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Circle criterion-based $\mathscr{H}_\infty$ observer design for Lipschitz and monotonic nonlinear systems – Enhanced LMI conditions and constructive discussions[☆]

Ali Zemouche [a,b], Rajesh Rajamani [c], Gridsada Phanomchoeng [d], Boulaïd Boulkroune [e], Hugues Rafaralahy [a], Michel Zasadzinski [a]

[a] University of Lorraine, CRAN UMR CNRS 7039, 54400 Cosnes et Romain, France
[b] EPI Inria DISCO, Laboratoire des Signaux et Systèmes, CNRS-CentraleSupélec, 91192 Gif-sur-Yvette, France
[c] Laboratory for Innovations in Sensing, Estimation, and Control, Department of Mechanical Engineering, University of Minnesota, Minneapolis, USA
[d] Department of Mechanical Engineering, Faculty of Engineering, Chulalongkorn University, Bangkok 10330, Thailand
[e] Einderpad, 20A, 3920 Lommel, Belgium

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ABSTRACT

A new LMI design technique is developed to address the problem of circle criterion-based $\mathscr{H}_\infty$ observer design for nonlinear systems. The developed technique applies to both locally Lipschitz as well as monotonic nonlinear systems, and allows for nonlinear functions in both the process dynamics and output equations. The LMI design condition obtained is less conservative than all previous results proposed in the literature for these classes of nonlinear systems. By judicious use of a modified Young's relation, additional degrees of freedom are included in the observer design. These additional decision variables enable improvements in the feasibility of the obtained LMI. Several recent results in the literature are shown to be particular cases of the more general observer design methodology developed in this paper. Illustrative examples are given to show the effectiveness of the proposed methodology. The application of the method to slip angle estimation in automotive applications is discussed and experimental results are presented.

1. Introduction and preliminaries

1.1. Introduction

Observer design for nonlinear systems has attracted much research interest in recent years. This is due to the important role of observers for the estimation of unmeasurable variables, that are increasingly present in modern real-world applications, such as intelligent vehicles (Rajamani, 2012), electrical machines (Khalil, 2015), position estimation in industrial systems (Henriksson, Norrlöf, Moberg, Wernholt, & Schön, 2009), and biomedical applications (Chong, Postoyan, Nesić, Kuhlmann, & Varsavsky, 2012). The emergence of automation in many real world applications renders the estimation problem very important. In addition to the estimation of unmeasurable variables, observers play critical roles in the fields of fault diagnosis, feedback control and automated event detection.

Although state observer design has been widely investigated in the literature and numerous methods have been established (Arcak & Kokotovic, 2001; Califano, Monaco, & Normand-Cyrot, 2003; Fan & Arcak, 2003; Gauthier, Hammouri, & Othman, 1992; Gauthier & Kupka, 1994; Khalil, 2002; Kravaris, Sotiropoulos, Georgiou, Kazantzis, Xiao, & Krener, 2004, 2007; Krener & Respondek, 1985; Simon, 2006; Thau, 1973), this issue remains a challenge for the control research community. Several new methods have been developed in the recent literature (Abbaszadeh & Marquez, 2010; Açikmese & Corless, 2011; Alessandri & Rossi, 2013, 2015; Andrieu, Praly, & Astolfi, 2009; Astolfi & Marconi, 2015; Astolfi, Marconi, & Teel, 2016; Ibrir, 2007; Phanomchoeng, Rajamani, & Piyabongkarn, 2011; Tsinias, 2008; Wang, Astolfi, Marconi, & Su, 2017; Zemouche & Boutayeb, 2013). All these techniques have been motivated by the lack of a general systematic method to deal with nonlinear systems. Even if many improvements have been proposed in the recent years (Açikmese & Corless, 2011;

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E-mail addresses: ali.zemouche@univ-lorraine.fr (A. Zemouche), rajamani@me.umn.edu (R. Rajamani), phano002@gmail.com (G. Phanomchoeng), boulkrb@gmail.com (B. Boulkroune), hugues.rafaralahy@univ-lorraine.fr (H. Rafaralahy), Michel.Zasadzinski@univ-lorraine.fr (M. Zasadzinski).

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Oueder, Farza, Abdennour, & M'Saad, 2012; Zemouche & Boutayeb, 2013), the estimation problem still remains open. Particularly, for the class of globally Lipschitz nonlinear systems, several LMI methods have been proposed where each method provides a new LMI technique. For instance, some techniques are based on the use of the S-Procedure lemma (Boyd, El Ghaoui, Feron, & Balakrishnan, 1994); others use Riccati equations (Raghavan & Hedrick, 1994), and finally some are based on the standard use of Young's inequality (Alessandri, 2004). A two degree-of-freedom observer design method has been proposed in Arcak and Kokotovic (2001), generalized by Zemouche and Boutayeb (2009). Despite all these new ways to overcome the effect of the nonlinearities, the proposed methods remain conservative for some classes of systems. To improve the existing results, an interesting method was proposed recently in Chong et al. (2012) by introducing a diagonal multiplier matrix as an additional degree of freedom. Such a technique has been shortly discussed in Fan and Arcak (2003) for a class of systems with monotonic nonlinearities. Although the introduction of a diagonal multiplier matrix is interesting and significant, some improvements remain possible. The main question that arises naturally is: why not a non diagonal multiplier matrix? The answer to this question is one of the main subjects of this paper. A short and preliminary version of this result has been presented in Zemouche, Rajamani, Boulkroune, Rafaralahy, and Zasadzinski (2016) as a conference paper. Indeed, a new relaxed LMI condition is provided to solve the problem of $\mathscr{H}_\infty$ observer synthesis by exploiting Young's relation in a judicious manner. This novel way to use Young's inequality allows to have additional degrees of freedom in the LMI and to avoid the diagonal form of the multiplier matrix. Further, the developed results are extended to nonlinear systems that are monotonic and not necessarily globally Lipschitz, and further to systems that contain nonlinearities in the output equation. To clarify the presentation of this paper, let us note that compared to the preliminary conference version paper (Zemouche et al., 2016), this extended version contains the following:

