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Functional observers design for descriptor systems via LMI: Continuous and discrete-time cases ☆

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ABSTRACT

This paper investigates the design of functional observers for linear time-invariant descriptor systems. A new method for designing these observers is given by using an LMI (Linear matrix inequality) formulation. The obtained result unifies the design, it considers the continuous-time and discrete-time cases and concerns the observers of various orders.

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1. Introduction

In the present paper, based on the results of Darouach (2012), a new functional observers design method for descriptor systems is presented. The novelty of the presented results lies in the formulation of the dynamics of the error which is represented in a descriptor system form. It permits the simplification of the design for the uncertain systems and gives more degrees of freedom for this design Yong-Yan & Zongli (2004) (see also Gao Huijun & Li, 2014 and references therein). It also unifies the design for different observer orders and concerns the continuous-time as well as the discrete-time systems. Necessary and sufficient conditions for the existence of these observers are given in LMIs form. The observers design in the LMI framework allows to integrate our approach with existing ones (optimal control, pole placement, etc.) that nowadays are all expressed in this way. The proposed approach also presents the advantage that LMIs can be easily tested by using standard convex optimization algorithms.

The paper is organized as follows. Section 2 presents some preliminary results on descriptor systems which will be used in the sequel of the paper. The functional observers design of different orders for descriptor systems, including particular cases and the procedure of this design, is given in Section 3. Section 4 concludes the paper.

2. Preliminary results

In this section we recall some results on linear descriptor systems which are used in the sequel of the paper (see Dai, 1989; Darouach, 2012). Consider the linear time-invariant multivariable system described by

$$E\sigma x(t) = Ax(t) + Bu(t) \tag{1a}$$

$$y(t) = Cx(t) \tag{1b}$$

$$z = Lx(t) \tag{1c}$$

where σ denotes the derivative operator $\sigma x(t) = dx(t)/dt$ for continuous-time systems and the forward-shift operator $\sigma x(k) = x(k+1)$, where $k \in \mathbb{Z}$, for discrete-time systems, $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^p$ are the semi state vector and the output vector of the system, $u \in \mathbb{R}^m$ is the known input and $z \in \mathbb{R}^r$ is the vector to be estimated, where $r \leqslant n$. Matrix $E \in \mathbb{R}^{n_1 \times n}$, when $n_1 = n$ matrix E is singular, matrices A, B, C, and L are known constant and of appropriate dimensions.

It is assumed that $u(t)$ and $x(0) = x_0$ are admissible, i.e they are such that there exists at least one trajectory satisfying (1a). By using the Laplace transform (z transform for the discrete-time case) we have the following solvability conditions Ishihara & Terra (2001):

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1. The pair $(u(t), x_0)$ is admissible if and only if $normal-rank\begin{bmatrix} sE-A & BU(s) & Ex_0\end{bmatrix} = normal-rank(sE-A)$ for the continuous-time case,
2. The pair $(u(t), x_0)$ is admissible if and only if $normal-rank\begin{bmatrix} zE-A & BU(z) & Ex_0\end{bmatrix} = normal-rank(zE-A)$ for the discrete-time case

where the normal-rank of the matrix pencil $(\lambda E - A)$ is defined as the $rank(\lambda E - A)$ for almost all $\lambda \in \mathbb{C}$. In the sequel of this paper we shall assume that the pair $(u(t), x_0)$ is admissible. Also we shall use the following notations:

The symbol Σ^+ denotes any generalized inverse of the matrix Σ, i.e. verifies $\Sigma\Sigma^+\Sigma = \Sigma$. This generalized inverse matrix is also denoted by Σ^- or $\Sigma^{(1)}$ in the literature. The symbol $E^\perp$ denotes a maximal row rank matrix such that $E^\perp E = 0$. When E is of full row rank, $E^\perp = 0$ by definition. Before presenting the observer design for system (1), we shall recall the following results, which extend the notion of impulse (causal for the discrete-time case) observability to the partial impulse (causal) observability (see reference Darouach, 2012).

Definition 1. The descriptor system (1) with $u(t) = 0$, or the triplet (C, E, A), is said to be partially impulse observable with respect to L, if $y(t)$ is impulse free for $t \geqslant 0$, only if $Lx(t)$ is impulse free for $t \geqslant 0$.

Definition 2. The descriptor system (1) with $u(t) = 0$, or the triplet (C, E, A), is said to be partially causal observable with respect to L, if $Lx(k)$ at any time point k is uniquely determined by the initial condition and the former measurement $y(i)$, $i = 0, 1, \ldots.., k$.

The following lemma gives the conditions for the partial impulse observability (partial causal observability for the discrete-time case) Darouach (2012).

Lemma 1. *The following statements are equivalent*

1. *The triplet (C, E, A) is partially impulse (causal) observable with respect to L ;*
2. $rank\begin{bmatrix} E & A\\ 0 & C\\ 0 & E\\ 0 & L\end{bmatrix} = rank\begin{bmatrix} E & A\\ 0 & C\\ 0 & E\end{bmatrix}$;
3. $rank\begin{bmatrix} L\\ E\\ E^\perp A\\ C\end{bmatrix} = rank\begin{bmatrix} E\\ E^\perp A\\ C\end{bmatrix}$.

