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Brief paper

Output regulation for general linear heterodirectional hyperbolic systems with spatially-varying coefficients☆

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ABSTRACT

This article presents a backstepping solution to the output regulation problem for general linear heterodirectional hyperbolic systems with spatially-varying coefficients. The disturbances can act at both boundaries, distributed in-domain or at the output to be controlled. The latter is defined at a boundary, distributed or pointwise in-domain and has not to be available for measurement. By utilizing backstepping coordinates it is shown that all design equations are explicitly solvable. This allows a simple determination of a state feedback regulator, that is implemented by a reference and a disturbance observer. Furthermore, an easy evaluation of the existence conditions for the resulting output feedback regulator is possible in terms of the plant transfer behaviour. In order to facilitate the parameterization of the regulator, the resulting closed-loop dynamics is directly related to the design parameters. The proposed backstepping-based design of the output feedback regulator is demonstrated for an unstable heterodirectional 4×4 hyperbolic system.

1. Introduction

In the last years the *backstepping approach* (see, e. g., Krstic and Smyshlyaev (2008) and Meurer (2013)) was further developed to provide systematic methods for the control of *hyperbolic systems*. These systems allow to describe transport phenomena with finite propagation speeds. Consequently, many processes arising in applications can be modelled by them. Examples are heat exchangers, transmission lines, open channel flows or plug flow chemical reactors (see Bastin and Coron (2016) for the modelling of these examples by hyperbolic systems). Starting with the pioneering work in Krstic (2009a) backstepping results are available for linear and quasilinear 2×2 hyperbolic systems (see Coron, Vazquez, Krstic, and Bastin (2013) and Vazquez, Krstic, and Coron (2011)) as well as for systems of $n+1$ coupled first-order hyperbolic linear PDEs (see Meglio, Vazquez, and Krstic (2013)). Recently, these results are formulated for a general class of linear and quasilinear heterodirectional hyperbolic systems with an arbitrary number of transport equations propagating in opposite directions (see Hu, Meglio, Vazquez, and Krstic (2016) and Hu, Vazquez, Meglio, and Krstic (submitted for publication)).

An important extension of the backstepping method is the design of stabilizing compensators, that additionally ensure the asymptotic tracking of reference inputs in the presence of disturbances. This amounts to solving an *output regulation problem* for distributed-parameter systems (DPS) with unbounded control and observation (see, e. g., Aulisa and Gilliam (2016), Natarajan, Gilliam, and Weiss (2014) and Paunonen and Pohjolainen (2014)). Thereby, *observer-based feedforward control* provides a flexible solution, as the output to be controlled need not be measured. These compensators contain a *state feedback regulator*, i. e., a state feedback with a feedforward of the state describing the exogenous signals. Then, the *output feedback regulator* is obtained by designing a reference and a disturbance observer for implementing the state feedback regulator. A constructive design procedure for these regulators is presented in Xu and Dubljevic (2016) for first-order hyperbolic systems with bounded control and unbounded observation. The corresponding stabilization problem was solved by applying a Riccati equation approach. Nevertheless, the backstepping approach is especially well-suited for systematically determining observer-based feedforward regulators. This was soon recognized by many researchers and led to the results in Aamo (2013), Anfinsen and Aamo (2015), Deutscher (2017) and Strecker and Aamo (2016) for 2×2 hyperbolic SISO systems and for $n+1$ hyperbolic SISO systems in Anfinsen and Aamo (2016) and Hasan (2014). Recently, the work Anfinsen and Aamo (2017) solves a disturbance rejection problem for a general class of heterodirectional hyperbolic MIMO systems with constant coefficients and a collocated measurement. Thereby, the disturbances and the outputs to be controlled are located at the uncontrolled boundary.

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By utilizing the results in Hu et al. (2016) an observer-based feedforward regulator is derived. Since the approach in Hu et al. (2016) only applies the backstepping transformation to the states of the boundary controlled PDEs, a less number of kernel equations has to be solved in the design.

