Automatica 85 (2017) 248–255

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Input-to-state stability of integral-based event-triggered control for linear plants[☆]

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ARTICLE INFO

Article history:
Received 6 August 2016
Received in revised form 14 June 2017
Accepted 5 July 2017

Keywords:
Input-to-state stability
Integral-based event-triggered control
Output feedback
Linear system

ABSTRACT

Integral-based event-triggered control utilizes the integrals of system states to construct the event conditions. By this means, the integral-based event-triggered control can relax the requirements on the derivative of the Lyapunov function, and then, may yield better sampling performance. In this paper, the effects of bounded disturbances on the integral-based event-triggered control systems are studied. Results on input-to-state stability with respect to the external disturbances are presented for linear plants with observer-based output feedbacks. An estimation on the upper bound of the input-to-state stability gain is given analytically. Then it is shown that for integral-based event-triggered control, a pre-specified upper bound of inter-event times is necessary to ensure the input-to-state stability. Furthermore, it is proved that increasing the pre-specified upper bound can only enlarge the input-to-state stability gain but cannot destroy the input-to-state stability. Moreover, a positive lower bound of inter-event times is provided to exclude Zeno behaviors. Finally, numerical examples are given to illustrate the efficiency and the feasibility of the proposed results.

1. Introduction

Traditionally, most digital control applications are implemented in a time-triggered manner where the control tasks are executed periodically (Laila, Nešić, & Astolfi, 2006). The time-triggered control (also known as periodic sampling), however, often leads to inefficient utilization of limited resources. As an alternative, event-triggered control has attracted more and more attention due to the advantages of saving communication and computation resources. In this control scheme, the control task execution is determined by a triggering condition, which is a designed rule depending on the current state of the plant (see Lunze & Lehmann, 2010; Tabuada, 2007, and the references therein). Therefore the event-triggered control is able to execute the control tasks when necessary. There are a number of literatures on the topic of event-triggered control, such as, the event-triggered control for state-feedback asymptotic stabilization (Tabuada, 2007; Yu & Hao, 2016a), for disturbance rejections (Donkers & Heemels, 2012), and for output feedbacks (Chen & Hao, 2013; Tallapragada & Chopra, 2012).

Note that, in all the aforementioned works, the designed triggering conditions require the Lyapunov function of the closed-loop system to decrease all the time, which is not necessary from the stability point of view. Some attempts (Dolk, Borgers, & Heemels, 2017; Girard, 2014; Postoyan, Tabuada, Nešić, & Anta, 2015; Wang & Lemmon, 2011) have been made to relax this requirement. Recently, Mousavi, Ghodrat, and Marquez (2015) proposed a new integral-based event-triggered control, which can be regarded as a special form of the scheme in Girard (2014), to deal with this issue as well. Literally, the integral-based event-triggered control is to utilize the integrals of the measurement signals to construct the triggering conditions. By this means, this control scheme can allow the Lyapunov function to be non-decrescent between two consecutive triggering instants. Consequently, as shown in Mousavi et al. (2015), the integral-based event-triggered control is more likely to yield better sampling performance than the scheme in Tabuada (2007). There are few works on the integral-based event-triggered control with disturbances. Dolk et al. (2017) proposed a new dynamic event-triggered control, extended from Girard (2014), to ensure $\mathcal{L}_p$-gain performance with $p \in [1, \infty)$. However, to our knowledge, the input-to-state stability (Sontag, 1989) of integral-based event-triggered control with respect to disturbances was not studied sufficiently in the previous works, which motivates this study.

[☆] This work was supported by National Nature Science Foundation of China under Grants 61573036 and 61174057, Beijing Natural Science Foundation under Grant 4112034, the Academic Excellence Foundation of BUAA for Ph.D. Students. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Sophie Tarbouriech under the direction of Editor Richard Middleton.

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http://dx.doi.org/10.1016/j.automatica.2017.07.068

A key problem of event-triggered control is to guarantee the existence of a positive lower bound on the inter-event times which denote the time intervals between two consecutive triggering instants. This property can exclude Zeno behaviors and prevent the control tasks being executed at an arbitrarily large frequency. This problem becomes particularly non-trivial for the systems with output feedbacks and/or external disturbances. The scheme in Tabuada (2007) was proved to be non-robust to external disturbances and/or output feedbacks since it would perform Zeno behaviors (see Borgers & Heemels, 2014). Therefore, some improvements have been made to ensure the positive minimum inter-event time. Either the time-regularized event-triggered control (see, e.g., Abdelrahim, Postoyan, Daafouz, & Nešić, 2017; Selivanov & Fridman, 2016; Tallapragada & Chopra, 2012), where the triggering condition is detected only after a specific time duration since the last sampling occurred, or the periodic event-triggered control (see, e.g., Heemels & Donkers, 2013; Yu & Hao, 2016b), where the triggering condition is checked at fixed periodic sampling time instants, was employed. A defect of the two schemes is that the corresponding event-triggered control may degenerate approximately to the time-triggered one in the presence of disturbances (see Dolk et al., 2017).

Based on the observations above, this paper studies the integral-based event-triggered control systems with the output feedbacks and the external disturbances. The main contributions are summarized as follows.

First, results on input-to-state stability for integral-based event-triggered control systems are presented. An estimation on the upper bound of the input-to-state stability gain (ISS-gain) is given analytically. The results are not trivial since some existing technical tools may be invalid to deal with this issue. On the one hand, conventionally, the input-to-state stability is often proved by using the derivative property of the ISS-Lyapunov function (see, e.g., Sontag & Wang, 1995). Since the integral-based triggering condition can only provide some inequalities based on integral signals, the derivative property of the ISS-Lyapunov function would not be ensured all the time. One the other hand, for the integral-based event-triggered control, Barbalat's Lemma (Khalil, 2002) is a powerful tool to prove the asymptotic stability in the absence of disturbances (see,e.g., Ghodrat & Marquez, 2015). However, the disturbances would yield the nonexistence of the limitations for the involved integral signals. As a result, Barbalat's Lemma is invalid for the systems with external disturbances.

