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Brief paper

Unknown input functional observability of descriptor systems with neutral and distributed delay effects☆

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ABSTRACT

In this paper a general class of linear systems with time-delays is considered, which includes linear classical systems, linear systems with commensurate delays, neutral systems and singular systems with delays. After given a formal definition of functional backward observability (BO), an easily testable condition is found. The fulfillment of the obtained condition allows for the reconstruction of the trajectories of the system under consideration using the actual and past values of the system output and some of its derivatives. The methodology we follow consists in an iterative algorithm based upon the classical Silverman algorithm used for inversion of linear systems. By using basic module theory we manage to prove that the proposed algorithm is convergent. A direct application of studying functional observability is that a condition can be derived for systems with distributed delays also, we do this as a case of study. The obtained results are illustrated by two examples, one is merely academic but illustrates clearly the kind of systems which the proposed methodology works for and the other is a practical system with distributed delays.

1. Introduction

The description of a variety of practical systems by means of descriptor systems (DS), also called singular, implicit, or differential algebraic systems, has been shown to be useful since several decades ago as it is well explained in Campbell (1980). The observability problem of DS has been studied in papers like Cobb (1984), Hou and Müller (1999) and Yip and Sincovec (1981). The same problem but including unknown inputs has been addressed in Bejarano, Perruquetti, Floquet, and Zheng (2013), Darouach and Boutat-Baddas (2008), Darouach, Zasadzinski, and Hayar (1996), Geerts (1993), Koenig (2005) and Paraskevopoulos, Koumboulis, Tzierakis, and Panagiotakis (1992). Such systems, as many others, may contain time-delay terms in the state, input, and/or system output (see, e.g. Bellen, Guglielmi, and Ruehli (1999), Mounier, Rouchon, and Joachim (1997) and Zheng and Frank (2002)). Some results on the observability problem of dynamical systems with time-delays can be found in Anguelova and Wennberg (2010) Bejarano and Zheng (2014) Fliess and Mounier (1998) Lee and Olbrot (1981) Malek-Zavarei (1982) Marquez-Martinez, Moog, and Velasco-Villa (2002) Olbrot (1981) Przyiuski and Sosnowski (1984) Sename (2001, 2005) Zheng, Barbot, Boutat, Floquet, and Richard (2011).

Descriptor systems with time-delays serve to describe several classes of systems, such as large scale interconnected systems, power systems, chemical processes, etc. For a more extensive revision and recent results on DS with time delays see Gu, Su, Shi, and Chu (2013) and references therein. However, despite the increasing research on problems as solvability, stability and controllability, up to the authors' knowledge, there are only few works dedicated to the study of the observability of descriptor systems with time-delays. For descriptor systems with a single time-delay in the state, a condition guaranteeing the observability of the system is given in Wei (2013). There, observability is interpreted as the reconstruction of the initial condition of the trajectories. However such a condition seems to be quite difficult to check. Two observers for particular types of linear time-delay descriptor systems with unknown inputs can be found in Khadhraoui, Ezzine, Messaoud, and Darouach (2014) and Perdon and Anderlucci (2006). In Rabah and Sklyar (2016), the exact observability of a class of linear neutral systems is tackled. In Bejarano and Zheng

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(2016), the conditions are given for the observability of singular systems with commensurate time-delays. Hence, we may say that the observability problem of descriptor time-delay systems has not completely been solved.

The main motivation of this work arises from the interest of tackling the observability problem of a general class of descriptor linear time-delay systems with neutral terms, namely continuous-time systems whose dynamics is governed by equations as these ones:

$$J\dot{x}(t) = \sum_{i=1}^{k_f} F_i\dot{x}(t-ih) + \sum_{i=1}^{k_a} A_i x(t-ih) + \sum_{i=1}^{k_b} B_i u(t-ih)$$
$$y(t) = \sum_{i=1}^{k_c} C_i x(t-ih) + \sum_{i=1}^{k_d} D_i u(t-ih)$$

where the matrices J, F_i, A_i, B_i, C_i, and D_i are all constant and J is not necessarily a square matrix. It is used to defining the backward shift operator $\delta : x(t) \mapsto x(t-h)$ for rewriting the above dynamic equations as

