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Brief paper

Quantum linear coherent controller synthesis: A linear fractional representation approach☆

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ABSTRACT

This paper is concerned with a linear fractional representation approach to the synthesis of linear coherent quantum controllers for a given linear quantum plant. The plant and controller represent open quantum harmonic oscillators and are modelled by linear quantum stochastic differential equations. The feedback interconnections between the plant and the controller are assumed to be established through quantum bosonic fields. In this framework, conditions for the stabilization of a given linear quantum plant via linear coherent quantum feedback are addressed using a stable factorization approach. The class of all stabilizing quantum controllers is parameterized in the frequency domain. Coherent quantum weighted $\mathscr{H}_2$ and $\mathscr{H}_\infty$ control problems for linear quantum systems are formulated in the frequency domain. Finally, a projected gradient descent scheme is outlined for the coherent quantum weighted $\mathscr{H}_2$ control problem.

1. Introduction

The main motivation for coherent quantum feedback control is based on avoiding the loss of quantum information in conversion to classical signals which occurs during measurement (Landau & Lifshitz, 1975; Lloyd, 2000). This approach builds on the technique of constructing a feedback network from the interconnection of quantum systems, for example, through field coupling; see Gough (2010) and Gough and James (2007). In this framework, coherent quantum control theory aims at developing systematic methods to design measurement-free interconnections of Markovian quantum systems modelled by quantum stochastic differential equations (QSDEs); for example, see James, Nurdin, and Petersen (2008), Nurdin, James, and Petersen (2009) and Petersen (2010). Owing to recent advances in quantum optics, the implementation of quantum feedback networks governed by linear QSDEs (Mabuchi, 2008; Parthasarathy, 1992; Petersen, 2010) is possible using quantum-optical components, such as optical cavities, beam splitters and phase shifters, provided the former represent open quantum harmonic oscillators (OQHOs) with a quadratic Hamiltonian and linear system-field coupling operators with respect to the state variables satisfying canonical commutation relations (Edwards & Belavkin, 2005; Gardiner & Zoller, 2004). This important class of linear QSDEs models the Heisenberg evolution of pairs of conjugate operators in a multi-mode quantum harmonic oscillator that is coupled to external bosonic fields. As a consequence, the notion of physical realizability (PR) addresses conditions under which a linear QSDE represents an OQHO. This condition is organized as a set of constraints on the coefficients of the QSDE (James et al., 2008) or, alternatively, on the quantum system transfer matrix (Shaiju & Petersen, 2012; Sichani & Petersen, in press) in the frequency domain. These constraints complicate the solution of the coherent quantum synthesis problems which are otherwise reducible to tractable unconstrained counterparts in classical control theory.

Coherent quantum feedback control problems, such as internal stabilization and optimal control design, are of particular interest in linear quantum control theory (James et al., 2008; Petersen, 2010). These problems are amenable to transfer matrix design methods (Gough, 2010; Petersen, 2010; Shaiju & Petersen, 2012; Yanagisawa & Kimura, 2003a, b). Among the transfer matrix approaches to the control problems for linear multivariable systems, the linear fractional representation approach to analysis and synthesis has been largely developed in the literature; see Vidyasagar (2011) and the references therein. The linear fractional representation approach is a cornerstone in the study of stabilization

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problems. The central idea of this approach is to represent the transfer matrix of a plant as fractions of stable rational matrices to generate stable factorizations. By combining the idea of the stable factorizations of a plant with the concept of coprimeness, necessary and sufficient conditions for internal stabilizability are derived in terms of Bézout equations (Vidyasagar, 2011). By solving these Bézout equalities, a parameterization of all stabilizing controllers, known as the Youla–Kučera parameterization, is obtained. This idea gives rise to a method which leads to the solution of several important control problems; see for example Vidyasagar (2011).

