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Brief paper

Distributed set-membership observers for interconnected multi-rate systems[☆]

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ABSTRACT

This paper presents a new distributed observer for interconnected multi-rate systems. The developed observer belongs to the family of set-membership estimators, and the use of zonotopes is proposed to mathematically describe the sets, a choice motivated by the available mathematical background in operations such as intersections, combinations and linear manipulations. The main features of the proposed distributed observer are (a) the actual state of any subsystem always belongs to the computed sets; (b) the volume of these sets is minimized in real time; (c) under equivalent assumptions, the performance of the observer approaches to that of an analogous distributed Kalman filter.

1. Introduction

The great development of communication technologies over the last two decades has led to flexible configurations that reduce installation and maintenance costs in automation. As a result, new communication architectures find their applications in modern interconnected systems such as power grids, intelligent buildings, or traffic control systems, to give some examples.

In such systems distributed state estimation plays a key role, for having accurate and reliable estimations is crucial both in monitoring and operation tasks. Within this context, Distributed Kalman Filtering (DKF) has shown itself as an efficient and flexible adaptation of the Kalman filter (Khan & Moura, 2008; Song, Zhu, Zhou, & You, 2007), and it has been combined with consensus (Battistelli & Chisci, 2016; Kamal, Ding, Song, Farrell, & Roy-Chowdhury, 2011; Olfati-Saber, 2009), and diffusion strategies (Cattivelli & Sayed, 2010). Consensus filters (Matei & Baras, 2012; Millán, Orihuela, Vivas, & Rubio, 2012; Orihuela, Millán, Vivas, & Rubio, 2013; Stanković, Stanković, & Stipanović, 2009), H_∞ filtering (Shen, Wang, & Hung, 2010) and moving-horizon techniques (Farina, Ferrari-Trecate, & Scattolini, 2010) have also been successfully applied to the distributed estimation problem.

A different approach to state estimation, first introduced by Schweppe (1968) for centralized problems, is the set-membership (SM) paradigm. This method relies on bounded uncertainties/disturbances and leads to estimators that provide, in real time, sets containing the state of the system with guarantees. To characterize these sets, different authors have resorted to variants of ellipses (Durieu, Walter, & Polyak, 2001; El Ghaoui & Calafiore, 2001; Savkin & Petersen, 1998), polyhedrons (Kuntsevich & Lychak, 1985), consistency techniques (Jaulin, 2002), interval analysis (Mazenc & Bernard, 2011; Raïssi, Ramdani, & Candau, 2004), or zonotopes (Alamo, Bravo, & Camacho, 2005; Combastel, 2015).

The latter approach, derived from parallelotopic descriptions (Chisci, Garulli, & Zappa, 1996), is very suitable for distributed implementations. The fact that the zonotopes can be represented in terms of vectors and matrices eases the transmission of information and reduces the mathematical calculations, simple enough to be carried out in distributed embedded systems with limited computation capabilities.

Despite these advantages, the literature concerned with zonotope-based distributed estimation is very scarce, being limited to some preliminary results of the authors of this paper (Garcia, Millan, Orihuela, Ortega, & Rubio, 2015), and also the recent work in Riverso, Rubini, and Ferrari-Trecate (2015). The latter paper is based on the concept of practical robust positive invariance, which allows the authors to guarantee convergence just in

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the absence of disturbances and measurement noises. In addition, the design of the observer gains is centralized, which hinders the implementation in distributed systems.

This paper develops a new distributed estimation observer for interconnected systems composed of a number of interacting subsystems, each of them obeying to different dynamics. The paper contains several contributions. Firstly, the coupled subsystems can be sampled or actuated at different rates, increasing the applicability of the proposed method to interconnected systems of distinct nature. Unlike (Riverso et al., 2015), the observer gains can be computed in a completely distributed fashion and it requires only to perform some simple matrix calculations. Besides providing guaranteed sets for the estimates, the proposed method minimizes the volume of the zonotopes, this meaning a reduction in estimation uncertainties. Finally, inspired by the work (Combastel, 2015), it is shown that the proposed observer yields equivalent results than the distributed Kalman filter developed in Roshany-Yamchi, Cychowski, Negenborn, De Schutter, Delaney, and Connell (2013) for the same kind of interconnected systems.

