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Intrinsic reduced attitude formation with ring inter-agent graph☆

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ABSTRACT

This paper investigates the reduced attitude formation control problem for a group of rigid-body agents using feedback based on relative attitude information. Under both undirected and directed cycle graph topologies, it is shown that reversing the sign of a classic consensus protocol yields asymptotical convergence to formations whose shape depends on the parity of the group size. Specifically, in the case of even parity the reduced attitudes converge asymptotically to a pair of antipodal points and distribute equidistantly on a great circle in the case of odd parity. Moreover, when the inter-agent graph is an undirected ring, the desired formation is shown to be achieved from almost all initial states.

1. Introduction

Multi-agent coordination (Beard, Lawton, & Hadaegh, 2001; Fax & Murray, 2004; Jadbabaie, Lin, & Morse, 2003; Olfati-Saber, Fax, & Murray, 2007) has gained increasing recognition and appreciation during the last decade. Following many significant results on consensus, how to effectively generate various multi-agent formations, patterns or subgroup divisions has attracted much attention. Among the problems studied, attitude formation of multiple rigid-body agents is of key importance with wide potential applications such as formation flying (Beard et al., 2001; Scharf, Hadaegh, & Ploen, 2004) and multi-camera surveillance (Tron & Vidal, 2014; Wang, 2013). Attitude synchronization or consensus, a special and simple formation pattern of attitudes, has been widely studied (Sarlette, Sepulchre, & Leonard, 2009; Thunberg, Song, Montijano, Hong, & Hu, 2014; Tron, Afsari, & Vidal, 2012, 2013).

The (full) attitude of a rigid-body agent can be represented by a rotation matrix that evolves on the Lie group $\mathcal{SO}(3)$. However, many attitude control applications do not require all three degrees of freedom of the full attitude to be determined. In rigid-body pointing applications, for example a body-fixed camera, the solar panel of a satellite or an antenna need to point towards some desired direction, the rotation about the pointing axis is irrelevant since this rotation does not change the direction in which the agent points. Moreover, in under-actuated situations where the rigid-body is actuated by only two independent control torques, for example due to the failure of a third actuator, the rotation about the unactuated axis is disregarded. The reduced attitude provides the proper framework to deal with such a situation (Bullo, Murray, & Sarti, 1995). All these applications lead to a reduced attitude control problem (Bullo et al., 1995; Chaturvedi, Sanyal, & McClamroch, 2011; Lee, Leok, & McClamroch, 2011; Mayhew & Teel, 2010), in which the reduced attitude of two degrees of freedom is naturally identified with a point on the 2-sphere $\mathcal{S}^2$.

Consider the attitude formation problem for a system of n rigid-body agents on the product manifold $\mathcal{SO}(3)^n$ or $(\mathcal{S}^2)^n$ under a continuous feedback control law based on relative attitude information. It can be shown that consensus states are intrinsic equilibria of the closed-loop system regardless of the topology of the inter-agent graph. The work Tron et al. (2012) and Pereira and Dimarogonas (2017) achieve this attitude synchronization for full attitudes and reduced attitudes respectively. However, due to the fact that $\mathcal{SO}(3)^n$ and $(\mathcal{S}^2)^n$ are compact manifolds without boundary (Bhat & Bernstein, 2000), continuous time-invariant feedback control also yields some other closed-loop equilibria that vary with the graph topology. These equilibria represent different attitude configurations of the system, which may include a desired formation depending on the application. A natural and interesting question thus arises: is it possible to achieve a desired formation by imposing some suitable inter-agent graph to the system and designing a feedback control with only relative attitude information that stabilizes the formation?

It is increasingly recognized that one of the important ideas in multi-agent systems is to design simple distributed control

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algorithms with not only cooperative but also antagonistic interactions between neighboring agents. Modulus consensus requires the moduli of all agent states to reach a common value but the agents may be separated into several antagonistic subgroups. The simplest case concerning two antagonistic subgroups, *bipartite consensus*, models the inter-agent connection as a signed graph (Altafini, 2013; Proskurnikov, Matveev, & Ming, 2014; Valcher & Misra, 2014). For a general modulus consensus case or even the extended *set surrounding* case, signed graphs are replaced by the graphs with complex weights expressed with complex adjacency matrices (Lou & Hong, 2015). These results demonstrate that antagonistic interactions are effective to generate new coordination or formation patterns, but the considered dynamics basically evolve in Euclidean spaces. Additionally, cooperative control of motion on the circle and sphere with both attractive and repulsive couplings are studied in Olfati-Saber (2006) and Li and Spong (2014), respectively, but the inter-agent graph is required to be undirected and complete.