The Youla–Kučera parameterization was developed originally in the frequency domain for finite-dimensional linear time-invariant systems using transfer function methods, see Youla, Bongiorno, and Jabr (1976) and Youla, Jabr, and Bongiorno (1976), and generalized to infinite-dimensional systems afterwards (Desoer, Liu, Murray, & Saeks, 1980; Quadrat, 2003; Vidyasagar, 2011). The state space representation of all stabilizing controllers has also been addressed for finite-dimensional, linear time-invariant (Nett, Jacobson, & Balas, 1984) and time-varying (Dale & Smith, 1993) systems, and the approach was shown to be applicable to a class of nonlinear systems (Anderson, 1998; Hammer, 1985; Paice & Moore, 1990). In the Youla–Kučera parameterization, the feedback loop involving the controller is redefined in terms of another parameter known as the Youla or Q parameter. The closed-loop map is then an affine function of Q, and so the optimal Q in standard optimal stabilization problems can be easily found. Moreover, some constraints, such as internal stability, are reduced to convex constraints on Q. Therefore, this approach provides a tool that allows us to better understand the dichotomy between tractable and intractable control synthesis problems in the presence of additional constraints on the controller; see for example Boyd and Barratt (1991).

In the present paper, we employ a stable factorization approach in order to develop a counterpart of the classical Youla–Kučera parameterization for describing the set of linear coherent quantum controllers that stabilize a linear quantum stochastic system (LQSS). In particular, we address the problem of coherent quantum stabilizability of a given linear quantum plant. The class of all stabilizing controllers is parameterized in the frequency domain. This approach allows weighted $\mathscr{H}_2$ and $\mathscr{H}_\infty$ coherent quantum control problems to be formulated for linear quantum systems in the frequency domain. In this way, the weighted $\mathscr{H}_2$ and $\mathscr{H}_\infty$ control problems are reduced to constrained optimization problems with respect to the Youla–Kučera parameter with convex cost functionals. Moreover, these problems are organized as a constrained version of the model matching problem (Francis, 1987). Finally, a projected gradient descent scheme is proposed to provide a conceptual solution to the weighted $\mathscr{H}_2$ coherent quantum control problem in the frequency domain.

The rest of this paper is organized as follows. Section 2 outlines the notation used in the paper. Linear quantum stochastic systems are described in Section 3. The coherent quantum feedback interconnection under consideration is described in Section 4. Section 5 revisits the PR conditions for linear quantum systems in the frequency domain. Sections 6 and 7 formulate a quantum version of the Youla–Kučera parameterization and provide relevant preparatory material. Also, a class of unstabilizable LQSS systems is presented. Coherent quantum weighted $\mathscr{H}_2$ and $\mathscr{H}_\infty$ control problems are introduced in Section 8. A projected gradient descent scheme for the quantum weighted $\mathscr{H}_2$ control problem is outlined in Section 9. Section 10 gives concluding remarks.

A preliminary version of this work, Sichani, Petersen, and Vladimirov (2015), has been published in the conference proceedings of the 10th Asian control conference. In comparison to the conference version, use is made of a modified version of the physical realizability condition for linear quantum stochastic systems in the frequency domain (Sichani & Petersen, in press) which leads to more complete and simple results. The changes include a real-valued parameterization of the linear coherent quantum stochastic feedback systems (without loss of generality) and the omission of technical assumptions in the main results of the paper. The main theorem, Theorem 8, in Sichani et al. (2015) has been modified to provide a parameterization of the set of *all* stabilizing linear coherent quantum controllers. A class of linear quantum systems is presented which cannot be stabilized by linear coherent quantum controllers. For complete proofs, complementary results and technical details see Sichani and Petersen (2017).