- Nonlinearities in the output equation while the conference paper had only a linear measurement equation;
- vehicle slip angle estimation which is a real-world application and further includes experimental results;
- additional example to show the role of the non diagonal multiplier matrices;
- extended discussions and some analytic comparisons.

The developed $\mathscr{H}_\infty$ observer can be applied for many practical problems. The vehicle slip angle estimation is one of the challenging problems which can be solved by the method. The feedback of vehicle slip angle is useful for Electronic Stability Control (ESC) systems. In situations on low-friction road surfaces, it is useful for the ESC system to control the vehicle slip angle and prevent the vehicle slip angle from being too high (Phanomchoeng et al., 2011). However, vehicle slip angle cannot be easily measured. The vehicle slip angle is also not easy to estimate due to the nonlinear tire model. Both the dynamic and measurement models of the system are highly nonlinear models (Phanomchoeng et al., 2011).

In this paper, the proposed $\mathscr{H}_\infty$ observer was used to estimate the vehicle slip angle based on a nonlinear vehicle model. The observer is shown to be suitable for a large range of operating conditions. The developed technique is validated with experimental measurements on a test vehicle, under different road conditions.

The remaining of the paper is organized as follows: after some useful preliminaries, the problem formulation and the preliminary result of Zemouche et al. (2016) are introduced in Section 2, in order to well position what we propose. The main contribution related to the new LMI observer design method extended to systems with nonlinear outputs is presented in Section 3. Section 4 presents discussions and comparisons with previous results in the literature. Two simple but relevant examples are proposed in Section 5 to show the efficiency of the proposed design methodology. Section 6 includes the design of an observer and experimental results for the application of slip angle estimation in automobiles. Finally, we end the paper by a conclusion in Section 7.

Notations: Throughout this paper, we use the following notations:

- $(\star)$ is used for the blocks induced by symmetry;
- A^T represents the transposed matrix of A;
- $\mathbb{I}_r$ represents the identity matrix of dimension r;
- for a square matrix S, $S > 0$ ($S < 0$) means that this matrix is positive definite (negative definite);
- $e_s(i) = \big(\underbrace{0, \dots, 0, \overbrace{1}^{i\text{th}}, 0, \dots, 0}_{s \text{ components}}\big)^T \in \mathbb{R}^s$, $s \geq 1$ is a vector of the canonical basis of $\mathbb{R}^s$.

1.2. Some preliminaries

We start by introducing some definitions and preliminaries which will be used throughout this paper.

Definition 1 (*Zemouche & Boutayeb, 2013*)**.** Consider two vectors

$$X = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \in \mathbb{R}^n \text{ and } Y = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} \in \mathbb{R}^n.$$

For all $i = 0, \dots, n$, we define an auxiliary vector $X^{Y_i} \in \mathbb{R}^n$ corresponding to X and Y as follows:

$$\begin{cases} X^{Y_i} = \begin{pmatrix} y_1 \\ \vdots \\ y_i \\ x_{i+1} \\ \vdots \\ x_n \end{pmatrix} \text{ for } i = 1, \dots, n \\ X^{Y_0} = X \end{cases} \quad (1)$$

Lemma 2 (*Zemouche & Boutayeb, 2013*)**.** *Consider a function $\Psi : \mathbb{R}^n \longrightarrow \mathbb{R}^n$. Then, the two following items are equivalent:*

- *Ψ is globally Lipschitz with respect to its argument, i.e.,*

$$\left\|\Psi(X) - \Psi(Y)\right\| \leq \gamma_\Psi \left\|X - Y\right\|, \quad \forall X, Y \in \mathbb{R}^n \quad (2)$$

- *for all $i, j = 1, \dots, n$, there exist functions*

$$\psi_{ij} : \mathbb{R}^n \times \mathbb{R}^n \longrightarrow \mathbb{R}$$

and constants $\underline{\gamma}_{\psi_{ij}}$ and $\bar{\gamma}_{\psi_{ij}}$ such that $\forall X, Y \in \mathbb{R}^n$

$$\Psi(X) - \Psi(Y) = \sum_{i=1}^{i=n} \sum_{j=1}^{j=n} \psi_{ij} \mathscr{H}_{ij} \Big(X - Y\Big) \quad (3)$$

and the functions $\psi_{ij}(.)$ are globally bounded from above and below as follows:

$$\underline{\gamma}_{\psi_{ij}} \leq \psi_{ij} \leq \bar{\gamma}_{\psi_{ij}} \quad (4)$$

where

$$\psi_{ij} \triangleq \psi_{ij}\Big(X^{Y_{j-1}}, X^{Y_j}\Big) \text{ and } \mathscr{H}_{ij} = e_n(i)e_n^T(j)$$

Proof. The proof is omitted. See Zemouche and Boutayeb (2013).