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Brief paper

An optimal fault detection approach for piecewise affine systems via diagnostic observers☆

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ABSTRACT

In this paper, the design issues of diagnostic observer-based fault detection (FD) for piecewise affine systems are investigated. To be specific, the diagnostic observer-based FD approaches are developed to satisfy the specified disturbance attenuation and optimize the fault detectability at the same time. Considering the fact that the measurements are generally corrupted with noises or disturbances, the proposed approaches are mainly dedicated to the case that the process and its FD system are non-synchronous in transition from one region to another. Moreover, aiming at improving the FD performance, a mode observer is integrated to relax the design condition for the residual generator. An example is given in the end to show the effectiveness of the proposed approaches.

1. Introduction

Driven by the considerably increasing demands for system reliability, studies on fault detection and fault-tolerant control have attracted remarkable attention from both the application and research domains (Basin & Pinsky, 1998; Chen & Patton, 1999; Ding, 2013; Zhou & Ren, 2001). Over the past decades, the framework for linear observer-based fault detection (FD) approaches has been well established. Most recently, significant research effort has been made to the observation/filtering issues for nonlinear systems (Basin & Calderon-Alvarez, 2009; Hernandez-Gonzalez & Basin, 2014), and to the application of the observer theory to deal with nonlinear FD issues (Li, Lam, Gao, & Xiong, 2016b; Wang, Wang, & Shi, 2009a; Yin, Gao, & Kaynak, 2016; Zhao, Lam, & Gao, 2009). For instance, sliding mode observer-based FD (Yan & Edwards, 2007), adaptive observer-based FD (Zhang, Polycarpou, & Parisini, 2010), fuzzy technique based FD (Li, Ding, Qiu, Yang,& Zhang, 2016a; Zhang, Jiang, & Shi, 2012) and diagnostic observer-based FD (Chadli, Abdo, & Ding, 2013) have been reported in the literature.

Piecewise affine systems have become an important research subject in control community, since considerable amount of practical control systems and chaotic circuits are embedded with piecewise components (Johansson, 2003). On the other hand, it has been shown in (Heemels, de. Schutter, & Bemporad, 2001; Tabatabaeipour, Ravn, Izadi-Zamanabadi, & Bak, 2010; Xu & Xie, 2014) that the piecewise affine dynamic modeling is an effective way to approximate hybrid systems, mixed logical dynamical systems, switched systems and general nonlinear systems with arbitrary accuracy. A wide range of nonlinear elements in mechanical and electrical systems, such as relay action, saturation, stiction and backlash, can be well described by piecewise affine approximation (Hu, Lin, & Chen, 2002). Most recently, the characterization of controllability and reachability for piecewise affine systems have been studied (Alur & Pappas, 2004; Xu & Xie, 2014). Then, enormous research efforts have been dedicated to the analysis and synthesis for the control and estimation of piecewise linear systems via piecewise quadratic Lyapunov functions (Feng, 2006; Wang, Wei, & Feng, 2009b). In the same period, the stabilization and filtering issues for piecewise affine systems have been studied in Feng (2005), Ferrari-Trecate, Cuzzola, Mignone, and Morari (2002), Imura (2004) and Xu and Xie (2014). In recent

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years, intensive attention has been attracted to the FD issues for piecewise systems, hybrid systems and mixed logical dynamical systems (Gholami, Schioler, & Bak, 2011; Morales-Morales, Morales-Rodriguez, Adam-Medina, & Vela-Vald, 2015; Qiu, Feng, Gao, & Fan, 2010; Tabatabaeipour & Bak, 2014; Tabatabaeipour et al., 2010; Xu, Lum, & Loh, 2007).

Proceedings from the above observations, the main objective of this paper is to investigate the weighted FD approach for piecewise affine processes via diagnostic observers. To this end, the diagnostic observer-based FD schemes are first proposed to optimize the fault detectability by weighting the residual signal of each local region with different weighting matrices. Then, a mode observer is developed to relax the design condition for the residual generator. The proposed approach can be applied to handle the case that the diagnostic observer and the process are not operating in the same region. Moreover, comparing with existing results on FD approaches for piecewise affine systems, the main advantages lie in (i) optimizing fault detection performance by weighting the residual signal of each region individually, and (ii) simplifying the switch rule for diagnostic observers via a mode observer.

The paper is organized as follows. In Section 2, the preliminaries and the problem to be addressed are given. The design schemes of non-synchronized diagnostic observer-based FD approaches are studied in Section 3. A mode observer is proposed in Section 4. In Section 5, an example is used to demonstrate the effectiveness of the proposed approaches. In Section 6, conclusions and future works are given.

Notations. In this paper, standard notations are adopted. $\|\omega(k)\|$ represents the Euclidean norm of the vector $\omega(k)$. $\mathcal{L}_2$-norm of $\omega(k)$ is defined as $\|\omega\|_2 = \left(\sum_{k=0}^{\infty}\|\omega(k)\|^2\right)^{1/2}$. $\mathcal{L}_{2,[0,\tau]}$-norm of $\omega(k)$ is defined as $\|\omega_\tau\|_2 = \left(\sum_{k=0}^{\tau}\|\omega(k)\|^2\right)^{1/2}$. In a symmetric matrix, $\star$ indicates the symmetric elements. M^+ denotes the left inverse of M, that is, $M^+M = \mathrm{I}$.