By extending the coordination studies with antagonistic interactions discussed in Euclidean spaces to those on compact manifolds, this paper provides a partial but affirmative answer to the aforementioned question via investigating reduced attitude formation with both undirected and directed ring inter-agent graph. In particular, we focus on the generation of attitude formation patterns using only relative attitude information of a group of rigid-body agents. Compared to the full attitude formation problem, the reduced attitude formation is more intuitive and easier to visualize.

In this paper, a simple angular velocity control for reduced attitude formation is proposed on the basis of antagonistic interactions between neighboring agents. Due to the geometry of the 2-sphere some interesting phenomena are observed: the closed-loop system behaves differently under the proposed distributed control when the parity of the total number of agents is different. Specifically, the antipodal formation is achieved when the number is even, and the cyclic formation is achieved when the number is odd. It is shown that these two reduced attitude formations are intrinsic in the sense that they result from the geometry of the 2-sphere and the topology of the inter-agent graph. It is worthwhile to mention that, in addition to the simple control structure, another strength of the proposed method is that we do not need to have the desired formation given beforehand or the formation errors in the control, in contrast to most existing methods (Beard et al., 2001; Song, Hong, & Hu, 2013; Wu, Flewelling, Leve, & Lee, 2013).

Comparing the control protocol and the resulting formation in this paper with that of Altafini (2013) and Valcher and Misra (2014) in Euclidean space, there are mainly two differences: (i) for neighboring agents with antagonistic interaction, only relative states of neighboring agents is utilized in reduced attitude control, while absolute position of neighboring agents is required in the control of Altafini (2013) and Valcher and Misra (2014); (ii) in the case of ring inter-agent graph, if the total number of agents is odd and all neighboring agents are antagonistic, cyclic reduced formation can be attained due to the geometry of the 2-sphere, while the positions of all agents in Altafini (2013) and Valcher and Misra (2014) reach consensus at the origin because the graph is unbalanced.

The rest of the paper is organized as follows: in Section 2, necessary preliminaries on the reduced attitude and the 2-sphere are introduced. In Section 3, the reduced attitude formation problem with the ring inter-agent graph is formulated and a distributed angular velocity control law is proposed. The antipodal formation and cyclic formation of reduced attitudes are discussed in Sections 4 and 5, respectively. Following that, illustrative examples are provided in Section 6, and the conclusions are given in Section 7.

2. Notations and preliminaries

This paper considers the reduced attitude control problem for a network of n ($n \geq 2$) rigid-body agents. In this section, we give some preliminaries on the reduced attitude and the 2-sphere.

Let the index set $\mathcal{V} = \{1, 2, \ldots, n\}$ represent the agents in the network. Denote $R_i \in \mathcal{SO}(3)$ as the attitude of agent $i \in \mathcal{V}$ relative to the inertial frame $\mathcal{F}$, where $\mathcal{SO}(3) = \{R \in \mathbb{R}^{3\times 3} : R^T R = I, \det(R) = 1\}$ is the rotation group of $\mathbb{R}^3$. The kinematics of R_i is governed by Murray, Li, and Sastry (1994)

$$\dot{R}_i = \widehat{\omega}_i R_i, \tag{1}$$

where $\omega_i \in \mathbb{R}^3$ is the angular velocity of agent i in the inertial frame $\mathcal{F}$, and the *hat* operator $(\cdot)^\wedge$ is defined by the equality that $\widehat{x}y = x \times y$ for any $x, y \in \mathbb{R}^3$.

