Automatica 85 (2017) 227–233

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief Paper

A data-driven approach to robust control of multivariable systems by convex optimization[☆]

Alireza Karimi, Christoph Kammer

Laboratoire d'Automatique, École Polytechnique Fédérale de Lausanne (EPFL), CH-1015 Lausanne, Switzerland

ARTICLE INFO

Article history:
Received 8 September 2016
Received in revised form 4 May 2017
Accepted 6 June 2017

Keywords:
Data-driven control
Robust control
Convex optimization

ABSTRACT

The frequency-domain data of a multivariable system in different operating points is used to design a robust controller with respect to the measurement noise and multimodel uncertainty. The controller is fully parameterized in terms of matrix polynomial functions and can be formulated as a centralized, decentralized or distributed controller. All standard performance specifications like H_2, H_∞ and loop shaping are considered in a unified framework for continuous- and discrete-time systems. The control problem is formulated as a convex–concave optimization problem and then convexified by linearization of the concave part around an initial controller. The performance criterion converges monotonically to a local optimum or a saddle point in an iterative algorithm. The effectiveness of the method is compared with fixed-structure controller design methods based on non-smooth optimization via multiple simulation examples.

1. Introduction

Recent developments in the fields of numerical optimization, computer and sensor technology have led to a significant reduction of the computational time of optimization algorithms and have increased the availability of large amounts of measured data during a system's operation. These progresses make computationally demanding data-driven control design approaches an interesting alternative to the classical model-based control problems. In these approaches, the controller parameters are directly computed by minimizing a control criterion which is a function of measured data. Therefore, a parametric model of the plant is not required and there are no unmodeled dynamics. The only source of uncertainty is the measurement noise, whose influence can be reduced significantly if the amount of measurement data is large.

Frequency-domain data is used in the classical loop-shaping methods for computing simple lead–lag or PID controllers for SISO stable plants. The Quantitative Feedback Theory (QFT) uses also the frequency response of the plant model to compute robust controllers (Horowitz, 1993). In these approaches the controller parameters are tuned manually using graphical methods. New optimization-based algorithms have also been proposed recently (Mercader, Åström, Baños, & Hägglund, 2016). The set of all stabilizing PID controllers with H_∞ performance is obtained using only the frequency-domain data in Keel and Bhattacharyya (2008). This method is extended to design of fixed-order linearly parameterized controllers in Parastvand and Khosrowjerdi (2015, 2016). The frequency response data are used in Hoogendijk, Den Hamer, Angelis, van de Molengraft, and Steinbuch (2010) to compute the frequency response of a controller that achieves a desired closed-loop pole location. A data-driven synthesis methodology for fixed structure controller design problems with H_∞ performance is presented in Den Hamer, Weiland, and Steinbuch (2009). This method uses the Q parameterization in the frequency domain and solves a non-convex optimization problem to find a local optimum. Another frequency-domain approach is presented in Khadraoui, Nounou, Nounou, Datta, and Bhattacharyya (2013) to design reduced order controllers with guaranteed bounded error on the difference between the desired and achieved magnitude of sensitivity functions. This approach also uses a non-convex optimization method.

Another direction for robust controller design based on frequency-domain data is the use of convex optimization methods. A linear programming approach is used to compute linearly parameterized (LP) controllers for SISO systems with specifications in gain and phase margin as well as the desired closed-loop bandwidth in Karimi, Kunze, and Longchamp (2007); Saeki (2014). A convex optimization approach is used to design LP controllers with loop shaping and H_∞ performance in Karimi and Galdos (2010). This method is extended to MIMO systems for computing decoupling LP-MIMO controllers in Galdos, Karimi, and Longchamp

[☆] The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Denis Arzelier under the direction of Editor Richard Middleton.

E-mail addresses: alireza.karimi@epfl.ch (A. Karimi), christoph.kammer@epfl.ch (C. Kammer).

http://dx.doi.org/10.1016/j.automatica.2017.07.063
0005-1098/

(2010). Recently, the necessary and sufficient conditions for the existence of data-driven H_∞ controllers for SISO systems have been proposed in Karimi, Nicoletti, and Zhu (2016).

The use of the frequency response for computing SISO-PID controllers by convex optimization is proposed in Hast, Aström, Bernhardsson, and Boyd (2013). This method uses the same type of linearization of the constraints as in Karimi and Galdos (2010) but interprets it as a convex–concave approximation technique. An extension of Hast et al. (2013) for the design of MIMO-PID controllers by linearization of quadratic matrix inequalities is proposed in Boyd, Hast, and Åström (2016) for stable plants. A similar approach, with the same type of linearization, is used in Saeki, Ogawa, and Wada (2010) for designing LP-MIMO controllers (which includes PID controllers as a special case). This approach is not limited to stable plants and includes the conditions for the stability of the closed-loop system.

In this paper, a new data-driven controller design approach is proposed based on the frequency response of multivariable systems and convex optimization. Contrarily to the existing results in Boyd et al. (2016); Galdos et al. (2010); Saeki et al. (2010), the controller is fully parameterized and the design is not restricted to LP or PID controllers. The other contribution is that the control specification is not limited to H_∞ performance. The H_2, H_∞ and mixed H_2/H_∞ control problem as well as loop shaping in two- and infinity-norm are presented in a unified framework for systems with multimodel uncertainty. A new closed-loop stability proof based on the Nyquist stability criterion is also given.

