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Brief paper

On guaranteeing point capture in linear *n*-on-1 endgame interception engagements with bounded controls☆

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ABSTRACT

A linearized endgame interception scenario along a line between a single evading target and *n* pursuers is considered, in which the adversaries' controls are bounded and have arbitrary-order dynamics, and the evader's maneuvers are not known a priori to the pursuing team. To determine the merit in utilizing multiple interceptors, in terms of their capability to impose point capture, a capturability analysis is performed, presenting necessary and sufficient conditions for the feasibility of point capture for any admissible evader maneuver. It is shown that the pursuing team is capable of guaranteeing point capture if and only if it consists of at least one pursuer capable of independently imposing point capture. This requirement is independent of the number of pursuers, leading to the conclusion that it cannot be relaxed by increasing the number of interceptors or by any manner of cooperation, in terms of coordinated motion, between the pursuers.

1. Introduction

Cooperative strategies are becoming more and more popular with the continuing evolution and advancement in decision making capabilities of autonomous vehicles. Utilizing multiple agents to perform a given task can be beneficial even in cases when the goal is achievable by a single agent. With regard to interception engagements of an evading target, through shared information and coordinated actions the capability requirements and/or the number of required agents may be relaxed and reduced, respectively. It is therefore a great point of interest, when analyzing interception engagements, to discern under what conditions the evader's capture can be guaranteed, and whether or not these conditions are dependent on the number of pursuers.

Isaacs, in his study of pursuit-evasion games (Isaacs, 1965), was the first to obtain explicit conditions for capture in conflicts between a single pursuer and a single evader. The construction of the capture zone's boundary provided the set of initial conditions from which the pursuer was capable of guaranteeing the evader's capture, given the engagement parameters. An example of a game between adversaries with maneuverability constraints was presented by Isaacs in the form of the so-called game of two cars. This planar engagement includes adversaries which have constant speeds, minimum turn radii and no control dynamics. Additionally, each adversary has knowledge only of its opponent's current position and attitude. In Isaacs' analysis of this game "capture" was defined by the pursuer reaching some non-negative distance from the evader and the pursuer was assumed to be faster than the evader. This was later completed in Meier (1969), in which the case of a slower pursuer was considered. The first to focus on the required capabilities for guaranteeing capture of an evading target was Cockayne (1967). He proved that the pursuer in the game of two cars can capture the evader (achieve position coincidence) from any initial geometry if and only if it has a speed advantage and is at least as maneuverable as the evader. Cockayne stated that these conditions should coincide with Isaacs' results in the game of two cars with the capture radius set to zero. Rublein (1972) later extended Cockayne's work to address motion in three dimensional space. He showed that a sufficient condition for guaranteeing point capture is the pursuer's superiority both in speed and in maneuverability. In Borowko and Rzymowski (1984) an inverse study to Cockayne (1967) was presented, concerning the capabilities required in order to guarantee successful evasion from a pursuer. The author proved that the evader in the game of two cars can avoid capture for any initial conditions if and only if one of the following holds: (a) the evader has a speed advantage and its maximal maneuver capability is greater than or equal to that of the pursuer times the pursuer-to-evader speed ratio, (b) the evader's speed is equal to the pursuer's and it has a maneuverability advantage. These results together with those presented in Cockayne (1967) lead to the conclusion that if the evader has a speed disadvantage,

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but its maximal maneuver capability is greater than that of the pursuer times the pursuer-to-evader speed ratio, then there exist some initial conditions from which the pursuer can guarantee the evader's capture. An extension of this work to a planar case of n pursuers vs. a single evader was presented in Rzymowski (1984). It was shown that if, relative to each pursuer, the evader has a speed advantage and its maximal maneuver capability is greater than or equal to that of any pursuer times the corresponding pursuer-to-evader speed ratio it can avoid capture indefinitely.

There have been various other publications discussing capturability of a single target by multiple pursuers, with varying engagement formulations and capture definitions. In Bopardikar, Bullo, and Hespanha (2009) a cooperative strategy is proposed for the confinement of a more maneuverable but slower evader by a team of identical pursuers. It is shown that, given the non-zero turn radius and capture radius of the pursuers and the evader/pursuer speed ratio, there exists a minimum number of pursuers required in order to guarantee successful confinement. A similar analysis is performed in Chen, Zha, Peng, and Gu (2016) for a different multi-player pursuit-evasion game: a team of pursuers endeavors to capture (impose position coincidence) a single target in the plane, all of which have constant speeds and instantaneous turn capability. In this case the capturability analysis is extended to include the initial conditions from which capture may be guaranteed. Another method used in the development of pursuit strategies for multiple pursuers intercepting a single target is Voronoi partitioning (Bakolas & Tsiotras, 2012; Zhou, Zhang, Ding, Huang, Stipanović, & Tomlin, 2016). In Bakolas and Tsiotras (2012) a capturability condition is derived for a simple planar pursuit-evasion scenario in which each of the adversaries has bounded speed and is capable of instantaneous turns. In Zhou et al. (2016) a cooperative pursuit strategy is developed for a similar problem in which motion is restricted to a convex planar domain. It is shown that under the proposed strategy capture (defined by the evader entering a finite radius) is guaranteed. Generally, as is the case in these presented works, the capturability conditions are dependent on the proposed strategies of the adversaries and may therefore be more stringent than actually needed under optimal play.