2. Notation

Unless specified otherwise, vectors are organized as columns, and the transpose $(\cdot)^{\mathrm{T}}$ acts on matrices with operator-valued entries as if the latter were scalars. For a vector X of self-adjoint operators $X_1, \ldots, X_r$ and a vector Y of operators $Y_1, \ldots, Y_s$, the commutator matrix is defined as an $(r\times s)$-matrix $[X, Y^{\mathrm{T}}] := XY^{\mathrm{T}} - (YX^{\mathrm{T}})^{\mathrm{T}}$ whose (j, k) th entry is the commutator $[X_j, Y_k] := X_jY_k - Y_kX_j$ of the operators X_j and Y_k. Furthermore, $(\cdot)^\dagger := ((\cdot)^\#)^{\mathrm{T}}$ denotes the transpose of the entry-wise operator adjoint $(\cdot)^\#$. When it is applied to complex matrices, $(\cdot)^\dagger$ reduces to the complex conjugate transpose $(\cdot)^* := (\overline{(\cdot)})^{\mathrm{T}}$. The positive semi-definiteness of matrices is denoted by $\succcurlyeq$, and $\otimes$ is the tensor product of spaces or operators (for example, the Kronecker product of matrices). Furthermore, $\mathbb{S}_r$, $\mathbb{A}_r$ and $\mathbb{H}_r := \mathbb{S}_r + i\mathbb{A}_r$ denote the subspaces of real symmetric, real antisymmetric and complex Hermitian matrices of order r, respectively, with $i := \sqrt{-1}$ the imaginary unit. Also, I_r denotes the identity matrix of order r, the identity operator on a space $\mathscr{H}$ is denoted by $\mathscr{I}_{\mathscr{H}}$, and the matrices $\mathrm{J} := \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $J_r := I_{\frac{r}{2}} \otimes \mathrm{J}$. The sets $\mathrm{O}(r) := \left\{\Sigma \in \mathbb{R}^{r\times r} : \Sigma^{\mathrm{T}}\Sigma = I\right\}$ and $\mathrm{Sp}(r, \mathbb{R}) := \left\{\Sigma \in \mathbb{R}^{r\times r} : \Sigma^{\mathrm{T}}J_r\Sigma = J_r\right\}$ refer to the group of orthogonal matrices and the group of symplectic real matrices of order r. The notation $\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$ refers to a state-space realization of the corresponding transfer matrix $\Gamma(s) := C(sI - A)^{-1}B + D$ with a complex variable $s \in \mathbb{C}$. The conjugate system transfer function $(\Gamma(-\bar{s}))^*$ is written as $\Gamma^\sim(s)$. The Hardy space of (rational) transfer functions of type $p = 2, \infty$ is denoted by $\mathscr{H}_p$ (respectively, $\mathscr{RH}_p$). The symbol $\otimes$ is used for the tensor product of spaces.

3. Linear quantum stochastic systems

We consider a Markovian quantum stochastic system interacting with an external boson field. The system has n dynamic variables $X_1(t), \ldots, X_n(t)$, where $t \geqslant 0$ denotes time. We generally suppress the time argument of operators, unless we are explicitly concerned with their time dependence, with the understanding that all operators are evaluated at the same time. The system variables are self-adjoint operators on an underlying complex separable Hilbert space $\mathscr{H}$ which satisfy the Heisenberg canonical commutation relations (CCRs)

$$[X, X^{\mathrm{T}}] = 2i\Theta \otimes \mathscr{I}_{\mathscr{H}}, \qquad X := \begin{bmatrix} X_1 \\ \vdots \\ X_n \end{bmatrix}, \tag{1}$$

on a dense domain in $\mathscr{H}$, where $\theta \in \mathbb{A}_n$ is nonsingular. In what follows, the matrix $\Theta \otimes \mathscr{I}_{\mathscr{H}}$ will be identified with Θ. The system variables evolve in time according to a Hudson–Parthasarathy QSDE (Parthasarathy, 1992) with identity scattering matrix (which eliminates from consideration the gauge, also known as conservation, processes associated with photon exchange between the fields):

$$\mathrm{d}X = f\,\mathrm{d}t + g\,\mathrm{d}W. \tag{2}$$