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Stabilization of infinite dimensional port-Hamiltonian systems by nonlinear dynamic boundary control ☆

Hector Ramirez [a], Hans Zwart [b,c], Yann Le Gorrec [a]

[a] FEMTO-ST UMR CNRS 6174, AS2M department, University of Bourgogne Franche-Comté, 24 rue Savary, F-25000 Besançon, France

[b] University of Twente, Faculty of Electrical Engineering, Mathematics and Computer Science, Department of Applied Mathematics, P.O. Box 217 7500 AE Enschede, The Netherlands

[c] Technische Universiteit Eindhoven, Department of Mechanical Engineering, P.O. Box 513, 5600 MB Eindhoven, The Netherlands

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ABSTRACT

The conditions for existence of solutions and stability, asymptotic and exponential, of a large class of boundary controlled systems on a 1D spatial domain subject to nonlinear dynamic boundary actuation are given. The consideration of such class of control systems is motivated by the use of actuators and sensors with nonlinear behavior in many engineering applications. These nonlinearities are usually associated to large deformations or the use of smart materials such as piezo actuators and memory shape alloys. Including them in the controller model results in passive dynamic controllers with nonlinear potential energy function and/or nonlinear damping forces. First it is shown that under very natural assumptions the solutions of the partial differential equation with the nonlinear dynamic boundary conditions exist globally. Secondly, when energy dissipation is present in the controller, then it globally asymptotically stabilizes the partial differential equation. Finally, it is shown that assuming some additional conditions on the interconnection and on the passivity properties of the controller (consistent with physical applications) global exponential stability of the closed-loop system is achieved.

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1. Introduction

In many physical processes the effects produced by distributed phenomena cannot be neglected. This is for instance the case for transmission lines, flexible beams and plates, tubular and nuclear fusion reactors and wave propagation to cite a few. These processes are hence modeled using partial differential equations (PDE) in which state variables and parameters are time and spatial dependent. In many relevant applications the measurement and the actuation occur on the spatial boundary of the system, hence what the controller actually imposes through the physical actuators are time varying boundary conditions. Formally this class of control systems are called boundary control systems (BCS).

In engineering applications BCS are often controlled using localized actuators which exhibit *nonlinear* behavior. These nonlinearities are for example related to large deformations of compliant structures (nonlinear springs) in mechanical systems or hysteresis behavior of ferro and piezo electrical materials in electro mechanical systems. This is for instance the case of silicon made nanotweezers built up from beams which are controlled using electrostatic comb drives and attached through nonlinear silicon made suspensions (thin beams) (Boudaoud, Haddab, & Le Gorrec, 2012), nonlinear fluid structure interaction, such as in distributed control of vibro-acoustic systems through nonlinear loudspeakers (Collet, David, & Berthillier, 2009) or the stability characterization of biomechanical processes such as the blood flow dynamics in bio-prosthetic heart valves (Borazjani, 2013) or the vocal cords dynamics (Ishizaka & Flanagan, 1972). The nonlinear components are generally associated to nonlinear constitutive laws of the driving forces, usually present in a potential energy term and to nonlinear damping phenomena related to nonlinear resistors and dampers, respectively.

In the linear case the existence of solutions, the stability and the design of stabilizing controllers can be tackled using linear semigroup theory and the associated abstract formulation based on unbounded input/output mappings (Curtain & Zwart, 1995). When asymptotic or exponential stability is concerned, the main difficulty remains in finding the appropriate Lyapunov function candidate to prove the stability. It is usually done on a case by case basis using physical considerations depending on the application field. When characterizing exponential stability, contrary

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E-mail addresses: hector.ramirez@femto-st.fr (H. Ramirez), h.j.zwart@utwente.nl, h.j.zwart@tue.nl (H. Zwart), legorrec@femto-st.fr (Y. Le Gorrec).

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to asymptotic stability, the conditions insuring the exponential convergence are quite rigid as the controller has to damp infinitely high frequency as well as all low frequency modes.