The paper is organized as follows. Section 2 introduces some notation and preliminaries. Section 3 formally presents the problem. The proposed distributed observer is presented in Section 4. The comparison with the analogous DKF is made in Section 5. Some examples illustrate the performance of the algorithm in Section 6. Finally, conclusions are drawn in Section 7.

2. Notation and preliminaries

Let $R \in \mathbb{R}^{n\times p}$ be a matrix of n rows and p columns. Then, $\|R\|_F = \sqrt{tr(R^TR)}$ is the Frobenius norm of R. Given matrices A, B of appropriate dimensions, operator cat$\{A, B\}$ implies the concatenation of the matrices, that is, cat$\{A, B\} = [A\ B]$.

A zonotope, represented with calligraphic letter $\mathcal{X}$, is defined by its center $c \in \mathbb{R}^n$ and a matrix $E \in \mathbb{R}^{n\times p}$:

$$\mathcal{X} = \langle c, E\rangle = \left\{ c + \sum_{i=1}^{p} \varsigma_i e_i : |\varsigma_i| \le 1 \right\},$$

being $e_i \in \mathbb{R}^n$ the generator vectors (the columns of E). The order of a zonotope is given by the number of generator vectors, its F-radius is the Frobenius norm of E, and its *covariation* is defined as $P_{\mathcal{X}} = EE^T$.

Let $\mathcal{X} = \langle c_x, E_x\rangle$ and $\mathcal{Y} = \langle c_y, E_y\rangle$ be two zonotopes and R a matrix of appropriate dimensions. A linear transformation of a zonotope is given by $R\mathcal{X} = \langle Rc_x, RE_x\rangle$, and the Minkowski sum of two zonotopes is obtained as $\mathcal{X} \oplus \mathcal{Y} = \langle c_x + c_y, \text{cat}\{E_x, E_y\}\rangle$. Given matrix A and vectors $x \in \mathcal{X}$, $w \in \mathcal{W}$, then $y \triangleq Ax + w \in A\mathcal{X} \oplus \mathcal{W}$.

3. Problem formulation

This paper considers linear interconnected subsystems with coupled dynamics. We assume that the input/output rates of each subsystem can be different. This means, different components of the input/output vectors might have different sampling rates. In order to model this, we use the framework in Roshany-Yamchi et al. (2013) and Scattolini and Schiavoni (1995) for multi-rate systems:

$$x_i(k+1) = A_{ii}x_i(k) + B_i\Delta u_i(k) + \sum_{j\in N_i} A_{ij}x_j(k) + D_iw_i(k) \quad (1)$$

$$\varphi_i(k) = \Upsilon_i(k)C_ix_i(k) + \Upsilon_i(k)v_i(k) \quad (2)$$

where $x_i \in \mathbb{R}^{n_i}$ is the state of subsystem i ($i = 1, 2 \dots, b$) with output $\varphi_i \in \mathbb{R}^{m_i}$, $\Delta u_i = \Psi_i(k)\vartheta_i(k) \in \mathbb{R}^{n_{u_i}}$ is a local control signal, and $w_i \in \mathbb{R}^{n_{w_i}}$, $v_i \in \mathbb{R}^{n_{v_i}}$ are disturbances or unmodeled dynamics and noises, respectively. The set N_i is comprised of those subsystems j that dynamically affect subsystem i. As detailed in Roshany-Yamchi et al. (2013) and Scattolini and Schiavoni (1995), the diagonal periodic matrices $\Upsilon_i(k)$, $\Psi_i(k)$ manage the input/output transmission instants, respectively.