$$\begin{aligned} J\dot{x}(t) &= F(\delta)\dot{x}(t) + A(\delta)x(t) + B(\delta)u(t) \\ y(t) &= C(\delta)x(t) + D(\delta)u(t) \end{aligned}$$

where, by definition, $F(\delta) = \sum_{i=0}^{k_f} F_i\delta^i$, $A(\delta) = \sum_{i=0}^{k_a} A_i\delta^i$, $C(\delta) = \sum_{i=0}^{k_c} C_i\delta^i$, and $D(\delta) = \sum_{i=0}^{k_d} D_i\delta^i$. The definition $E(\delta) = J - F(\delta)$ yields the following representation of the previous system equations

$$\begin{aligned} E(\delta)\dot{x}(t) &= A(\delta)x(t) + B(\delta)u(t) \\ y(t) &= C(\delta)x(t) + D(\delta)u(t). \end{aligned} \tag{1}$$

In Bejarano and Zheng (2014), the observability of the system (1) was studied considering that $E(\delta)$ is equal to an identity matrix. There sufficient conditions were given by using algebraic tools like the Smith normal form of a matrix. The aim of this paper is to extend those results to the case when the matrix $E(\delta)$ is not an invertible matrix and when only some variables are required to be reconstructed.

The following notation will be used along the paper. The limit from below of a time valued function is denoted as $f(t_-)$. $\mathbb{R}$ is the field of real numbers. $\mathbb{R}[\delta]$ is the polynomial ring over $\mathbb{R}$. I_n is the identity matrix of dimension n by n. Since hereinafter mostly matrices with terms in the polynomial ring $\mathbb{R}[\delta]$ will be needed, instead of using the symbol (δ) behind a matrix to indicate that its elements are within $\mathbb{R}[\delta]$, we prefer to use a more compact notation. As for, we express the polynomial ring as $\mathfrak{R} = \mathbb{R}[\delta]$. Thus, $\mathfrak{R}^{r\times s}$ means the set of all matrices whose dimension is r by s and whose entries are within $\mathfrak{R}$. A square matrix T whose terms belong to $\mathfrak{R}$ is called unimodular (or invertible) if its determinant is a nonzero constant. A matrix $M \in \mathfrak{R}^{r\times s}$ is called left unimodular (invertible) if there exists a matrix $M^+ \in \mathfrak{R}^{s\times r}$ such that $M^+M = I_r$. For a matrix F (with terms in $\mathfrak{R}$), rank F denotes the rank of F over $\mathfrak{R}$. The degree of a polynomial $p(\delta)$ is denoted by $\deg p(\delta)$. For a matrix F with elements in $\mathfrak{R}$, $\deg F$ denotes the greatest degree of all entries of F. By $\mathrm{Inv}_s F$ we denote the set of invariant factors of the matrix F.

2. Formulation of the problem

We consider the class of systems that can be represented by the following equations

$$E\dot{x}(t) = Ax(t) + Bu(t) \tag{2a}$$
$$y(t) = Cx(t) + Du(t) \tag{2b}$$
$$z(t) = \Psi x(t) \tag{2c}$$

where, $x(t) \in \mathbb{R}^n$, $y(t) \in \mathbb{R}^p$, $u(t) \in \mathbb{R}^m$, and $z(t) \in \mathbb{R}^q$. The function $u(t)$ is assumed to be **unknown**, but piecewise continuous. The vector $z(t)$ is attempted to be reconstructed. The dimension of the matrices is as follows, $E \in \mathfrak{R}^{\bar{n}\times n}$, $A \in \mathfrak{R}^{\bar{n}\times n}$, $B \in \mathfrak{R}^{\bar{n}\times m}$, $C \in \mathfrak{R}^{p\times n}$, $D \in \mathfrak{R}^{p\times m}$, $\Psi \in \mathfrak{R}^{q\times n}$ (with $\bar{n} \le n$). Notice that E does not have to be a square matrix. According to the notation defined previously, where $\mathfrak{R} = \mathbb{R}[\delta]$, we use δ as the shift backward operator, i.e., $\delta : x(t) \mapsto x(t-h)$, where h is a non negative real number. The initial condition $\varphi(t)$ is a piecewise continuous function $\varphi : [-kh, 0] \to \mathbb{R}^n$, where k is the greatest degree of all polynomial terms of the matrices involved in system (2), hence $x(t) = \varphi(t)$ on $[-kh, 0]$. For $x(t;\varphi,u)$ we mean the solution of (2a) (assuming it exists) for the initial condition $\varphi = \varphi(t)$ and the input $u = u(t)$; $y(t;\varphi,u)$ and $z(t;\varphi,u)$ are defined analogously by (2b), (2c).