It should be mentioned that the problem is convexified using the same type of approximation as the one used in Boyd et al. (2016); Saeki et al. (2010). Therefore, like other fixed-structure controller design methods (model-based or data-driven), the results are local and depend on the initialization of the algorithm.

2. Preliminaries

The system to be controlled is a Linear Time-Invariant Multi-Input Multi-Output (LTI-MIMO) system represented by a multivariable frequency response model $G(e^{j\omega}) \in \mathbb{C}^{n\times m}$, where n is the number of outputs and m the number of inputs. The frequency response model can be identified using the Fourier analysis method from m sets of input/output sampled data as Pintelon & Schoukens (2001):

$$G(e^{j\omega}) = \left[\sum_{k=0}^{N-1} y(k)e^{-j\omega T_s k}\right]\left[\sum_{k=0}^{N-1} u(k)e^{-j\omega T_s k}\right]^{-1} \tag{1}$$

where N is the number of data points for each experiment, $u(k) \in \mathbb{R}^{m\times m}$ includes the inputs at instant k, $y(k) \in \mathbb{R}^{n\times m}$ the outputs at instant k and T_s is the sampling period. Note that at least m different experiments are needed to extract G from the data (each column of $u(k)$ and $y(k)$ represents respectively the input and the output data from one experiment). We assume that $G(e^{j\omega})$ is bounded in all frequencies except for a set B_g including a finite number of frequencies that correspond to the poles of G on the unit circle. Since the frequency function $G(e^{j\omega})$ is periodic, we consider:

$$\omega \in \Omega_g = \left\{\omega \,\middle|\, -\frac{\pi}{T_s} \le \omega \le \frac{\pi}{T_s}\right\} \setminus B_g \tag{2}$$

A fixed-structure matrix transfer function controller is considered. The controller is defined as $K = XY^{-1}$, where X and Y are polynomial matrices in s for continuous-time or in z for discrete-time controller design. This controller structure, therefore, can be used for both continuous-time or discrete-time controllers. The matrix X has the following structure:

$$X = \begin{bmatrix} X_{11} & \dots & X_{1n} \\ \vdots & \ddots & \vdots \\ X_{m1} & \dots & X_{mn} \end{bmatrix} \circ F_x \tag{3}$$

where X and F_x are $m \times n$ polynomial matrices and $\circ$ denotes the element by element multiplication of matrices. The matrix F_x represents the fixed known terms in the controller that are designed to have specific performance, e.g. based on the internal model principle. For discrete-time controllers, we have:

$$X(z) = X_p z^p + \dots + X_1 z + X_0 \tag{4}$$

where $X_i \in \mathbb{R}^{m\times n}$ for $i = 0, \dots, p$ contain the controller parameters. In the same way the matrix polynomial Y can be defined as:

$$Y = \begin{bmatrix} Y_{11} & \dots & Y_{1n} \\ \vdots & \ddots & \vdots \\ Y_{n1} & \dots & Y_{nn} \end{bmatrix} \circ F_y \tag{5}$$

where Y and F_y are $n \times n$ polynomial matrices. The matrix F_y represents the fixed terms of the controller, e.g. integrators or the denominator of other disturbance models. The set of frequencies of all roots of the determinant of F_y on the stability boundary (imaginary axis for continuous-time controllers or the unit circle for the discrete-time case) is denoted by B_y.

The matrix Y for discrete-time case can be written as:

$$Y(z) = Iz^p + \dots + Y_1 z + Y_0 \tag{6}$$

where $Y_i \in \mathbb{R}^{n\times n}$ for $i = 0, \dots, p-1$ contain the controller parameters. In order to obtain low-order controllers, a diagonal structure can be considered for Y that makes its inversion and implementation easier too. Note that $Y(e^{j\omega})$ should be invertible for all $\omega \in \Omega = \Omega_g \setminus B_y$.

The control structure defined in this section is very general and covers centralized, decentralized and distributed control structures. The well-known PID control structure for MIMO systems is also a special case of this structure.

3. Control performance

It is shown in this section that classical control performance constraints can be transformed to constraints on the spectral norm of the system and in general can be reformulated as:

$$F^*F - P^*P < \gamma I \tag{7}$$

where $F \in \mathbb{C}^{n\times n}$ and $P \in \mathbb{C}^{n\times n}$ are linear in the optimization variables and $(\cdot)^*$ denotes the complex conjugate transpose. This type of constraint is called convex–concave constraint and can be convexified using the Taylor expansion of P^*P around $P_c \in \mathbb{C}^{n\times n}$ which is an arbitrary known matrix (Dinh, Gumussoy, Michiels, & Diehl, 2012):

$$P^*P \approx P_c^*P_c + (P - P_c)^*P_c + P_c^*(P - P_c). \tag{8}$$

It is easy to show that the left hand side term is always greater than or equal to the right hand side term, i.e. :

$$P^*P \ge P^*P_c + P_c^*P - P_c^*P_c. \tag{9}$$

This can be obtained easily by development of the inequality $(P - P_c)^*(P - P_c) \ge 0$.

3.1. H_∞ performance

Constraints on the infinity-norm of any weighted sensitivity function can be considered. For example, consider the mixed sensitivity problem:

$$\min_K \left\| \begin{matrix} W_1 S \\ W_2 KS \end{matrix} \right\|_\infty \tag{10}$$