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Brief paper

Output regulation for a class of linear boundary controlled first-order hyperbolic PIDE systems☆

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ABSTRACT

This manuscript addresses the output regulation problem for a class of scalar boundary controlled first-order hyperbolic partial integro-differential equation (PIDE) systems with Fredholm integrals. In particular, with the advantage of the backstepping approach, simple structure systems can be obtained such that regulator equations for the state feedback regulator design are analyzed and solved in backstepping coordinates. Moreover, the finite time output regulation is achieved. In the observer-based output feedback regulator design, it is not necessary that the outputs to be controlled belong to the available output measurements and these outputs can be distributed, point-wise and/or boundary in nature, while the boundary placed measurements are used for regulator design. For the observer gains design, a transformation of the ODE–PDE system into an ODE–PDE cascade is considered. It is also shown that the separation principle holds for the output feedback regulator design and the exponential output regulation is realized for the resulting stable closed-loop system. Finally, the output regulation results are illustrated with two numerical simulations: a Korteweg–de Vries-like equation and a PDE–ODE interconnected system.

1. Introduction

Due to the wide area of applications, boundary control and observation of hyperbolic partial differential equations (PDE) systems have been active research topics during the last decade, e.g., Diagne, Bastin, and Coron (2012) and Krstic and Smyshlyaev (2008). The pioneering backstepping approach developed for parabolic PDEs (Bošković, Krstić, & Liu, 2001) has been applied to boundary controlled hyperbolic systems (Krstic & Smyshlyaev, 2008). The backstepping based stabilization method was applied to two coupled first order hyperbolic systems in Vazquez, Krstic, and Coron (2011), and to the general $n+1$ case in Di Meglio, Vazquez, and Krstic (2013) and the more general $n+m$ case in Hu, Di Meglio, Vazquez, and Krstic (2015). Along the same line, minimum time control law was developed for $n+m$ coupled hyperbolic PDEs in Auriol and Di Meglio (2016). Moreover, for the disturbance rejection problem, adaptive observers were constructed for hyperbolic PDEs in Aamo (2013) and Anfinsen and Aamo (2015). The linear first-order hyperbolic PIDEs considered in this manuscript were introduced in Krstic and Smyshlyaev (2008), which usually arise from two coupled PDEs with one being suitably perturbed. In Bernard and Krstic (2014), an adaptive output feedback controller was designed to deal with stabilization of PIDEs with unknown parameters and in Bribiesca-Argomedo and Krstic (2015) the boundary control concept was extended to the PIDEs setting with Fredholm operators that do not exhibit a strict feedback structure. In this work, state feedback and output feedback regulator design problems for PIDE systems are addressed.

Recently, state feedback regulators were designed to address the robust regulation problems for 2×2 hyperbolic and wave equation systems in Deutscher (2016) and Deutscher and Kerschbaum (2016). Along the line of contributions associated with results on output feedback regulator designs, the results in Deutscher (2015) on parabolic systems were extended to construct finite-time output regulators for 2×2 hyperbolic systems in Deutscher (2017). To complement this effort, in this work, the state and output feedback (using the measurement $y_m(t)$ and the reference signal $y_r(t)$) regulators are designed for a class of the first-order hyperbolic PIDE systems and cover a large number of transport processes. The state feedback regulator problem is solved by constructing the regulator equations in the backstepping coordinates and the corresponding solvability conditions are discussed. The solution of the output feedback regulator design problem yields a reference and disturbance observer design. To end this, this amounts to the stabilization of the disturbance observer error system in form of a coupled ODE–PDE system. Motivated by results

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in Aamo (2013) for 2×2 hyperbolic systems and Deutscher (2015), a transformation into an ODE–PDE cascade is considered in this work. The proposed disturbance observer design is based on a transformation of the PDE observer error subsystem into new coordinates using the backstepping methods. Then, the transformed observer error system is decoupled into a triangular system in the backstepping coordinates, so that the ODE and PDE subsystems can be stabilized independently. In new coordinates, the stabilization of the ODE–PDE observer error subsystem becomes very simple. This also yields explicit existence conditions for the exponential convergence of the disturbance observer and these existence conditions can be checked explicitly in backstepping coordinates. Finally, it is worth mentioning that the exosystem is extended to generate polynomial type reference signals and the corresponding approach solving regulator equations is provided in this work.