Suppose that $b_i \in \mathcal{S}^2$ is a constant pointing direction in the body-fixed frame of agent i, where $\mathcal{S}^2 = \{x \in \mathbb{R}^3 : \|x\| = 1\}$ is the 2-sphere and $\|\cdot\|$ is the Euclidean norm. Let $\Gamma_i \in \mathcal{S}^2$ denote the same pointing direction resolved in the inertial frame $\mathcal{F}$, then, $\Gamma_i = R_i b_i$. Γ_i is referred to as the *reduced attitude* of agent i since the rotation of agent i about b_i is ignored. In the paper, we use a parametrization of Γ_i given as follows

$$\Gamma_i = \begin{bmatrix} \cos(\psi_i)\cos(\varphi_i) \\ \sin(\psi_i)\cos(\varphi_i) \\ \sin(\varphi_i) \end{bmatrix} \tag{2}$$

where $\varphi_i \in [-\pi/2, \pi/2]$ and $\psi_i \in [-\pi, \pi)$. In fact, when $b_i = [1, 0, 0]^T$, the two angles $-\varphi_i$ and ψ_i are the respective pitch and yaw angles of the rotation R_i. By the kinematics (1) of the full attitude, the kinematics of Γ_i is governed by Lee et al. (2011)

$$\dot{\Gamma}_i = \widehat{\omega}_i \Gamma_i. \tag{3}$$

The tangent space of $\mathcal{S}^2$ at a point $\Gamma \in \mathcal{S}^2$ is given by $T_\Gamma \mathcal{S}^2 = \{x \in \mathbb{R}^3 : x^T \Gamma = 0\}$. Rotating $\Gamma \in \mathcal{S}^2$ about a unit axis $u \in T_\Gamma \mathcal{S}^2$ through an arbitrary angle β transforms it to another point $\exp(\beta\widehat{u})\Gamma \in \mathcal{S}^2$, where $\exp(\cdot)$ is the matrix exponential. For any two reduced attitudes $\Gamma_i, \Gamma_j \in \mathcal{S}^2$, define $\theta_{ij} \in [0, \pi]$ and $k_{ij} \in \mathcal{S}^2$ as

$$\theta_{ij} = \arccos(\Gamma_i^T \Gamma_j), \quad k_{ij} = \widehat{\Gamma}_i \Gamma_j / \sin(\theta_{ij}).$$

It holds that $\Gamma_j = \exp(\theta_{ij}\widehat{k}_{ij})\Gamma_i$. Notice that the above equation for the unit axis k_{ij} is valid only when $\theta_{ij} \in (0, \pi)$. When $\theta_{ij} = 0$ or π, we stipulate that k_{ij} is chosen as any unit vector orthogonal to Γ_i.

The geodesic distance between any two points $\Gamma_i, \Gamma_j \in \mathcal{S}^2$, denoted as $d_{\mathcal{S}^2}(\Gamma_i, \Gamma_j)$, is the length of the shorter arc on the great circle of $\mathcal{S}^2$ joining the two points. Therefore,

$$d_{\mathcal{S}^2}(\Gamma_i, \Gamma_j) = \theta_{ij}.$$

The following lemma gives the relationship among geodesic distances of any three points on the 2-sphere, which can be verified using spherical cosine formula. More details about the geometry of the 2-sphere can be found in Ferreira, Iusem, and Németh (2014) and Todhunter (1859).

Lemma 2.1. *For any three points $\Gamma_i, \Gamma_j, \Gamma_k \in \mathcal{S}^2$,*

$$\cos(\theta_{ij}) = \cos(\theta_{ik})\cos(\theta_{jk}) + \sin(\theta_{ik})\sin(\theta_{jk}) k_{ik}^T k_{jk},$$
$$\theta_{ij} + \theta_{ik} + \theta_{jk} \leq 2\pi.$$

Furthermore, $\Gamma_i, \Gamma_j, \Gamma_k$ lie on a great circle of $\mathcal{S}^2$ if and only if $\theta_{ij} = |\theta_{ik} - \theta_{jk}|$, $\theta_{ij} = \theta_{ik} + \theta_{jk}$ or $\theta_{ij} + \theta_{ik} + \theta_{jk} = 2\pi$.

Denote the state space of the system (3) as the product manifold $(\mathcal{S}^2)^n$, which is the n-fold Cartesian product of $\mathcal{S}^2$ with itself. We use $\mathbf{\Gamma} = \{\Gamma_i\}_{i \in \mathcal{V}} \in (\mathcal{S}^2)^n$ to denote the state of the system, and use the metric in $(\mathcal{S}^2)^n$ as

$$d_{(\mathcal{S}^2)^n}(\mathbf{\Gamma}, \bar{\mathbf{\Gamma}}) = \max_{i \in \mathcal{V}} d_{\mathcal{S}^2}(\Gamma_i, \bar{\Gamma}_i), \quad \forall \mathbf{\Gamma}, \bar{\mathbf{\Gamma}} \in (\mathcal{S}^2)^n.$$