In this paper, the output regulation problem is solved for general linear heterodirectional hyperbolic systems consisting of an arbitrary number of transport equations propagating in both directions. A systematic solution of this problem is obtained for this general class of hyperbolic MIMO systems by extending the previous results in Deutscher (2017) for 2×2 hyperbolic SISO systems. Thereby, spatially-varying coefficients are allowed so that the results are applicable to a wide range of DPS. For the regulator design the backstepping approach in Hu et al. (submitted for publication) concerning general linear hyperbolic systems with spatially-varying coefficients is utilized. Therein, all system states are included in the backstepping transformation. As a consequence, a larger number of kernel equations as in Hu et al. (2016) has to be solved for determining the integral kernel. However, this approach has the advantage that it allows a very straightforward solution of the output regulation problem for general classes of disturbances. In particular, the disturbances can act at both boundaries, distributed in-domain and at the output to be controlled. This is much more involved when applying the backstepping coordinates defined in Hu et al. (2016), because integral terms appear in the resulting target systems. Similar to Deutscher (2017), the design of the state feedback regulator is significantly simplified by solving the *regulator equations* in the backstepping coordinates. Thereby, the employed backstepping coordinates allow to take a general class of outputs to be controlled into account. More precisely, they can be defined distributed, pointwise in-domain or at the boundaries.

As Hu et al. (submitted for publication) only consider the case of state feedback control, an extension of these results to the observer design for an anticollocated measurement is proposed. With this, a disturbance observer is derived for the estimation of the plant and disturbance model states. The solvability conditions for the output regulation problem in question are derived in backstepping coordinates. Hence, they can easily be evaluated on the basis of the plant transfer behaviour.

It is shown that all equations for determining the output feedback regulator in the backstepping coordinates are solvable in closed-form. Furthermore, the target systems utilized in the backstepping design allow to determine the pointwise closed-loop solution explicitly. Hence, the resulting closed-loop dynamics and thus the output regulation can be directly related to the design parameters. This allows a very transparent parameterization of the output feedback regulator. As a result, a systematic and general output regulation design method is obtained for a large class of hyperbolic systems.

The next section formulates the considered output regulation problem. Then, Section 3 presents the state feedback regulator design. In order to estimate the unknown states, a reference and a disturbance observer is determined in Section 4. These results are combined in Section 5 to obtain an output feedback regulator. The proposed backstepping solution of the posed output regulation problem is illustrated for an unstable heterodirectional 4×4 hyperbolic system.

2. Problem formulation

Consider the *general linear hyperbolic system*

$$\partial_t x(z,t) = \Lambda(z)\partial_z x(z,t) + A(z)x(z,t) + G_1(z)d(t) \tag{1a}$$

$$x_2(0,t) = Q_0 x_1(0,t) + G_2 d(t), \qquad t > 0 \tag{1b}$$

$$x_1(1,t) = Q_1 x_2(1,t) + u(t) + G_3 d(t), \qquad t > 0 \tag{1c}$$

$$\eta(t) = x_1(0,t), \qquad t \geq 0 \tag{1d}$$

$$y(t) = \mathcal{C}[x(t)] + G_4 d(t), \qquad t \geq 0, \tag{1e}$$

that consists of n coupled *transport PDEs* (1a) defined on the domain $(z,t) \in (0,1) \times \mathbb{R}^+$ with the state $x(z,t) = [x^1(z,t) \ \ldots \ x^n(z,t)]^T \in \mathbb{R}^n$, the input $u(t) \in \mathbb{R}^p$ and the unmeasurable disturbance $d(t) \in \mathbb{R}^q$. In (1) and in the sequel the shorthand notations $\partial_t := \partial/\partial t$ and $\partial_z := \partial/\partial z$ are utilized for the partial derivatives as well as $\mathrm{d}_z := \mathrm{d}/\mathrm{d}z$ for the ordinary derivative. The matrix $\Lambda(z)$ in (1a) is given by

$$\Lambda(z) = \mathrm{diag}(\lambda_1(z), \ldots, \lambda_n(z)) \tag{2}$$

with $\lambda_1(z) > \ldots > \lambda_p(z) > 0 > \lambda_{p+1}(z) > \ldots > \lambda_n(z), z \in [0,1]$ and $\lambda_i \in C^1[0,1], i = 1,2,\ldots,n$. This suggests to introduce the matrices

$$E_1 = \begin{bmatrix} I_p \\ 0 \end{bmatrix} \in \mathbb{R}^{n \times p} \quad \text{and} \quad E_2 = \begin{bmatrix} 0 \\ I_m \end{bmatrix} \in \mathbb{R}^{n \times m} \tag{3}$$