In this manuscript, after the problem formulation in Section 2, the state feedback regulator problem is solved in Section 3. Section 4 introduces the design of the output feedback regulator, according to the separation principle. Finally, the results are demonstrated through illustrative simulations in Section 5.

2. Problem formulation

We consider the following hyperbolic PIDE systems on the domain $\left\{t \in \mathbb{R}^+, z \in (0,1)\right\}$ presented in Bribiesca-Argomedo and Krstic (2015):

$$\begin{aligned} \partial_t x(z,t) &= \partial_z x(z,t) + f(z)x(0,t) \\ &\quad + \int_0^z g(z,\xi)x(\xi,t)d\xi \\ &\quad + \int_z^1 h(z,\xi)x(\xi,t)d\xi + g_1(z)d_1(t) \end{aligned} \tag{1}$$

$$x(1,t) = u(t) + g_2 d_2(t) \tag{2}$$

$$y(t) = \mathcal{C}x(t) \tag{3}$$

$$y_m(t) = x(0,t) \tag{4}$$

with the input $u(t) \in \mathbb{R}$. $d_1(t) \in \mathbb{R}$ and $d_2(t) \in \mathbb{R}$ are unmeasurable process and boundary input disturbances, respectively. f, g and h are real-valued continuous functions. $g_1 \in C[0,1]$ and $g_2 \in \mathbb{R}$ in (1)–(2) are known functions that characterize the distribution of disturbances. $x(\cdot,t) \in H = L_2(0,1)$, $\forall t \in \mathbb{R}^+$ denotes the state variable and then $x(\cdot,t)$ at the point z is $x(z,t)$. $H = L_2(0,1)$ is a real Hilbert space equipped with the inner product $\langle h_1, h_2\rangle = \int_0^1 h_1(z)h_2(z)dz$, $\forall h_1, h_2 \in H$. Then, the norm is given by $\|x\|_2 = \langle x, x\rangle$, $\forall x \in H$. In (3), $y(t) \in \mathbb{R}$ is the output to be controlled. The corresponding output operator $\mathcal{C}$ may describe point-wise or distributed in domain outputs, i.e.

$$y(t) = \mathcal{C}x(t) = \int_0^1 c(z)x(z,t)dz \tag{5}$$

where $c(z) = \sum_{i=1}^N c_i\delta(z-z_i)$, $z_i \in (0,1)$ and $c_i \in \mathbb{R}$, or $c(z) \in L_2(0,1)$. The measurement $y_m(t) \in \mathbb{R}$ is different from the controlled output $y(t)$. In particular, it is not necessary that the controlled output $y(t)$ can be measured.

The following scalar hyperbolic PIDE system:

$$\begin{aligned} \partial_t x(z,t) &= v(z)\partial_z x(z,t) + \alpha(z)x(z,t) \\ &\quad + \bar{f}(z)x(0,t) + \int_0^z \bar{g}(z,\xi)x(\xi,t)d\xi \\ &\quad + \int_z^1 \bar{h}(z,\xi)x(\xi,t)d\xi + g_1(z)d_1(t) \end{aligned}$$

on the domain $(z,t) \in (0,1) \times (0,T]$ can be transformed into (1)–(4) by applying an appropriate change of variables, see Bribiesca-Argomedo and Krstic (2015). Concomitantly, the resulting boundary conditions and outputs remain the same as in (2)–(4). Hence, the following results of this manuscript are also valid for this general system class that describes many transport processes.