The next assumption is used in the sequel of the paper.

Assumption I : We assume that system (1) is partial impulse (causal) observable with respect to L.

Now we can give the following definitions for descriptor systems (see Dai, 1989; Xu & Lam, 2006).

Definition 3. Let us consider the following descriptor system :

$E\sigma x(t) = Ax(t)$ where $E \in \mathbb{R}^{n\times n}$ and $A \in \mathbb{R}^{n\times n}$, then we have the following definitions

1. The pair (E, A) is regular if $det(\lambda E - A)$ is not identically zero.
2. The pair (E, A) is impulse free (causal for the discrete-time case) if $deg(det(\lambda E - A)) = rankE$.
3. The pair (E, A) is said to be stable if the roots of $det(\lambda E - A) = 0$ have negative real parts (are inside the unit circle for the discrete-time case).
4. The pair (E, A) is said to be admissible if it is regular, impulse free (causal for the discrete-time case) and stable.

The following lemma gives the necessary and sufficient conditions for the admissibility of the pair (E, A), in the strict LMI formulation (see Xu & Lam, 2006).

Lemma 2. *The pair (E, A) is admissible if and only if there exist matrices $X > 0$ and Q such that*

$$(XE + E^{\perp T}Q)^T A + A^T(XE + E^{\perp T}Q) < 0 \tag{2}$$

for the continuous-time case, and

$$A^TXA - E^TXE + QE^\perp A + A^TE^{\perp T}Q^T < 0 \tag{3}$$

for the discrete-time case.

Let us consider the following reduced order observer.

$$\sigma\zeta(t) = N\zeta(t) + F\begin{bmatrix} -E^\perp Bu(t)\\ y(t)\end{bmatrix} + Hu(t) \tag{4a}$$

$$\widehat{z}(t) = P\zeta(t) + Q\begin{bmatrix} -E^\perp Bu(t)\\ y(t)\end{bmatrix} \tag{4b}$$

where $\zeta(t) \in \mathbb{R}^q$ is the state of the observer, $\widehat{z}(t) \in \mathbb{R}^r$ is the estimate of $z(t)$. Matrices N, F, H, P and Q are constant and of appropriate dimensions to be determined such that $\lim_{t\to\infty}(\widehat{z}(t) - z(t)) = 0$.

The following theorem gives the conditions for system (4) to be a qth order observer for the functional $z(t)$ in system (1).

Theorem 1. *The qth-order observer (4) will estimate (asymptotically) $z(t)$ if there exists a matrix parameter T such that the following conditions hold*

(1) $NTE - TA + F\begin{bmatrix} E^\perp A\\ C\end{bmatrix} = 0$,

(2) $\begin{bmatrix} P & | & Q\end{bmatrix}\begin{bmatrix} TE\\ \hline E^\perp A\\ C\end{bmatrix} = L$,

(3) $H = TB$.

(4) *The pair $(\mathbf{E}, \mathbf{A})$ is admissible, where $\mathbf{E} = \begin{bmatrix} I & 0\\ 0 & 0\end{bmatrix}$ and $\mathbf{A} = \begin{bmatrix} N & 0\\ P & -I\end{bmatrix}$.*

Proof. Let $\epsilon(t)$ be the error between $\zeta(t)$ and $TEx(t)$, i.e $\epsilon(t) = \zeta(t) - TEx(t)$, then its dynamic is given by

$$\sigma\epsilon(t) = N\epsilon(t) + \left(NTE - TA + F\begin{bmatrix} E^\perp A\\ C\end{bmatrix}\right)x(t) + (H - TB)u(t). \tag{5}$$

On the other hand from (4b) the estimate of $z(t)$ can be written as

$$\widehat{z}(t) = P\epsilon(t) + \begin{bmatrix} P & | & Q\end{bmatrix}\begin{bmatrix} TE\\ \hline E^\perp A\\ C\end{bmatrix}x(t). \tag{6}$$

If conditions (1), (2) and (3) are satisfied, then (5) and (6) reduce to $\sigma\epsilon(t) = N\epsilon(t)$, and $e(t) = \widehat{z}(t) - z(t) = \widehat{z}(t) - Lx(t) = P\epsilon(t)$, which can be written as

$$\mathbf{E}\sigma\xi(t) = \mathbf{A}\xi(t) \tag{7}$$

where $\xi(t) = \begin{bmatrix} \epsilon(t)\\ e(t)\end{bmatrix}$. In addition if condition (4) is satisfied we have $\lim_{t\to\infty}\epsilon(t) = 0$ and $\lim_{t\to\infty}e(t) = 0$ for any $x(0)$, $\widehat{z}(0)$, and $u(t)$. Hence $\widehat{z}(t)$ in (4) is an estimate of $z(t)$. This completes the proof.

From Theorem 1, the design of the observer (4) is reduced to find the matrices T, N, P, Q, F and H such that conditions (1)–(4) are satisfied.

In the sequel of this paper we shall use the same notations as in Darouach (2012). Define the following matrices $\Gamma = \begin{bmatrix} E\\ \hline E^\perp A\\ C\end{bmatrix}$

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