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Brief paper

Event-triggered cooperative robust practical output regulation for a class of linear multi-agent systems ☆

Wei Liu, Jie Huang

Department of Mechanical and Automation Engineering, The Chinese University of Hong Kong, Shatin, N.T., Hong Kong

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ABSTRACT

In this paper, we consider the event-triggered cooperative robust practical output regulation problem for a class of linear minimum-phase multi-agent systems. We first convert our problem into the cooperative robust practical stabilization problem of a well defined augmented system based on the distributed internal model approach. Then, we design a distributed event-triggered output feedback control law together with a distributed output-based event-triggered mechanism to stabilize the augmented system, which leads to the solvability of the cooperative robust practical output regulation problem of the original plant. Our distributed control law can be directly implemented in a digital platform provided that the distributed triggering mechanism can monitor the continuous-time output information from neighboring agents.

1. Introduction

Over the past few years, event-triggered control has attracted extensive attention from the control community. Compared with conventional periodical time-triggered control, event-triggered control samples the system's state or output aperiodically and reduces the number of control task executions while maintaining the control performance. As noted in Heemels, Johansson, and Tabuada (2012), event-triggered control is reactive and generates sensor sampling and control actuation when, for instance, the plant state or output deviates more than a certain threshold from a desired value. Various event-triggered control problems have been studied for several types of systems in Donkers and Heemels (2012); Girard (2015); Liu and Jiang (2015); Liu and Huang (2017b); Tabuada (2007); Tallapragada and Chopra (2013) and the references therein. One of the main challenges for event-triggered control is to avoid the Zeno behavior, which means that the execution times become arbitrarily close and result in an accumulation point Tabuada (2007). In Tabuada (2007), the stabilization problem of a given system by a state-based event-triggered control law was linked to the input-to-state stabilizability (ISS) of the system, and it was shown that the Zeno behavior can be excluded when the measurement state is not equal to zero. In Girard (2015), a dynamic state-based event-triggered mechanism was further proposed to study the stabilization problem for the same class of nonlinear systems as that in Tabuada (2007). Reference Tallapragada & Chopra, (2013) studied the asymptotic tracking problem of a control system by a stated-based event-triggered controller and gave a stated-based event-triggered controller that was able to achieve the uniform ultimate boundedness of the tracking error. Reference Liu & Jiang (2015) further studied the robust stabilization problem for a class of systems subject to external disturbances by a state-based event-triggered control based on the small-gain theorem. In Donkers & Heemels (2012), an output-based event-triggered control law was proposed to analyze the stability and $\mathcal{L}_\infty$ performance for a class of linear systems. In Liu & Huang (2017b), the robust practical output regulation problem for a class of linear systems was studied by an output-based event-triggered control law. Some other contributions can also be found in Abdelrahim, Postoyan, Daafouz, and Nesic (2016); Dolk, Borgers, and Heemels (2017); Postoyan, Tabuada, Nesic, and Anta (2015) etc.

In this paper, we further consider the cooperative robust practical output regulation problem for a class of linear minimum-phase multi-agent systems by a dynamic distributed output-based event-triggered control law. The problem can be viewed as an extension of the result in Liu and Huang (2017b) from a single system to a multi-agent system. Compared with Liu & Huang (2017b), the main challenge for this paper is that the control of each subsystem and hence the triggering mechanism are subject to some communication constraints described by a digraph. We need

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E-mail addresses: wliu@mae.cuhk.edu.hk (W. Liu), jhuang@mae.cuhk.edu.hk (J. Huang).

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to specifically design an output-based event-triggered control law that satisfies the communication constraints. Thus, our problem is more challenging than that of Liu and Huang (2017b). In what follows, a control law and an event-triggered mechanism satisfying the communication constraints are called a distributed control law and a distributed event-triggered mechanism, respectively.

Several other cooperative control problems of multi-agent systems by event-triggered control have also been studied. For example, the consensus problem of single-integrator and double-integrator multi-agent systems by event-triggered control was studied in Dimarogonas, Frazzoli, and Johansson (2012); Fan, Feng, Wang, and Song (2013); Seyboth, Dimarogonas, and Johansson (2013), and (Li, Liao, Huang, & Zhu, 2015; Mu, Liao, & Huang, 2015), respectively. Reference Zhang, Feng, Yan, & Chen (2014) studied the consensus problem for the general linear multi-agent systems by an observer-based output feedback event-triggered distributed control law. Reference Wang & Ni (2012) studied the cooperative output regulation problem for linear multi-agent system by a centralized event-triggered mechanism. Reference Hu & Liu (2017) further studied the cooperative output regulation problem for linear multi-agent systems by a distributed event-triggered mechanism based on the feedforward design method. Other relevant results for the general linear multi-agent systems can be found in Cheng and Ugrinovskii (2016); Wang, Ni, and Ma (2015); Zhu, Jiang, and Feng (2014) etc.