In the last decade an approach based on the extension of the Hamiltonian formulation to open distributed parameter systems (van der Schaft & Maschke, 2002) has been developed for modeling and control. It has been shown that distributed port-Hamiltonian systems encompass a large class of physical systems, including mechanical, electrical, electro-mechanical, hydraulic and chemical systems to mention some. See Duindam, Macchelli, Stramigioli, and Bruyninckx (2009) for an extensive exposition and a large list of references. Regarding the extension of the Hamiltonian formulation to stabilizing control of BCS, in the 1D linear case it gave rise to the definition of boundary control port-Hamiltonian systems (BC-PHS) (Le Gorrec, Zwart, & Maschke, 2004) and allowed to parametrize, by using simple matrix conditions, the boundary conditions that define a well-posed problem (Le Gorrec, Zwart, & Maschke, 2005). Different variations around these first results can be found in Villegas (2007) and in Jacob and Zwart (2012). Well-posedness and stability have been investigated in open-loop and for static boundary feedback control in Zwart, Le Gorrec, Maschke, and Villegas (2010), Villegas, Zwart, Le Gorrec, and Maschke (2009) and Villegas, Zwart, Le Gorrec, Maschke, and van der Schaft (2005) respectively, and linear dynamic boundary control has been studied in Augner and Jacob (2014), Macchelli, Le Gorrec, Ramirez, and Zwart (2017), Ramirez, Le Gorrec, Macchelli, and Zwart (2014) and Villegas (2007).

In this paper the results on existence of solution and stabilization of linear dynamic boundary control of BC-PHS are generalized to the case of nonlinear boundary control. This class of systems is of real practical interest since the controllers are often implemented with actuators and sensors with nonlinear behavior, due for instance to large deformations, the use of smart materials or saturation phenomena. The same kind of problem has already been studied in Miletić, Stürzer, Arnold, and Kugi (2016) and in Augner (2016) from a theoretical point of view. In Miletić et al. (2016) LaSalle's invariance principle is used and precompactness of trajectories is established but asymptotic stability was only shown for a dense set of initial conditions. In Augner (2016) nonlinear contraction semigroups are used leading to quite strong assumptions on the class of considered nonlinearities. This approach differs from the methods that we use in this paper, which are based on nontrivial extensions of the asymptotic and exponential stability results presented in Zwart, Ramirez, and Le Gorrec (2016) and Ramirez et al. (2014), respectively, allowing to deal with very large class of nonlinearities. More precisely, a general class of passive boundary controllers, with nonlinear potential energy function and damping matrix is considered. This class of controllers encompasses mechanical, electrical and electro-mechanical systems among others. First it is shown that under natural assumptions on the nonlinear potential function and damping matrix the solutions of the PDE with this class of nonlinear dynamic boundary conditions exist globally. Then, it is shown that the most general form of this class of passive controllers globally asymptotically stabilizes the closed loop system (PDE + nonlinear ODE). Finally, it is shown that by restricting the nonlinear potential energy to functions with quasi quadratic bound and a full rank condition on the feedthrough term of the controller global exponential stability is achieved. The first part of this work, dealing with asymptotic stability, has been illustrated on the particular example of pure nonlinear damper in Zwart et al. (2016).

The paper is organized as follows. In Section 2 the definition and main properties of the considered class of PDE and nonlinear dynamic boundary controller are given. The existence and the uniqueness of the solutions of the PDE are established in Section 3. The asymptotic stability is studied in Section 4 while the exponential stability is addressed in Section 5. Finally some concluding remarks and comments to future work are given in Section 6.

2. Port-Hamiltonian systems with nonlinear boundary control

Throughout this article we assume that our distributed parameter system is modeled by a PDE of the following form

$$\frac{\partial x}{\partial t}(t,\zeta) = P_1 \frac{\partial}{\partial \zeta}\left(\mathcal{H}(\zeta)x(t,\zeta)\right) + (P_0 - G_0)\mathcal{H}(\zeta)x(t,\zeta), \tag{1}$$

with $\zeta \in (a,b)$, $P_1 \in M_n(\mathbb{R})$[1] a nonsingular symmetric matrix, $P_0 = -P_0^\top \in M_n(\mathbb{R})$, $G_0 \in M_n(\mathbb{R})$ with $G_0 \geq 0$ and x taking values in $\mathbb{R}^n$. Furthermore, $\mathcal{H}(\cdot) \in L_\infty((a,b); M_n(\mathbb{R}))$ is a bounded and measurable, matrix-valued function satisfying for almost all $\zeta \in (a,b)$, $\mathcal{H}(\zeta) = \mathcal{H}(\zeta)^\top$ and $\mathcal{H}(\zeta) > mI$, with m independent from ζ.

