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Brief paper

Second order sliding mode control for nonlinear affine systems with quantized uncertainty☆

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ABSTRACT

This paper deals with the design of a Second-Order Sliding Mode (SOSM) control algorithm able to enhance the closed-loop performance depending on the current working conditions. The novelty of the proposed approach is the design of a nonsmooth switching line, based on the quantization of the uncertainties affecting the system. The quantized uncertainty levels allow one to define nested box sets in the auxiliary state space, i.e., the space of the sliding variable and its first time derivative, and select suitable control amplitudes for each set, in order to guarantee the convergence of the sliding variable to the sliding manifold in a finite time. The proposed algorithm is theoretically analyzed, proving the existence of an upperbound of the reaching time to the origin through the considered quantization levels.

1. Introduction

Nowadays, Sliding Mode Control (SMC) is one of the most effective solution to control systems characterized by hard uncertainties (Edwards & Spurgeon, 1998; Utkin, 1992). SMC is able to guarantee robustness against a wide class of disturbances, above all in case of matched uncertainties, i.e., uncertainties acting on the same channel of the control variable (Edwards & Spurgeon, 1998). Yet, because of the discontinuous nature of the control law, the so-called chattering effect (Boiko, Fridman, Pisano, & Usai, 2007; Levant, 2010) can be produced, i.e., high frequency oscillations of the controlled variable which can be disturbing for the actuators.

However, in the literature, several methods to perform chattering alleviation have been proposed, such as filtered sliding mode (Tseng & Chen, 2010), boundary layer sliding mode (Burton & Zinober, 1986) or fractional order sliding mode control (Corradini, Giambò, & Pettinari, 2015). Among these methodologies, the so-called Higher Order Sliding Mode (HOSM) control approaches, which involve not only the sliding variable, but also its time derivatives up to a certain order $r-1$ (Bartolini, Ferrara, & Usai, 1997; Bartolini, Ferrara, Usai, & Utkin, 2000; Dinuzzo & Ferrara, 2009b), consist in confining the discontinuity, necessary to steer the so-called sliding variable to zero, to a derivative of the control variable, so that the control signal actually fed into the plant is continuous. Because of the continuous nature of the control action, HOSM control approaches are appropriate to be applied even to electrical, electromechanical or mechanical systems (Bartolini, Pisano, Punta, & Usai, 2003; Utkin, Guldner, & Shi, 1999), as testified by Capisani and Ferrara (2012), Cucuzzella, Incremona, and Ferrara (2015), Cucuzzella, Incremona, and Ferrara (2017), Cucuzzella, Rosti, Cavallo, and Ferrara (2017), Cucuzzella, Trip, De Persis, and Ferrara (2017), Ferrara and Incremona (2015), Incremona, Cucuzzella, and Ferrara (2016) and Incremona, De Felici, Ferrara, and Bassi (2015).

In the classical formulation of SMC, the uncertain terms are assumed to be bounded with known bounds. It is also reasonable assuming that uncertainties can be linked to the system states because of state-dependent disturbances or different levels of confidence in the system model in different operating conditions (Tanelli & Ferrara, 2013). This can imply a quantization of the uncertain terms such that different compact box sets can be defined in the state space. In the paper, the convergence to the origin of the auxiliary state space is proved, and an upperbound of the convergence time with respect to the worst realization of the uncertainties is analytically provided.

The present proposal provides a simple way to tune the amplitude of the discontinuous control action depending on the uncertainties quantization levels. Other interesting tuning mechanisms are presented in Pisano, Tanelli, and Ferrara (2015) and Tanelli and Ferrara (2013). Yet, they differ from the proposed approach since they are based on the a priori subdivision of the auxiliary state space into regions. Moreover, they rely on the use of the Suboptimal SOSM control law, while in this paper the SOSM control law

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with optimal reaching (Dinuzzo & Ferrara, 2009b) is used inside each level, so that for each uncertainty quantization level a minimum time passage through the corresponding set is featured by the auxiliary state trajectory. In fact, the corresponding nonsmooth surface is the combination of different switching lines which result in being attractive with optimal reaching for the auxiliary state trajectories.

The paper is organized as follows. In Section 2 the problem is formulated, while in Section 3 the proposed strategy based on a nonsmooth switching line is presented. In Section 4 the stability analysis is discussed and an academic example is reported in Section 5. Some conclusions in Section 6 end the paper.

2. Problem formulation

Consider a plant which can be described by the single-input system affine in the control variable

$$\dot{x}(t) = a(x(t)) + b(x(t))u(t) \tag{1}$$

where $x \in \Omega$ ($\Omega \subset \mathbb{R}^n$ bounded) is the state vector, the value of which at the initial time instant t_0 is $x(t_0) = x_0$, and $u \in \mathbb{R}$ is a scalar input subject to the saturation $[-\alpha, \alpha]$, while $a(x(t)) : \Omega \to \mathbb{R}^n$ and $b(x(t)) : \Omega \to \mathbb{R}^n$ are uncertain functions of class $C^1(\Omega)$.

Define a suitable output function $\sigma(x) : \Omega \to \mathbb{R}$ of class $C^2(\Omega)$. This function will play the role of "sliding variable" in the following, that is $\sigma(x)$ is the variable to steer to zero in a finite time in order to solve the control problem, according to classical sliding mode control theory (Utkin, 1992). The sliding variable $\sigma(x)$ has to be selected such that the following assumption holds.

Assumption 1. If $u(t)$ in (1) is designed so that, in a finite time t_r (ideal reaching time), $\sigma(x(t_r)) = 0 \; \forall x_0 \in \Omega$ and $\sigma(x(t)) = 0 \; \forall t > t_r$, then $\forall t \geq t_r$ the origin is an asymptotically stable equilibrium point of (1) constrained to $\sigma(x(t)) = 0$.