Compared with the existing results on event-triggered cooperative control problems, this paper needs to overcome some specific challenges. First, our problem formulation generalizes the existing event-triggered cooperative control problems, say, in Cheng and Ugrinovskii (2016); Dimarogonas et al. (2012); Fan et al. (2013); Li et al. (2015); Mu et al. (2015); Seyboth et al. (2013); Zhang et al. (2014); Zhu et al. (2014), in the sense that we achieve not only asymptotical tracking but also disturbance rejection. Second, since our system contains unknown parameters, we need to adopt the distributed internal model approach, which leads to a robust stabilization problem for a more complicated augmented system. Third, our event-triggered mechanism is output-based and distributed in the sense that the event-triggered mechanism of each agent only depends on the output information of its neighbors and itself. Finally, our control law is piecewise constant, which lends itself to a direct implementation in a digital platform.

Due to the space limit, no example is included in this paper. Readers are referred to Liu and Huang (2017a) for a numerical example.

Throughout this paper, we use the following notation: $\mathbb{Z}^+$ denotes the set of all nonnegative integers. For any column vectors $a_i, i = 1, \ldots, s$, we denote $\mathrm{col}(a_1, \ldots, a_s) = [a_1^T, \ldots, a_s^T]^T$. For any matrices $X \in \mathbb{R}^{n\times m}$, we denote $\mathrm{vec}(X) = [X_1^T, \ldots, X_m^T]^T$ where X_i with $i = 1, \ldots, m$ is the ith column of X. The notation $\|x\|$ denotes the Euclidean norm of vector x. The notation $\|A\|$ denotes the induced norm of matrix A by the Euclidean norm. The notation $\otimes$ denotes the Kronecker product of matrices. Denote the base of the natural logarithm by $\mathbf{e}$. Denote the maximum eigenvalue and the minimum eigenvalue of a symmetric real matrix A by $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$, respectively.

2. Problem formulation and preliminaries

Consider the following linear multi-agent systems

$$\begin{aligned}
\dot{z}_i &= A_{1i}(w)z_i + A_{2i}(w)\xi_{1i} + E_{0i}(w)v\\
\dot{\xi}_{si} &= \xi_{(s+1)i}, \quad s = 1, \ldots, r-1\\
\dot{\xi}_{ri} &= A_{3i}(w)z_i + \sum_{s=1}^{r} c_{si}(w)\xi_{si} + E_{ri}(w)v + b_i(w)u_i\\
y_i &= \xi_{1i}, \ i = 1, \ldots, N,
\end{aligned} \tag{1}$$

where $z_i \in \mathbb{R}^{n_i-r}$ and $\xi_i = \mathrm{col}(\xi_{1i}, \ldots, \xi_{ri}) \in \mathbb{R}^r$ are the states, $u_i \in \mathbb{R}$ is the input, $y_i \in \mathbb{R}$ is the output, $w \in \mathbb{R}^{n_w}$ is an unknown parameter vector, $b_i(w) > 0$ for all $w \in \mathbb{R}^{n_w}$, and $v(t) \in \mathbb{R}^{n_v}$ is an exogenous signal representing both reference input and disturbance and is assumed to be generated by the following linear system

$$\dot{v} = Sv \tag{2}$$

where S is some known constant matrix. Let $y_0 = F(w)v \in \mathbb{R}$ be the output of the exosystem (2). Then, the regulated error of each subsystem is defined as $e_i = y_i - y_0$ for $i = 1, \ldots, N$.

For $i = 1, \ldots, N$ and $s = 1, \ldots, r$, let $A_{1i}(w) = A_1 + w_{A_{1i}} \in \mathbb{R}^{(n_i-r)\times(n_i-r)}$, $A_{2i}(w) = A_2 + w_{A_{2i}} \in \mathbb{R}^{(n_i-r)\times 1}$, $A_{3i}(w) = A_3 + w_{A_{3i}} \in \mathbb{R}^{1\times(n_i-r)}$, $E_{0i}(w) = E_0 + w_{E_{0i}} \in \mathbb{R}^{(n_i-r)\times n_v}$, $E_{ri}(w) = E_r + w_{E_{ri}} \in \mathbb{R}^{1\times n_v}$, $F(w) = F + w_F \in \mathbb{R}^{1\times n_v}$, $c_{si}(w) = c_s + w_{c_{si}} \in \mathbb{R}$, $b_i(w) = b + w_{b_i} \in \mathbb{R}$, $w_{A_1} = \mathrm{col}(\mathrm{vec}(w_{A_{11}}), \ldots, \mathrm{vec}(w_{A_{1N}}))$, $w_{A_2} = \mathrm{col}(\mathrm{vec}(w_{A_{21}}), \ldots, \mathrm{vec}(w_{A_{2N}}))$, $w_{A_3} = \mathrm{col}(\mathrm{vec}(w_{A_{31}}), \ldots, \mathrm{vec}(w_{A_{3N}}))$, $w_{E_0} = \mathrm{col}(\mathrm{vec}(w_{E_{01}}), \ldots, \mathrm{vec}(w_{E_{0N}}))$, $w_{E_r} = \mathrm{col}(\mathrm{vec}(w_{E_{r1}}), \ldots, \mathrm{vec}(w_{E_{rN}}))$, $w_{c_s} = \mathrm{col}(w_{c_{s1}}, \ldots, w_{c_{sN}})$, $w_b = \mathrm{col}(w_{b_1}, \ldots, w_{b_N})$, $w = \mathrm{col}(w_{A_1}, w_{A_2}, w_{A_3}, w_{E_0}, w_{E_r}, w_{c_1}, \ldots, w_{c_r}, w_b, w_F^T)$, where $A_1, A_2, A_3, E_0, E_r, F, c_s$ and b denote the nominal values, and w denotes the unknown parameter.

