Automatica 85 (2017) 362–373

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Compensation of input delay that depends on delayed input ☆

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ARTICLE INFO

Article history:
Received 11 September 2016
Received in revised form 7 April 2017
Accepted 28 June 2017

Keywords:
Nonlinear control
Delay systems
Predictor-feedback
Backstepping

ABSTRACT

For nonlinear systems, we develop a PDE-based predictor-feedback control design, which compensates actuator dynamics, governed by a transport PDE with outlet boundary-value-dependent propagation velocity. Global asymptotic stability under the predictor-feedback control law is established assuming spatially uniform strictly positive transport velocity. The stability proof is based on a Lyapunov-like argument and employs an infinite-dimensional backstepping transformation that is introduced. An equivalent representation of the transport PDE/nonlinear ODE cascade via a nonlinear system with an input delay that is defined implicitly through an integral of the past input is also provided and the equivalent predictor-feedback control design for the delay system is presented. The validity of the proposed controller is illustrated applying a predictor-feedback "bang–bang" boundary control law to a PDE model of a production system with a queue. Consistent simulation results are provided that support the theoretical developments.

1. Introduction

Cascades of partial and ordinary differential equations are widely used for modeling of complex dynamics in various engineering applications, such as screw extrusion processes in 3D printing (Diagne & Krstic, 2015), metal cutting processes (Otto & Radons, 2013), moisture in convective flows (Bresch-Pietri & Coulon, 2015), populations (Smith, 1993), transport phenomena in gasoline engines (Bresch-Pietri, Chauvin, & Petit, 2014; Detwiler & Wang, 2006; Guzzella & Onder, 2009; Jankovic & Magner, 2011; Kahveci & Jankovic, 2010), crushing-mills (Richard, 2003), production of commercial fuels by blending (Chebre, Creff, & Petit, 2010), and of stick–slip instabilities during oil drilling (Bekiaris-Liberis & Krstic, 2014; Cai & Krstic, 2015, 2016; Krstic, 2009), to name only a few. Depending on the application, the PDE state may evolve on a time-varying domain (Cai & Krstic, 2015, 2016; Diagne & Krstic, 2015; Diagne, Shang, & Wang, 2016a,b) or its transport coefficient may vary with time (Bresch-Pietri et al., 2014; Otto & Radons, 2013).

The nonlinear predictor-feedback concept, which enables one to design efficient feedback laws that compensate constant input delays arising in nonlinear systems was originally introduced in Krstic (2010a, c) where the PDE backstepping methodology combined with a Lyapunov analysis was exploited to establish stability results. For nonlinear systems with time-varying and state-dependent delays, the analogous control design methodology was developed in Bekiaris-Liberis, Jankovic, and Krstic (2012) and Bekiaris-Liberis and Krstic (2012, 2013a, b), . For linear systems Karafyllis, Malisoff, de Queiroz, Krstic and Yang (2015) and Mazenc and Malisoff (2015) proposed alternative prediction-based approaches. Later, the method was extended to deal with the stabilization problem of nonlinear systems with actuator dynamics governed by a wave PDE with moving boundary that depends on the ODE state (Cai & Krstic, 2015, 2016).

However, the problem of design of predictor-feedback controllers for compensation of input delays that depend on the control input itself is left out in most of the existing contributions. As described in Richard (2003), the design of delay-compensating control laws for such systems of transport PDE/ODE cascades with input-dependent transport coefficient (that appear for example when describing the dynamics of crushing-mill processes (Richard, 2003), recycling CSTR (Albertos & Garcia, 2012), and single-phase marine cooling systems (Hansen, Stoustrup, & Bendtsen, 2013)) remains an open problem. To our knowledge, the result in Bresch-Pietri et al. (2014), which is motivated by the dynamical model of fuel to air ratio (FAR) in gasoline engines, is perhaps the only contribution that covers this particular subject on delay compensation. Due to the dependency of the prediction horizon on the future input values a design that completely compensate the input delay does not seem possible.

☆ The material in this paper was partially presented at the 2017 American Control Conference, May 24–26, 2017, Seattle, WA, USA. This paper was recommended for publication in revised form by Associate Editor Hiroshi Ito under the direction of Editor Daniel Liberzon.

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http://dx.doi.org/10.1016/j.automatica.2017.07.069
0005-1098/

The present work deals with the problem of compensation of transport PDE actuator dynamics with boundary-value-dependent propagation speed in nonlinear systems. Equivalently, the nonlinear ODE's actuator dynamics are described as a *delayed-input-dependent input delay*. Here, the delay function is implicitly given by an integral equation, similarly to Bresch-Pietri et al. (2014), but is dependent on the delayed rather than the current input.

The predictor-feedback control law for both the PDE and the delay system representations of the PDE–ODE cascade system is developed. Our contribution stands as the first one in which actual *compensation* of a delayed-input-dependent input delay is achieved. A global stability result of the closed-loop system is established. The designed compensator is employed for control of PDE models of production systems with a finite buffer size at the end of the production chain (Borsche, Colombo, & Garavello, 2010; Herty, Klar, & Piccoli, 2007; Sun & Dong, 2008).

The paper is organized as follows. In Section 2 the general problem is described and the main result together with a global stability proof, based on a PDE representation of the predictor-feedback control law, is presented in Section 3. An alternative representation of the actuator dynamics as an implicitly defined delayed-input-dependent input delay and the associated delay compensator are given in Section 4. Section 5 is dedicated to the application of the designed control law to a PDE model of production systems enabling a delay-compensating "bang–bang" feedback law. Concluding remarks are stated in Section 7.