In scenarios where during the endgame the adversaries' motion is near their respective collision courses the kinematics of the engagement can be linearized relative to some fixed frame (Zarchan, 1994). For such cases a point of reference with regard to capturability is once again solutions to games of pursuit. Existing solutions to linear pursuit-evasion games of a single pursuer vs. a single evader with bounded controls also include variations on the order of the players' control dynamics as well as the number of control inputs (Gutman, 1979; Gutman & Leitmann, 1976; Qi, Liu, & Tang, 2011; Shima, 2005; Shima & Golan, 2006; Shima & Shinar, 2002; Shinar, 1981; Turetsky & Shinar, 2003). An analysis of a class of linear time-varying feedback pursuit strategies in the same framework was presented in Turetsky (2008), focusing on scenarios in which point capture is guaranteed.

These previous studies have yielded important conclusions with regard to the necessary and sufficient requirements from interceptors in 1-on-1 engagements. Following these works, and considering interception scenarios with multiple pursuers, it is of interest to examine the necessary and sufficient conditions for capture in a general n-on-1 engagement, the results of which have important implications on the merits of utilizing a multiplicity of pursuers.

This paper presents an analytical study of the conditions for the feasibility of exact capture in an n-on-1 linearized endgame engagement along a line in which the adversaries' kinematics and control dynamics are represented together by arbitrary-order time-variant linear systems. Rather than solving a general n-on-1 pursuit-evasion game, we adopt a reachability approach, thereby

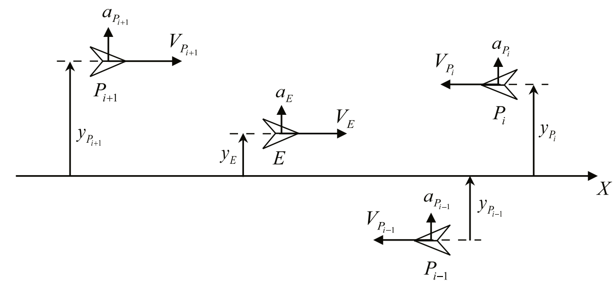

Fig. 1. Linear n-on-1 engagement scheme.

avoiding the need to first define optimal strategies for the adversaries. It is shown that the pursuing team is capable of guaranteeing point capture if and only if it consists of at least one pursuer capable of independently imposing point capture. This requirement is independent of the number of pursuers, leading to the conclusion that it cannot be relaxed by increasing the number of interceptors or by any manner of cooperation, in terms of coordinated motion, between the pursuers.

The remainder of the paper is structured as follows: In Section 2 the formulation of the interception engagement and its mathematical model are presented. Next, general conditions for the existence of a capture zone are derived in Section 3, followed by concluding remarks in Section 4.

2. Linear *n*-on-1 engagement formulation

Consider the endgame of a planar interception engagement of a single evader by a group of n pursuers, in which it is assumed (as is common in endgame missile interception engagements, see Shima & Shinar, 2002; Zarchan, 1994) that

- the adversaries can be regarded as point masses with linear arbitrary-order control dynamics, having multiple decoupled bounded control inputs, the bounds of which are known functions of time,
- the adversaries' motion is restricted to a plane and their speeds are known functions of time, which are not necessarily equal,
- the adversaries are all near head-on or tail-chase and their motion can be linearized around a common fixed reference line,

as depicted in Fig. 1. V and a denote the speed along X and the acceleration normal to X, respectively, and y denotes the normal displacement relative to X. We define the group of adversaries $G = \{P_1, P_2, \ldots, P_n, E\}$, the control input tags $N_c^j = \{1, 2, \ldots, n_c^j\}$ for each $j \in G$ and the pursuer tags $N = \{1, 2, \ldots, n\}$. Under these assumptions the lateral maneuvers do not affect the horizontal velocities, but only the vertical speeds, relative to the reference line. Such would be the case in realistic scenarios which include multiple pursuers launched from a single platform (e.g. aircraft) and point defense (e.g. ballistic missile defense). As a result, the kinematics along X are solved, yielding

$$r_i(t) = r_i^o - \int_{t_o}^{t} V_{c_i}(\xi)d\xi, \quad i \in N, \tag{1}$$

where r_i is the ith pursuer-to-evader range along X. t_o is the initial time and r_i^o and $V_{c_i}(t)$ are, respectively, the initial ith pursuer-to-evader range and the positive closing speed between the ith pursuer and the evader, both measured along the common reference line (for near head-on $V_{c_i}(t) \approx V_{P_i}(t) + V_E(t)$ and for near tail-chase $V_{c_i}(t) \approx V_{P_i}(t) - V_E(t)$, where for the latter $V_{P_i}(t) > V_E(t\ \forall t \geq$