Note that Assumption 1 guarantees that the sliding mode control law to design is stabilizing.

Now consider the input–output map

$$\begin{cases} \dot{x}(t) = a(x(t)) + b(x(t))u(t) \\ y(t) = \sigma(x(t)) \\ x(t_0) = x_0 \; . \end{cases} \tag{2}$$

Assume that (2) is complete in Ω and has a uniform relative degree equal to 2. Moreover, assume that system (2) admits a global normal form in Ω, i.e., there exists a global diffeomorphism of the form $\Phi(x) : \Omega \to \Phi_\Omega \subset \mathbb{R}^n$,

$$\Phi(x) = \begin{pmatrix} \Psi(x) \\ \sigma(x) \\ a(x)\cdot\nabla\sigma(x) \end{pmatrix} = \begin{pmatrix} x_r \\ \xi \end{pmatrix}$$

$$\Psi : \Omega \to \mathbb{R}^{n-2}, \quad x_r \in \mathbb{R}^{n-2}, \quad \xi = \begin{pmatrix} \sigma(x) \\ \dot{\sigma}(x) \end{pmatrix} \in \mathbb{R}^2 \; ,$$

such that,

$$\begin{cases} \dot{x}_r = a_r(x_r, \xi) & \text{(a)} \\ \dot{\xi}_1 = \xi_2 & \text{(b)} \\ \dot{\xi}_2 = f(x_r, \xi) + g(x_r, \xi)u & \text{(c)} \\ y = \xi_1 & \text{(d)} \\ \xi(t_0) = \xi_0 & \text{(e)} \end{cases} \tag{3}$$

with

$$a_r = \frac{\partial \Psi}{\partial x}(\Phi^{-1}(x_r, \xi))a(\Phi^{-1}(x_r, \xi))$$
$$f = a(\Phi^{-1}(x_r, \xi)) \cdot \nabla(a(\Phi^{-1}(x_r, \xi)) \cdot \nabla\sigma(\Phi^{-1}(x_r, \xi)))$$
$$g = b(\Phi^{-1}(x_r, \xi)) \cdot \nabla(a(\Phi^{-1}(x_r, \xi)) \cdot \nabla\sigma(\Phi^{-1}(x_r, \xi)))$$

where the obvious dependence on time is omitted. Note that, as a consequence of the uniform relative degree assumption, it yields

$$g(x_r, \xi) \neq 0, \quad \forall (x_r, \xi) \in \Phi_\Omega \; . \tag{4}$$

In the literature, see for instance Bartolini, Ferrara, and Usai (1998), making reference to the previous system, subsystem (3)(b)–(e) is called "auxiliary system". Since $a_r(\cdot), f(\cdot), g(\cdot)$ (the latter is assumed to be positive definite, for the sake of simplicity) are continuous functions and Φ_Ω is a bounded set, one has that

$$\exists F > 0 : |f(x_r, \xi)| \leq F \; \forall (x_r, \xi) \in \Phi_\Omega \tag{5}$$
$$\exists G_{\max} > 0 : g(x_r, \xi) \leq G_{\max} \; \forall (x_r, \xi) \in \Phi_\Omega \tag{6}$$
$$\exists G_{\min} > 0 : g(x_r, \xi) \geq G_{\min} \; \forall (x_r, \xi) \in \Phi_\Omega \; . \tag{7}$$

Note that, instead of (6) and (7), if $g(\cdot)$ was negative definite, one could analogously have the opposite inequalities. Moreover, the following assumption on the internal dynamics (3)(a) holds.

Assumption 2. Given the auxiliary system (3), the internal dynamics (3)(a) does not present finite time escape phenomena and the corresponding zero dynamics $a_r(x_r, 0)$ is globally asymptotically stable.

Relying on (3)–(7) and Assumptions 1 and 2, the control problem to solve is hereafter introduced.

Problem 1. Design a feedback control law

$$u(t) = \kappa(\sigma(x(t)), \dot{\sigma}(x(t))) \tag{8}$$

such that $\forall x_0 \in \Omega, \exists \, t_r \geq 0 : \sigma(x(t)) = \dot{\sigma}(x(t)) = 0, \forall t \geq t_r$ in spite of the uncertainties.

The proposed control strategy has the merit to allow one to reformulate the control problem of stabilizing a nonlinear uncertain system, into a simpler control problem: that of stabilizing the auxiliary system (3)(b)–(e) forced by a bounded input. In fact, it is sufficient to suitably select the sliding variable σ according to Assumption 1, to be able to determine (3)(b)–(e), so that the explicit knowledge of Ψ is not actually necessary to solve the problem.

Remark 1. Note that, if the sliding variable σ is steered to zero, this directly implies the asymptotic stability of the origin of the closed-loop system (1) since, by assumption, the zero dynamics (3)(a) of system (1), transformed via the diffeomorphism $\Phi(x)$, is globally asymptotically stable.

In the present work, in order to reduce the control effort of the input fed into the plant, relying on the 2-relative degree of system (2), a gain tuning mechanism is combined with the SOSM control strategy giving rise to a new control algorithm.

3. Nonsmooth switching line based SOSM control

We are now in a position to introduce the proposed SOSM control algorithm based on a nonsmooth switching line.

3.1. Design of the switching line

Making reference to the SOSM algorithm with optimal reaching presented in Dinuzzo and Ferrara (2009b), let α_r be the reduced control amplitude, which is the minimum amplitude of $\ddot{\sigma}$ in presence of the maximum realization of the uncertainty terms when $u = \pm\alpha$ is applied, i.e.,

$$\alpha_r = G_{\min}\alpha - F > 0 \; , \tag{9}$$