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Moving horizon estimation for discrete-time linear systems with binary sensors: Algorithms and stability results ☆

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ABSTRACT

The paper addresses state estimation for linear discrete-time systems with binary (threshold) measurements. A *Moving Horizon Estimation* (MHE) approach is followed and different estimators, characterized by two different choices of the cost function to be minimized and/or by the possible inclusion of constraints, are proposed. Specifically, the cost function is either quadratic, when only the information pertaining to the threshold-crossing instants is exploited, or piece-wise quadratic, when all the available binary measurements are taken into account. Stability results are provided for the proposed MHE algorithms in the presence of unknown but bounded disturbances and measurement noise. Performance of the proposed techniques is also assessed by means of simulation examples.

1. Introduction

Binary (threshold) sensors, whose output can take two possible values according to whether the sensed variable exceed or not a given threshold, are nowadays commonly exploited for monitoring/control aims in a wide range of application domains. A non-exhaustive list of existing binary sensors includes: industrial sensors for brushless dc motors, liquid levels, pressure switches; chemical process sensors for vacuum, pressure, gas concentration and power levels; switching sensors for exhaust gas oxygen (EGO or lambda sensors), ABS, shift-by-wire in automotive applications; gas content sensors (CO, CO_2, H_2, etc.) for gas & oil industry; traffic condition indicators for *asynchronous transmission mode* (ATM) networks; medical sensors/analyses with dichotomous outcomes. In some applications, binary sensors represent the only viable solution for real-time monitoring. In any case, they provide a remarkably more cost-effective alternative to traditional (continuous-valued) sensors at the price of an accuracy deterioration which can, however, be compensated by using many binary sensors (for different variables and/or thresholds) in place of a single one or few traditional sensors. Moreover, binary (threshold) measurements arise naturally in the context of networked state estimation when, in order to save bandwidth and reduce the energy consumption due to data transmission, the measurements collected by each remote sensor are compared locally with a (possibly time-varying) threshold and only information pertaining to the threshold-crossing instants is transmitted to the fusion center. This latter setting falls within the framework of event-based or event-triggered state estimation (Battistelli, Benavoli, & Chisci, 2012; Shi, Chen, & Shi, 2014; Sijs & Lazar, 2012), and is more challenging as compared to the usually addressed settings due to the minimal information exchange.

The above arguments, as well as the difficulties due to the very limited information provided by binary measurements, have motivated the work on the exploitation of binary measurements for estimation purposes. In particular, Wang, Li, Guo, and Xu, (2011) and Wang, Xu, and Yin (2008) investigated observability and observer design for linear time-invariant (LTI) continuous-time systems under binary-valued output observations. The work in Wang, Yin, and Zhang (2006) and Wang, Zhang, and Yin (2003) addressed system identification using binary sensors. Specific attention was also devoted to state estimation of hybrid nonlinear systems with binary/quantized sensors (Koutsoukos, 2003) and to target tracking with binary sensor networks (Aslam et al., 2003). A possible solution for coping with the high nonlinearity associated with binary measurements within a stochastic framework is particle filtering (Djuric, Vemula, & Bugallo, 2008; Ristic, Gunatilaka, & Gailis, 2015). However such techniques, while effective in many contexts, suffer from the so-called curse of dimensionality (i.e., the exponential growth of the computational complexity as the state

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dimension increases) and from the lack of guaranteed stability and performance (being based on Monte Carlo integration).

The present paper addresses state estimation for linear discrete-time systems with binary (threshold) output measurements by following a *moving horizon estimation* (MHE) approach. MHE techniques were originally introduced to deal with uncertainties in the system knowledge (Jazwinski, 1968) and, in recent years, have gathered an increasing interest thanks to their capability of taking explicitly into account constraints on state and disturbances in the filter design (Rao, Rawlings, & Lee, 2001), and on the possibility of having guaranteed stability and performance even in the nonlinear case (Alessandri, Baglietto, & Battistelli, 2008; Alessandri, Baglietto, Battistelli, & Zavala, 2010; Rao, Rawlings, & Mayne, 2003). In fact, MHE has been successfully applied in many different contexts, ranging from switching and large-scale systems (Alessandri, Baglietto, & Battistelli, 2005; Farina, Ferrari-Trecate, & Scattolini, 2010b; Guo & Huang, 2013; Haber & Verhaegen, 2013; Schneider, Hannemann-Tams, & Marquardt, 2015) to networked systems (Farina, Ferrari-Trecate, & Scattolini, 2010a, 2012; Liu, Yu, Zhang, & Chen, 2013).

In this paper, the state estimation problem with binary measurements is cast in a deterministic framework, in the sense that no probabilistic description of the plant disturbance and noises is supposed to be available. The estimates are computed by minimizing suitable cost functions defined over a given time-horizon (advancing in time) of finite length, possibly subject to linear inequality constraints accounting for the threshold measurements. Specifically, two different approaches are proposed and analyzed. In the first approach, only the threshold-crossing instants are taken into account in the definition of the cost function, by penalizing the distance of the expected continuous outputs (based on the state estimates) from the threshold at those instants. The main advantage of this solution is that the resulting cost function is quadratic. The second approach, instead, exploits all the available information by defining a piece-wise quadratic cost function which accounts for all the available binary measurements, but requires the solution of a convex optimization problem at each time instant. Both unconstrained and constrained MH state estimators will be presented for the two different choices of the cost function and stability results will be proved, assuming unknown but bounded disturbances.