Second, a pre-specified upper bound of inter-event times is introduced to improve the reliability of the proposed schemes. An example is provided to show the necessity of introducing this upper bound. It is further proved that the stability can be preserved with arbitrarily large pre-specified upper bound, and thus, there is almost no restriction on the selection of the upper bound from the stability point of view.

Third, a positive lower bound of inter-event times is provided to exclude Zeno behaviors. Unlike the time-regularized manner, the proposed integral-based triggering condition can automatically guarantee a positive minimum inter-event time. Hence, the sampling time sequence would not degenerate to the periodic one even in the presence of disturbances.

2. Preliminaries

The set of real numbers is denoted by $\mathbb{R}$. The set of nonnegative integers is denoted by $\mathbb{Z}_{\geq 0}$. The transpose of a matrix A is denoted by A^{T}. 2-norm of a vector x is denoted by $\|x\|$, and the matrix induced 2-norm of a matrix A is denoted by $\|A\| := \sqrt{\max \lambda\{A^{\mathrm{T}}A\}}$, where $\lambda\{A^{\mathrm{T}}A\}$ denotes all the eigenvalues of $A^{\mathrm{T}}A$. The maximum and minimum eigenvalues of a symmetric matrix P are denoted by $\lambda_M(P)$ and $\lambda_m(P)$, respectively. An asterisk $(*)$ in matrix is used to represent symmetry block. A positive (negative) definite matrix $P \in \mathbb{R}^{n\times n}$ is denoted by $P > 0$ $(P < 0)$. The supremum norm of $f(t)$ over $[t_1, t_2]$ is denoted by $\|f\|_{[t_1,t_2]} := \sup_{t_1 \le t \le t_2} \|f(t)\|$ and the space of all bounded signals of dimension n is represented by $\mathcal{L}_\infty^n$.

Consider the following linear time-invariant (LTI) system with bounded external input $w(t) \in \mathcal{L}_\infty^r$,

$$\dot{x}_p(t) = Ax_p(t) + Dw(t), \tag{1}$$

where $x_p(t) \in \mathbb{R}^n$ is the state vector. A and D are constant matrix with appropriate dimensions. Then the LTI system (1) is said to be (exponential) input-to-state stable with respect to the input $w(t)$ if there exist constants $c, \theta, \kappa > 0$ such that the solution of (1) satisfies, for all $w(t) \in \mathcal{L}_\infty^r$ and $x_p(t_0) \in \mathbb{R}^n$,

$$\|x_p(t)\| \le ce^{-\theta(t-t_0)} \|x_p(t_0)\| + \kappa \|w\|_{[t_0,t]} \tag{2}$$

for all $t \ge t_0$, where t_0 denotes the initial instant. The constant κ is referred to as the (linear) ISS-gain. As is well known, the LTI system (1) is input-to-state stable with respect to $w(t)$ if and only if the matrix A is Hurwitz.

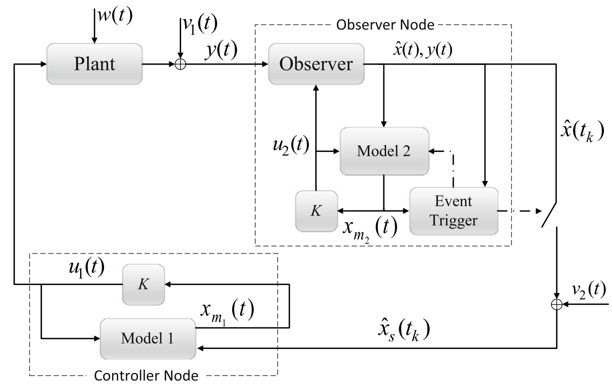

Fig. 1. Configuration of observer-based event-triggered control systems.

3. Problem formulation

For clarity, the observer-based event-triggered output feedback control system is illustrated in Fig. 1 at first, and the specific details will be given later.

Consider the following LTI plant

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu_1(t) + Dw(t), \\ y(t) &= Cx(t) + v_1(t), x(t_0) = x_0, \end{aligned} \tag{3}$$

where $x(t) \in \mathbb{R}^n$ and $x_0 \in \mathbb{R}^n$ are the plant's state vector and the initial condition, respectively. $u_1(t) \in \mathbb{R}^m$ is the control input to the plant and $y(t) \in \mathbb{R}^q$ is the output signal. $w(t) \in \mathcal{L}_\infty^r$ and $v_1(t) \in \mathcal{L}_\infty^q$ are the unknown bounded disturbances. A, B, C, D are constant matrices with appropriate dimensions. (A, B) is stabilizable and (A, C) is detectable.

Then the observer is defined as

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu_2(t) + F(C\hat{x}(t) - y(t)), \hat{x}(t_0) = \hat{x}_0, \tag{4}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the observer state and $u_2(t) \in \mathbb{R}^m$ is the input signal to the observer. $\hat{x}_0 \in \mathbb{R}^n$ is the initial state of the observer. $F \in \mathbb{R}^{n\times q}$ is the observer gain matrix such that $A + FC$ is Hurwitz.

Both the controllers in the two nodes are implemented in a model-based manner (see Montestruque & Antsaklis, 2003, for more details on model-based control) i.e.,

$$u_i(t) = Kx_{m_i}(t), t \in [t_k, t_{k+1}), i \in \{1, 2\}, \tag{5}$$