2. Preliminaries and problem formulation

Consider the following piecewise affine discrete-time system

$$\begin{aligned} &x(k+1) = A_i x(k) + a_i + B_i u(k) + D_i \omega(k)\\ &y(k) = Cx(k) + G\omega(k), \quad x(k) \in \Omega_i, \ i \in \Theta \end{aligned} \tag{1}$$

where $u(k) \in \mathcal{R}^{k_u}$, $y(k) \in \mathcal{R}^{k_y}$, $x(k) \in \mathcal{R}^{k_x}$ represent the input, output and state of the process, respectively; $\omega(k) \in \mathcal{R}^{k_\omega}$ denotes the disturbance which is assumed to be $\mathcal{L}_2$-bounded; A_i, B_i, D_i, C, G denote the local system matrices that are of appropriate dimensions; a_i represents the additional offset; $\Theta = \{1, 2, \ldots, \nu\}$ denotes the set of all region indices; ν represents the number of the regions for the piecewise affine systems; $\{\Omega_i\}_{i\in\Theta} \in \mathcal{R}^{k_x}$ stands for the partition of the state space into a set of closed polyhedral regions.

It is noted that the piecewise affine system is generally partitioned based on the measurement space. However, the noise or disturbance will lead to the deviation of the measurable signals. To deal with this issue, the state $x(k)$, instead of $y(k)$, is adopted for region partition. In this paper, $\mathcal{S}$ is used to represent all possible transitions from one region to another region

$$\mathcal{S} = \{(i, l) | x(k) \in \Omega_i, \ x(k+1) \in \Omega_l\} \tag{2}$$

which can be determined by the reachability analysis (Ferrari-Trecate et al., 2002). It is noteworthy that $l = i$ indicates that the trajectories of $x(k)$ stay in the same region Ω_i, and $l \neq i$ indicates that the trajectories of $x(k)$ transit from region Ω_i to region Ω_l.

Note that for each region that includes the origin, we have $a_i = 0$. Thus, the regions Θ can be further divided into two classes $\Theta = \Theta_0 \bigcup \Theta_1$, where Θ_0 represents the set of region indices that contain the origin, and Θ_1 denotes the set of the region that do not contain the origin. That is, for all $i \in \Theta_1$, $a_i \neq 0$.

Remark 1. The piecewise systems described by (1) are in fact affine dynamic systems rather than linear systems. By invoking the additional offset term a_i, (1) can be used to describe/approximate more general class of nonlinear systems.

In general, an observer-based FD system is composed by an observer-based residual generator, a residual evaluation function, and a decision making unit with a threshold. It is important to mention that the state variables of the process are not always fully measurable in practice. To deal with this issue, the state estimation $\hat{x}(k)$ is used instead to determine the region in which the residual generator is evolving. As a consequence, it is not always in the situation that the residual generator and the plant synchronize in transition from one region to another, in particular for the initial stage of the residual generator. For our purpose, we adopt the following diagnostic observer as the residual generator:

$$\begin{aligned} &z(k+1) = S_j z(k) + N_j a_j + M_j u(k) + L_j y(k)\\ &\hat{x}(k) = z(k) - K_j y(k)\\ &\hat{y}(k) = C\hat{x}(k)\\ &r(k) = W_j(y(k) - \hat{y}(k)), \quad \hat{x}(k) \in \Omega_j, j \in \Theta \end{aligned} \tag{3}$$

where $S_j, M_j, N_j, L_j, K_j, j \in \Theta$ represent the matrices of the diagnostic observer; and $W_j, j \in \Theta$ denote the weighting matrices. $z(k)$ denotes the auxiliary state of the observer; $\hat{x}(k)$ represents the estimation of $x(k)$; $\hat{y}(k)$ indicates the estimation of $y(k)$; and $r(k)$ denotes the residual signal.

The residual evaluator $J(r)$ is generally a positive definite function of the residual signal. By setting the threshold J_{th} as the maximal value of $J(r)$ in the fault-free case, the following detection logic leads to reliable observer-based FD system

$$\begin{cases} J(r) > J_{\mathrm{th}} \Longrightarrow \text{faulty}\\ J(r) \leq J_{\mathrm{th}} \Longrightarrow \text{fault-free.} \end{cases} \tag{4}$$

It is worth mentioning that the fault detectability is one of the key indicators for evaluating FD performance. Denote the set of all detectable faults by

$$\mathcal{F}_{(r,J(r),J_{th})} = \{f | f \neq 0 \text{ and } J(r) > J_{th}\} \tag{5}$$

where f represents the fault vector. It is evident that the domain of $\mathcal{F}_{(r,J(r),J_{th})}$ characterizes the fault detectability of the FD systems. Thus, the main objective of this paper is to develop the diagnostic observer-based FD systems for piecewise affine processes (1) such that the fault detectability is optimized while the following robustness specification is guaranteed

$$\|r_\tau\|_2^2 < \|\omega_\tau\|_2^2 + \|u_\tau\|_2^2 + \gamma(x(0), \hat{x}(0)) \tag{6}$$

where $\gamma(x(0), \hat{x}(0))$ is the positive definite function with respect to the initial conditions. Moreover, a mode observer is designed to deal with the case that the dynamics of the process is non-synchronized with the dynamics of the FD system in transition from one region to another.

3. Non-synchronized fault detection system design

In this section, the design approach of non-synchronized fault detection system is proposed for piecewise affine systems.