Summarizing, the paper provides the following contributions.

- Design of novel receding-horizon state estimators for linear discrete-time systems subject to binary (threshold) measurements using either a quadratic or a piecewise quadratic cost function to be minimized and, independently, either including or not constraints.
- Stability analysis showing that all proposed estimators, irrespectively of the cost being used and of the inclusion of constraints, guarantee an asymptotically bounded estimation error under bounded disturbances and suitable observability assumptions.
- Performance comparison demonstrating the effectiveness, in terms of both estimation accuracy and computational cost, of our approach.

Some of the results of this paper have been preliminarily presented, without proof, in Battistelli, Chisci, and Gherardini (2015).

The rest of the paper is structured as follows. Section 2 formulates the estimation problem of interest. Section 3 discusses how to solve the problem by means of the MHE approach, with different variants depending on the choice of the cost function as well as on the inclusion or not of constraints. Section 4 deals with the stability analysis of the proposed MH estimators. In Section 5, some numerical examples are presented in order to evaluate and compare the proposed estimators. Finally, Section 6 ends the paper with concluding remarks and perspectives for future work.

2. Problem formulation and preliminary considerations

The following notation will be used throughout the paper: $col(\cdot)$ denotes the matrix obtained by stacking its arguments one on top of the other; $diag(m_1, \ldots, m_q)$ denotes the diagonal matrix whose diagonal elements are the scalars $m_1, \ldots, m_q$; further, given a matrix M, $vec(M)$ denotes the linear transformation which converts the matrix M into a column vector and $\|v\|_M \triangleq v'Mv$. Finally, $\otimes$ denotes the Kronecker product.

Let us consider the problem of recursively estimating the state of the discrete-time linear dynamical system

$$\begin{aligned} x_{t+1} &= Ax_t + Bu_t + w_t \\ z_t^i &= C^i x_t + v_t^i, \quad i = 1, \ldots, p \end{aligned} \tag{1}$$

from *binary (threshold)* measurements

$$y_t^i = h^i(z_t^i) = \begin{cases} +1, & \text{if } z_t^i \geq \tau^i \\ -1, & \text{if } z_t^i < \tau^i. \end{cases} \tag{2}$$

In (1)–(2): $x_t \in \mathbb{R}^n$ is the state to be estimated; $u_t \in \mathbb{R}^m$ is a known input; $z_t = col(z_t^i)_{i=1}^p \in \mathbb{R}^p$; τ^i is the threshold of the ith binary sensor; $A, B, C = col(C^i)_{i=1}^p$ are matrices of compatible dimensions; w_t and $v_t = col(v_t^i)_{i=1}^p$ are the process and, respectively, measurement noises assumed *unknown but bounded*. Notice from (1)–(2) that sensor i provides a binary measurement $y_t^i \in \{-1, +1\}$ (two-level measurement quantization) according to whether the noisy linear function of the state $z_t^i = C^i x_t + v_t^i$ falls below or above the threshold τ^i. The problem (1)–(2) clearly includes, as a special instance, the case of quantized sensors with an arbitrary number of levels. In fact, a d-level, for generic $d \geq 2$, quantizer can be easily realized by using $d-1$ binary (threshold) sensors for the same physical variable but with appropriate different thresholds. The considered setting with multiple binary sensors (which can measure the same physical variable with different thresholds but also different physical variables) is clearly more general.

It is worth to point out that the system (1)–(2) represents a very special instance of a linear system with output nonlinearity, i.e. a *Wiener* system (Westwick & Verhaegen, 1996). However, due to the discontinuous nature of the measurement function (2), all state estimation techniques for Wiener systems that require a certain smoothness of the output nonlinearity (see for example (Glaria López & Sbarbaro, 1996) and the references therein) cannot be applied. In fact, while general-purpose nonlinear estimators accounting for such a discontinuity (e.g., the particle filter) could be used, the peculiar nature of the considered output nonlinearity deserves special attention and, for optimal exploitation of the poor available information, the development of ad-hoc receding-horizon estimators that will be presented in the sequel.

Before addressing the estimation problem, some preliminary considerations on the information provided by binary multisensor observations are useful. With this respect, it has been pointed out in Wang et al. (2011) that, in the continuous-time case, the information provided by a binary sensor of the form (2) is strictly related to the threshold-crossing instants. In fact, in this case, at every instant corresponding to a discontinuity of the binary signal y^i, it is known that the signal z^i is equal to the threshold value τ^i, implying that the linear measurement $z^i = \tau^i$ is available. Hence, observability with binary sensors for continuous-time linear systems can be analyzed within the more general framework of observability for irregularly sampled systems (Wang et al., 2011). In particular, observability can be ensured when the number of threshold-crossing instants (which corresponds to the number of available irregularly sampled linear measurements) is sufficiently large.

The situation is, however, different for discrete-time systems. To see this, consider a generic time instant k in which the binary