To ensure that the plant (1) is stabilizable in finite time, the following assumption providing sufficient conditions for the coefficients of (1) is given (Bribiesca-Argomedo & Krstic, 2015):

Assumption 1. Define the triangles

$$\mathcal{T}_l = \{(z,\xi) \in [0,1] \times [0,1], z \geq \xi\}$$

$$\mathcal{T}_u = \{(z,\xi) \in [0,1] \times [0,1], z \leq \xi\}$$

and the spaces $X_l = C(\mathcal{T}_l;\mathbb{R})$ and $X_u = C(\mathcal{T}_u;\mathbb{R})$ equipped with the norms

$$\|h\|_{X_i} = \sup_{(z,\xi)\in\mathcal{T}_i} |h(z,\xi)|, \forall h \in X_i, i = l, u$$

then the coefficients in (1) satisfy: $f \in C([0,1];\mathbb{R})$, $g \in X_l$ and $h \in X_u$. Moreover, f, g and h satisfy: $\max\left\{\sup_{\zeta\in[0,1]}|f(\zeta)|, \|g\|_{X_l}, \|h\|_{X_u}\right\} < 0.25$. In particular, if $f(z) \equiv 0$, then the coefficients g and h satisfy: $\max\left\{\|g\|_{X_l}, \|h\|_{X_u}\right\} < 0.5$.

Actually, by introducing Assumption 1, the plant is limited into a certain class of systems with heavily bounded coefficients. However, for some systems, even though the plant coefficients are larger than the sufficient conditions, these systems still can be stabilized, see Section II-*E* in Bribiesca-Argomedo and Krstic (2015). Moreover, when some coefficients such as f and h are zero functions, this limitation is relaxed. For example, in (1), when $h(z,\xi) \equiv 0$, the plant reduces to the system in Krstic and Smyshlyaev (2008) and is always stabilizable in finite time. Furthermore, for the case that g and h are only functions of z, i.e., $g(z,\xi) = g(z)$ and $h(z,\xi) = h(z)$, sufficient and necessary conditions were studied and provided in Coron, Hu, and Olive (2016).

The disturbances $d_1(t)$ and $d_2(t)$ in (1), (2) and the reference signal $y_r(t) \in \mathbb{R}$ to be asymptotically tracked by the controlled output $y(t)$ can be modeled by the known finite-dimensional exosystem:

$$\dot{v}(t) = Sv(t),\ v(0) = v_0 \in \mathbb{C}^{n_v} \tag{6}$$

$$d_1(t) = p_{d_1}^T v(t) = r_{d_1}^T v_d(t), \quad t \in \mathbb{R}^+ \tag{7}$$

$$d_2(t) = p_{d_2}^T v(t) = r_{d_2}^T v_d(t), \quad t \in \mathbb{R}^+ \tag{8}$$

$$y_r(t) = q^T v(t) = q_r^T v_r(t), \quad t \in \mathbb{R}^+ \tag{9}$$

where S is a block diagonal matrix $S = bdiag(S_d, S_r)$ having all its eigenvalues on the imaginary axis, i.e. iw_k where $i = \sqrt{-1}$, $k = 1,\ldots,n_v$ and w_k can have zero values. Correspondingly, $v = col(v_d, v_r)$ with the signal models $\dot{v}_d(t) = S_d v_d(t)$, $v_d(0) = v_{d0} \in \mathbb{C}^{n_d}$, and $\dot{v}_r(t) = S_r v_r(t)$, $v_r(0) = v_{r0} \in \mathbb{C}^{n_r}$, $n_d + n_r = n_v$.

In particular, we can design the above matrix S to have the form: $S = bdiag(S_d, S_r) = bdiag(S_m, S_n)$ and the block S_n is a nilpotent matrix with dimension n_n, i.e. its spectrum: $\sigma(S_n) = 0$. In this manuscript, we assume S_n is a sub-block matrix in the matrix S_r. The matrix S_m is a diagonalizable matrix with dimension n_m. Obviously, we have $n_n + n_m = n_v$. In particular, in this manuscript, S_n is given by

$$S_n = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & (n_n-1) & 0 \end{bmatrix}. \tag{10}$$