2. Problem statement and controller design

We consider the transport PDE/nonlinear ODE cascade system with boundary-value-dependent propagation speed defined as

$$\dot{X}(t) = f\left(X(t), u(0,t)\right), \tag{1}$$

$$\partial_t u(x,t) = v\left(u(0,t)\right)\partial_x u(x,t), \quad x \in (0,D), \tag{2}$$

$$u(D,t) = U(t). \tag{3}$$

where $X \in \mathbb{R}^n$, $f : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ is continuously differentiable with $f(0,0) = 0$, and $v : \mathbb{R} \to \mathbb{R}_+$ is continuously differentiable with respect to its argument. Eq. (2) represents the actuation path for the plant (1), located at the boundary $x = 0$, with an actuation device acting at the boundary $x = D$. The initial condition along the actuation path (2) is defined as

$$u(x,0) = u_0(x). \tag{4}$$

We design the following predictor-feedback controller for system (1)–(3)

$$U(t) = \kappa(p(D,t)), \tag{5}$$

$$p(x,t) = X(t) + \int_0^x \frac{1}{v(u(y,t))} f(p(y,t), u(y,t))dy. \tag{6}$$

The implementation of the control law (5), (6) requires measurements of the PDE state $u(x,t)$, $x \in [0,D]$. We emphasize that in the recent papers (Karafyllis, 2011; Karafyllis & Krstic, 2014), the implementation issue of predictor feedback is discussed in detail and various numerical schemes are developed for computation of predictor feedback laws.

For the system (1)–(3), we state the following assumptions:

Assumption 1. The delayed input-dependent propagation speed $v : \mathbb{R} \to \mathbb{R}_+$ is continuously differentiable and there exists a positive constant ε such that

$$v(\alpha) \geq \varepsilon, \quad \text{for all } \alpha \in \mathbb{R}. \tag{7}$$

Assumption 2. There exist a smooth positive definite function Θ and class $\mathcal{K}_\infty$ functions $\mathscr{K}_1$, $\mathscr{K}_2$, and $\mathscr{K}_3$ such that for the plant $\dot{X} = f(X,\omega)$ such that $f(0,0) = 0$, the following hold

$$\mathscr{K}_1(|X|) \leq \Theta(X) \leq \mathscr{K}_2(|X|) \tag{8}$$

$$\frac{\partial \Theta(X)}{\partial X} f(X,\omega) \leq \Theta(X) + \mathscr{K}_3(|\omega|), \tag{9}$$

for all $(X,\ \omega)^T \in \mathbb{R}^{n+1}$.

Assumption 2 guarantees that system $\dot{X} = f(X,\omega)$ is strongly forward complete with respect to ω.

Assumption 3. System $\dot{X} = f(X, \kappa(X) + \omega)$ is input-to-state stable (ISS) with respect to ω. Moreover, the feedback law $\kappa : \mathbb{R}^n \to \mathbb{R}$ is continuously differentiable with $\kappa(0) = 0$.

The definitions of strong forward completeness and input-to-state stability are those from Krstic (2010b) and Sontag (1995), respectively.

3. Main result and stability proof

Theorem 1. *Consider system (1)–(3) together with the control law (5), (6). Under Assumption 1–3, there exists a class $\mathcal{KL}$ function $\mathscr{L}_0$ such that for all initial conditions for which $u_0(x)$ is locally Lipschitz on $[0,D]$ and which satisfy the compatibility condition $u_0(D) = \kappa\left(p(D,0)\right)$, there exists a unique solution to the closed-loop system with $X(t) \in C^1[0,\infty)$ and $u(x,t)$ locally Lipschitz on $[0,D] \times [0,\infty)$, and the following holds for all $t \geq 0$*

$$|X(t)| + \sup_{x\in[0,D]} |u(x,t)| \leq \mathscr{L}_0\left(|X(0)| + \sup_{x\in[0,D]} |u_0(x)|, t\right). \tag{10}$$

The proof of Theorem 1 is established with the help of the following lemmas.

Lemma 1. *The infinite-dimensional backstepping transformation*

$$w(x,t) = u(x,t) - \kappa\left(p(x,t)\right), \tag{11}$$

where $p(x,t)$ is defined in (6), combined with the control law defined in (5), (6), maps the system (1), (2) with the boundary condition (3) into the following target system

$$\dot{X}(t) = f\left(X(t), \kappa\left(X(t)\right) + w(0,t)\right), \tag{12}$$

$$\partial_t w(x,t) = v(w(0,t) + \kappa\left(X(t)\right))\partial_x w(x,t), \quad x \in [0,D] \tag{13}$$

$$w(D,t) = 0. \tag{14}$$

Proof. Differentiation of (6) with respect to t gives

$$\begin{aligned}\partial_t p(x,t) = {} & f(p(0,t), u(0,t)) - \int_0^x f(p(y,t), u(y,t)) \\ & \times \left(\frac{v'(u(y,t))}{v^2(u(y,t))}\partial_t u(y,t)\right) dy \\ & + \int_0^x \frac{1}{v(u(y,t))}\partial_p f(p(y,t), u(y,t))\partial_t p(y,t)dy \\ & + \int_0^x \frac{1}{v(u(y,t))}\partial_u f(p(y,t), u(y,t))\partial_t u(y,t)dy.\end{aligned} \tag{15}$$

Differentiation of (6) with respect to x leads to the following relation

$$\begin{aligned}\partial_x p(x,t) = {} & -\int_0^x f(p(y,t), u(y,t))\left(\frac{v'(u(y,t))}{v^2(u(y,t))}\partial_y u(y,t)\right) dy \\ & + \int_0^x \frac{1}{v(u(y,t))}\partial_p f(p(y,t), u(y,t))\partial_y p(y,t)dy\end{aligned}$$