It is assumed that system (2a) admits at least one solution. However, it is worth noticing that the solvability and the observability problems are not related to each other, that is, the system could have more than one solution but still may be observable. That is why, for the observability analysis to be carried out, Eq. (2a) is allowed for having more than one solution. We assume also that any solution of (2a) is piecewise differentiable.

The following definition is taken as the starting point for the observability analysis that will be done further.

Definition 1. *The vector $z(t)$ in (2) is called backward unknown input observable (BUIO) on $[t_1, t_2]$ if for every $\tau \in [t_1, t_2]$ there exist $\bar{t}_1$ and $\bar{t}_2$, with $\bar{t}_1 < \bar{t}_2 \le \tau$, such that, for any input $u(t)$ and any initial condition $\varphi(t)$, the identity $y(t;\varphi,u) = 0$ for all $t \in [\bar{t}_1, \bar{t}_2]$ is true only if $z(\tau_-;\varphi,u) = 0$.*

The above definition of backward observability is related to the final observability given in Lee and Olbrot (1981), and it means that the reconstruction of $x(t)$ depends only on previous and actual values of $y(t)$ and some of its derivatives.

3. Like Silverman–Molinari algorithm

The BUIO will be checked by means of the matrix N_{k^*} which will be defined further. Firstly, let us select a unimodular matrix $S_0 \in \mathfrak{R}^{\bar{n}\times\bar{n}}$ so that we obtain the identity,

$$S_0 \begin{bmatrix} -I & E \end{bmatrix} = \begin{bmatrix} J_0 & R_0 \\ H_0 & 0 \end{bmatrix} \text{ such that } R_0 \in \mathfrak{R}^{\beta_0\times n} \tag{3}$$

where $\beta_0 = \operatorname{rank}(E)$.

Now, let us consider the following matrices, $G_0 = C$, $F_0 = D$. For the k-th step ($k \ge 1$) the matrices Δ_k, N_k and H_k are generated by using the following general procedure. Let us select a unimodular matrix T_k so that

$$T_k \begin{bmatrix} H_{k-1}A & H_{k-1}B \\ G_{k-1} & F_{k-1} \end{bmatrix} = \begin{bmatrix} G_k & F_k \\ \Delta_k & 0 \end{bmatrix} \text{ such that } F_k \in \mathfrak{R}^{\alpha_k\times m} \tag{4}$$

where $\alpha_k = \operatorname{rank}\begin{bmatrix} H_{k-1}B \\ F_{k-1} \end{bmatrix}$. With the matrix Δ_k defined explicitly by Eq. (4), a new matrix, denoted as N_k, is formed with the concatenation of the matrices $\Delta_1, \Delta_2, \ldots, \Delta_k$, that is,

$$N_k = \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \vdots \\ \Delta_k \end{bmatrix}. \tag{5}$$

The construction of the matrix H_k is done by means of the following equation

$$S_k \begin{bmatrix} -I & E \\ 0 & N_k \end{bmatrix} = \begin{bmatrix} J_k & R_k \\ H_k & 0 \end{bmatrix} \text{ such that } R_k \in \mathfrak{R}^{\beta_k\times n} \tag{6}$$

where $\beta_k = \operatorname{rank}\begin{bmatrix} E \\ N_k \end{bmatrix}$. The matrix S_k must be chosen to be unimodular and so that (6) is satisfied.