so that the states $x_1(z,t) = E_1^T x(z,t) \in \mathbb{R}^p$ describe the propagation in the negative direction of the spatial coordinate z with the transport velocities $\lambda_i(z), i = 1,2,\ldots,p$. Accordingly, the remaining states $x_2(z,t) = E_2^T x(z,t) \in \mathbb{R}^m$ with $p + m = n, p, m \geq 1$ and the velocities $|\lambda_i(z)|, i = p+1,\ldots,n$, take the transport in the z-direction into account. As a consequence, the system (1) is called *heterodirectional* (see Hu et al. (2016)). The matrix $A(z) = [A_{ij}(z)]$ in (1a) satisfies $A_{ij} \in C^1[0,1], i,j = 1,2,\ldots,n$ and $A_{ii}(z) = 0$, $z \in [0,1], i = 1,2,\ldots,n$. Note that the latter condition means no loss of generality (see, e. g., Hu et al. (2016)). The known disturbance input locations are characterized by $G_1 \in (C[0,1])^{n \times q}$, $G_2 \in \mathbb{R}^{m \times q}$ and $G_i \in \mathbb{R}^{p \times q}, i = 3,4$. Furthermore, $Q_0 \in \mathbb{R}^{m \times p}$ and $Q_1 \in \mathbb{R}^{p \times m}$ are arbitrary matrices. The *initial conditions (ICs)* of (1) are $x(z,0) = x_0(z) \in \mathbb{R}^n, z \in [0,1]$. As measurement the *anticollocated output* $\eta(t) \in \mathbb{R}^p$ in (1d) is utilized and $y(t) \in \mathbb{R}^p$ represents the output to be controlled, that need not be measured. This output can be defined at a boundary, pointwise or distributed in-domain, which is modelled by the formal *output operator*

$$\mathcal{C}[h] = \sum_{i=1}^{l} C_i h(z_i) + \int_0^1 C(z)h(z)\mathrm{d}z \tag{4}$$

for $h(z) \in \mathbb{C}^n$ with $C_i \in \mathbb{R}^{p \times n}, z_i \in [0,1], i = 1,2,\ldots,l$, and $C(z) = [c_{ij}(z)] \in \mathbb{R}^{p \times n}$ with c_{ij} piecewise continuous functions. In the following it is assumed that the resulting output y in (1e) is independent of the boundary conditions (BCs) (1b) and (1c) if $C(z) = 0, z \in [0,1]$, in (4).

It is assumed that only the actual value $r(t) \in \mathbb{R}^p$ of the *reference input* is available for the compensator, i.e., an *online-specification* of r is considered. Thereby, this exogenous signal and the disturbances can be represented by the solution of the finite-dimensional *signal model*

$$\dot{v}(t) = Sv(t), \qquad t > 0, \ v(0) = v_0 \in \mathbb{R}^{n_v} \tag{5a}$$

$$d(t) = P_d v(t) = \bar{P}_d v_d(t), \qquad t \geq 0 \tag{5b}$$

$$r(t) = P_r v(t) = \bar{P}_r v_r(t), \qquad t \geq 0 \tag{5c}$$

with $P_d \in \mathbb{R}^{q \times n_v}$ and $P_r \in \mathbb{R}^{p \times n_v}$. Therein, the *spectrum* $\sigma(S)$ of the diagonalizable matrix S only contains eigenvalues on the imaginary axis. This allows the modelling of bounded and persistently acting exogenous signals. In particular, the exogenous signals can be constant or trigonometric functions of time as well as linear combinations of both signal forms. By introducing $S = \mathrm{bdiag}(S_r, S_d)$ and $v = \mathrm{col}(v_r, v_d)$ one obtains the *reference model* $\dot{v}_r(t) = S_r v_r(t), v_r(0) = v_{r,0} \in \mathbb{R}^{n_r}$, and the *disturbance model* $\dot{v}_d(t) = S_d v_d(t), v_d(0) = v_{d,0} \in \mathbb{R}^{n_d}, n_r + n_d = n_v$. Consequently, $\bar{P}_d \in \mathbb{R}^{q \times n_d}$ and $\bar{P}_r \in \mathbb{R}^{p \times n_r}$ have to hold in (5). Furthermore, it is