For simplicity $\mathcal{H}(\zeta)x(t,\zeta)$ will be denoted by $(\mathcal{H}x)(t,\zeta)$. For the above PDE we assume that some boundary conditions are homogeneous, whereas others are controlled. Thus we consider two matrices $W_{B,1}$ and $W_{B,2}$ of appropriate sizes such that

$$u(t) = W_{B,1}\begin{bmatrix}(\mathcal{H}x)(t,b)\\(\mathcal{H}x)(t,a)\end{bmatrix} \tag{2}$$

and

$$0 = W_{B,2}\begin{bmatrix}(\mathcal{H}x)(t,b)\\(\mathcal{H}x)(t,a)\end{bmatrix}. \tag{3}$$

Furthermore, the boundary output is given by

$$y(t) = W_C\begin{bmatrix}(\mathcal{H}x)(t,b)\\(\mathcal{H}x)(t,a)\end{bmatrix}. \tag{4}$$

To study the existence and uniqueness of solution to the above controlled PDE, we follow the semigroup theory, see also Le Gorrec et al. (2005) and Jacob and Zwart (2012). Therefore we define the state space $X = L_2((a,b);\mathbb{R}^n)$ with inner product $\langle x_1, x_2\rangle_{\mathcal{H}} = \langle x_1, \mathcal{H}x_2\rangle$ and norm $\|x\|_{\mathcal{H}} = \sqrt{\langle x, x\rangle_{\mathcal{H}}}$. Note that due to the assumptions on $\mathcal{H}$ this is a norm on X and equivalent to the L_2 norm. Hence X is a Hilbert space. The reason for selecting this space is that $\|\cdot\|^2_{\mathcal{H}}$ is related to the energy function of the system, i.e., the total energy of the system equals $E = \frac{1}{2}\|x\|^2_{\mathcal{H}}$. The Sobolev space of order p is denoted by $H^p((a,b),\mathbb{R}^n)$.

Associated to the (homogeneous) PDE, *i.e.*, to the case $u(t) = 0$, we define the operator $Ax = P_1\frac{d}{d\zeta}(\mathcal{H}x) + (P_0 - G_0)\mathcal{H}x$ with domain

$$D(A) = \left\{\mathcal{H}x \in H^1((a,b);\mathbb{R}^n) \;\middle|\; \begin{bmatrix}(\mathcal{H}x)(b)\\(\mathcal{H}x)(a)\end{bmatrix} \in \ker W_B\right\},$$

where $W_B = \begin{bmatrix}W_{B,1}\\W_{B,2}\end{bmatrix}$. For the rest of the paper we make the following hypothesis.

Assumption 1. For the operator A and the pde (1)–(4) the following hold:

1. The matrix W_B is an $n \times 2n$ matrix of full rank;
2. For $x_0 \in D(A)$ we have $\langle Ax_0, x_0\rangle_{\mathcal{H}} \leq 0$.
3. The number of inputs and outputs are the same, k, and for classical solutions of (1)–(4) there holds $\dot{E}(t) \leq u(t)^\top y(t)$ with $E(t) = \frac{1}{2}\|x(t)\|^2_{\mathcal{H}}$.

It follows from Assumption 1, points 1 and 2, that the system (1)–(4) is a boundary control system (see Le Gorrec et al., 2005; Jacob & Zwart, 2012; Jacob, Morris, & Zwart, 2015), and so for $u \in C^2([0,\infty);\mathbb{R}^k)$, $\mathcal{H}x(0) \in H^1((a,b);\mathbb{R}^n)$, satisfying (2) and (3) (for $t = 0$), there exists a unique classical solution to (1)–(4), Jacob and Zwart, (2012, Theorem 11.2). Thus for this dense (in X) set of initial conditions and inputs, point 3 of Assumption 1 makes sense. We remark that the internal damping operator G_0 will hardly play

[1] $M_n(\mathbb{R})$ denote the space of real $n \times n$ matrices.

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