Throughout the paper, we consider that disturbances and noises affecting the dynamics always belong to known bounded sets, that is, $w_i(k) \in \mathcal{W}_i$ and $v_i(k) \in \mathcal{V}_i$, for $i = 1, 2 \dots, b$. These sets are modeled as follows:

$$\mathcal{W}_i = \langle 0, Q_i\rangle,\ \ \mathcal{V}_i = \langle 0, R_i\rangle. \quad (3)$$

This paper deals with the design of a set-membership distributed observer for the plant described in (1)–(2) affected by disturbances and noises characterized by (3), operating under any known control strategy $\vartheta_i(k)$, whose design is out of the scope of the paper.

Let us consider also a set of b agents, each one associated to a different subsystem and implementing a local estimator (filter/predictor):

$$\hat{\mathcal{X}}_i(k|k) = f_i(\hat{\mathcal{X}}_i(k|k-1), \varphi_i(k), \mathcal{V}_i), \quad (4)$$

$$\hat{\mathcal{X}}_i(k+1|k) = g_i(\hat{\mathcal{X}}_i(k|k), \hat{\mathcal{X}}_j(k|k)(j \in N_i), \vartheta_i(k), \mathcal{W}_i). \quad (5)$$

The *set-membership distributed observation problem* consists in finding functions $f_i(\cdot)$, $g_i(\cdot)$ such that:

(1) $x_i(k) \in \hat{\mathcal{X}}_i(k|k)$, $\forall k, i$;
(2) $x_i(k+1) \in \hat{\mathcal{X}}_i(k+1|k)$, $\forall k, i$;
(3) The F-radius of $\hat{\mathcal{X}}_i(k|k)$ is minimized, $\forall k, i$.

4. Distributed set-membership observer

Consider that at time instant k each agent has an initial prediction $\hat{\mathcal{X}}_i(k|k-1) = \langle c_i(k|k-1), E_i(k|k-1)\rangle$, and measures its local output $\varphi_i(k)$. In the filtering step each agent computes the set $\hat{\mathcal{X}}_i(k|k) = \langle c_i(k|k), E_i(k|k)\rangle$ as follows:

$$c_i(k|k) = c_i(k|k-1) + L_i(k)(\varphi_i(k) - \Upsilon_i(k)C_ic_i(k|k-1)), \quad (6)$$

$$E_i(k|k) = [M_i(k)E_i(k|k-1) \quad - L_i(k)\Upsilon_i(k)R_i], \quad (7)$$

where $L_i(k)$ is the observer gain to be designed and $M_i(k) \triangleq I - L_i(k)\Upsilon_i(k)C_i$.

Now, every agent communicates to its neighbors $j : i \in N_j$ its filtered set $\hat{\mathcal{X}}_i(k|k)$, and receives $\hat{\mathcal{X}}_j(k|k)$ from all $j \in N_i$. With this information, each agent computes its prediction zonotopes as follows:

$$c_i(k+1|k) = A_{ii}c_i(k|k) + \sum_{j\in N_i} A_{ij}c_j(k|k) + B_i\Psi_i(k)\vartheta_i(k), \quad (8)$$

$$E_i(k+1|k) = \left[A_{ii}E_i(k|k)\ \ \underset{j\in N_i}{\text{cat}}\{A_{ij}E_j(k|k)\}\ \ D_iQ_i \right]. \quad (9)$$

Observe that the center of the zonotope has the same dynamical equation, both in the filtering and prediction steps, as the distributed Kalman filter proposed in Roshany-Yamchi et al. (2013).

Note that the order of the zonotopes grows with both operations (7)–(9), increasing the computational requirements. It is a common practice in zonotope-based set-membership observers to iteratively reduce the order of the zonotopes such that it remains upper-bounded. This operation is defined in Combastel (2015). When reduction operations take place, prediction sets in (9) have to be computed using the reduced-order zonotopes.