I_q & 0 \\ 0 & 0 \end{bmatrix}$, $\tilde{f}(x) = S\bar{f}(Tx)$ and $\tilde{h}(x) = \bar{h}(Tx)$, or is equivalent to the following decomposition:

$$\begin{cases} \dot{x}_1 = \tilde{f}_1(x_1, x_2) \\ 0 = \tilde{f}_2(x_1, x_2) \\ \bar{y} = \tilde{h}(x_1, x_2) \end{cases} \tag{9}$$

where $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $x_1 \in \mathbb{R}^q, x_2 \in \mathbb{R}^{n-q}, \tilde{f} = \begin{bmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{bmatrix}$.

Remark 2. For system (9), if

$$\text{rank}\frac{\partial \tilde{f}_2(x)}{\partial x_2} = n - q, \forall x \in \Omega_x \tag{10}$$

then implicit function theorem ensures that there exists a function α such that $x_2 = \alpha(x_1)$. By inserting it back into (9), the following reduced regular model can be obtained:

$$\begin{cases} \dot{x}_1 = \tilde{f}_1(x_1, \alpha(x_1)) \\ \bar{y} = \tilde{h}(x_1, \alpha(x_1)) \end{cases}$$

Then the classical differential geometric method can be used to analyze observability and design observer. However, condition (10) is in some sense a little strong since it requires that the algebraic constrain contains all information of x_2. Otherwise, the reduced-order regularized system cannot be obtained. From (9), it can be seen clearly that $\bar{y}$ contains as well the information of x_2. Therefore, one natural way to relax condition (10) is to take into account as well the output of (9).