System (1) is called a linear multi-agent system in the normal form and it is said to be minimum phase if the matrix $A_{1i}(w)$ is Hurwitz for all w. Systems (1) and (2) together can be viewed as a multi-agent system with (2) as the leader system, and the N subsystems of (1) as N followers. It is noted that the cooperative robust output regulation problem for the multi-agent system composed of (1) and (2) was studied by the continuous distributed control law in Su and Huang (2014).

As in Su and Huang (2014), given the plant (1) and the exosystem (2), we can define a digraph $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$[1], where $\bar{\mathcal{V}} = \{0, 1\cdots, N\}$ with 0 associated with the leader system and with $i = 1, \ldots, N$ associated with the N followers, respectively, and $\bar{\mathcal{E}} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$ for all $t \geq 0$. For all $t \geq 0$, each $j = 0, 1, \ldots, N$, $i = 1, \ldots, N$, and $i \neq j$, $(j, i) \in \bar{\mathcal{E}}$ if and only if the control $u_i(t)$ can make use of $y_i(t) - y_j(t)$ for feedback control. Let $\bar{\mathcal{N}}_i = \{j, (j, i) \in \bar{\mathcal{E}}\}$ denote the neighbor set of node i.

To define our control law, we recall that the adjacency matrix of the digraph $\bar{\mathcal{G}}$ is a nonnegative matrix $\bar{\mathcal{A}} = [\bar{a}_{ij}] \in \mathbb{R}^{(N+1)\times(N+1)}$ where $\bar{a}_{ii} = 0$, $\bar{a}_{ij} = 1 \Leftrightarrow (j, i) \in \bar{\mathcal{E}}$, and $\bar{a}_{ij} = 0 \Leftrightarrow (j, i) \notin \bar{\mathcal{E}}$ for $i, j = 0, 1, \ldots, N$. Like in Su and Huang (2014), define the virtual output for the ith subsystem as $e_{vi}(t) = \sum_{j=0}^{N} \bar{a}_{ij}(y_i(t) - y_j(t))$ for $i = 1, \ldots, N$. Let $e_0 = 0$, $e = \mathrm{col}(e_1, \ldots, e_N)$, $e_v = \mathrm{col}(e_{v1}, \ldots, e_{vN})$, and $H = [h_{ij}]_{i,j=1}^N$ with $h_{ii} = \sum_{j=0}^{N} \bar{a}_{ij}$ and $h_{ij} = -\bar{a}_{ij}$ for $i \neq j$. It can be verified that $e_v = He$.

For any $k \in \mathbb{Z}^+$ and $i = 1, \ldots, N$, consider the following control law

$$\begin{aligned}
u_i(t) &= F_1\eta_i(t_k^i) + F_2\zeta_i(t_k^i)\\
\dot{\eta}_i(t) &= G_1\eta_i(t) + G_2\eta_i(t_k^i) + G_3\zeta_i(t_k^i)\\
\dot{\zeta}_i(t) &= G_4\zeta_i(t) + G_5 e_{vi}(t_k^i), \ \forall t \in [t_k^i, t_{k+1}^i)
\end{aligned} \tag{3}$$

where $F_1, F_2, G_1, \ldots, G_5$ are some real matrices with proper dimensions, the η_i subsystem is the so-called internal model, the ζ_i subsystem is a dynamic compensator, and t_k^i denotes the triggering time instants of agent i with $t_0^i = 0$ and is generated by the following event-triggered mechanism:

$$t_{k+1}^i = \inf\{t > t_k^i \,|\, h_i(\tilde{e}_{vi}(t), \tilde{\eta}_i(t), \tilde{\zeta}_i(t), e_{vi}(t), \zeta_i(t), t) \geq \delta\} \tag{4}$$

where $h_i(\cdot)$ is some nonlinear function, $\delta > 0$ is a real number and $\tilde{e}_{vi}(t) = e_{vi}(t_k^i) - e_{vi}(t)$, $\tilde{\eta}_i(t) = \eta_i(t_k^i) - \eta_i(t)$, $\tilde{\zeta}_i(t) = \zeta_i(t_k^i) - \zeta_i(t)$ for any $t \in [t_k^i, t_{k+1}^i)$.

[1] See Su & Huang (2014) for a summary of digraph.