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Brief paper

Switched systems with average dwell time: Computation of the robust positive invariant set[☆]

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ABSTRACT

In this paper, computational results of robust positive invariant (RPI) sets for switched systems with amplitude bounded, persistent, additive disturbance and average dwell time (ADT) switching are developed. Towards this end, new idea for facilitating the analysis of the ADT switching signal is conceptualized by a series of predefined stages and sufficient conditions for the existence of an ADT RPI set of the considered systems are established. By utilizing the structure characteristics at each stage of the ADT switching signal, an exact computational algorithm of the ADT RPI set of the considered system is developed. The effectiveness of the proposed algorithms are illustrated by a numerical example.

1. Introduction

The set invariant theory, as an efficient tool in dealing with the robust control problem of constraint dynamical systems, has gained its popularity over the last decades. Typical applications of the set invariant theory include model predictive control (Zhang, Zhuang, & Braatz, 2016), tracking system (Di Cairano & Borrelli, 2016), persistent additive disturbance analysis (Blanchini, 1999), etc. The computation of the robust positive invariant (RPI) set for a certain system is an essential topic in the control community. To this end, stability of discrete linear inclusion has been investigated for multi-mode system in literature (Gurvits, 1995). The notion of positive invariance is proposed to derive the stopping criterion of the computation of an approximate state reachable set for discrete time systems by Hinrata et al. in Hirata and Ohta (2003). By employing the technique of partial sums, outer approximations of the minimal invariant set of linear systems with bounded persistent additive disturbance has been computed in Ong and Gilbert (2006). Other efficient approaches for obtaining the approximations of the minimal RPI set of linear discrete time system can be found in Kouramas (2003) and references therein. The most recent contributions on this issue can be found in Trodden (2016) in which the linear programming approach is employed to compute the polytope RPI set of the considered system.

Switched systems, due to their effectiveness in modeling a large number of hybrid systems, have attracted a lot of attention from many researchers around the world. The invariant set theory for switched systems with arbitrary switching signals has been well established in Blanchini and Miani (2007). Recently, slowly switching signals such as dwell time (DT) switching signal (Dehghan & Ong, 2012a; Dehghan, Ong, & Chen, 2011), modal DT (MDT) switching signal (Dehghan & Ong, 2013), average DT (ADT) switching signal (Hespanha, 2004), modal ADT (MADT) switching signal (Fei, Shi, Zhao, & Wu, 2017; Zhao, Zhang, Shi, & Liu, 2012), persistent DT (PDT) switching signal (Zhang, Zhuang, & Shi, 2015a) and modal PDT (MPDT) switching signal (Zhang, Zhuang, Shi, & Zhu, 2015b) have been proposed to depict the certain switching behaviors. Thanks to the certain minimal running time between any two consecutive switchings, the computation approach and characteristics of DT RPI set have been worked out for switched system in Dehghan and Ong (2012a, 2012b). Corresponding results for the MDT switched systems can be found in Dehghan and Ong (2013). On the basis of these results, the computational algorithm of MPDT RPI set has been provided in Zhang et al. (2015b). It is worth noticing that continuous switchings are generally not allowed by DT switching signal and MDT switching signal. As for the MPDT switching signal, the time intervals in which continuous switchings are allowed are explicit. However, the number of continuous switchings that is allowed by an ADT switching signal is unknown a priori. This significantly aggravates the difficulty of computing a RPI set for the ADT switched systems. Moreover, to

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the best of our knowledge, no result has been published on the RPI set of ADT switched system. As the motivation of literature (Dehghan & Ong, 2012a, 2012b; Zhang et al., 2015b), though the well-established discrete linear inclusion technique such as Gurvits (1995); Hirata and Ohta (2003); Kouramas, Raković, Kerrigan, Allwright, and Mayne (2005) can be used to compute the RPI set of ADT switched system, the conservative remains too large. This motivates our current work.

In this paper, the computation of RPI set is investigated for ADT switched system. A kind of piecewise ADT (PADT) switching signal is proposed to facilitate the subsequent developments. Next, unlike the length-based approach proposed in literature (Dehghan & Ong, 2012a) by which the DT switching signal is divided into pieces with different length, a sequence-based technique, by which an ADT switching signal is divided into uniform stages such that all the admissible switching sequences at each stage can be exhausted, is proposed in this paper. With the novel proposed sequence-based technique, existence of a minimal RPI set is established and explicit algorithm is proposed to compute the generalized RPI (GRPI) set for ADT switched system. Main characteristics of the obtained ADT GRPI set are discussed.

Notations: Notations employed in this paper are standard. The term $\mathbb{R}^n$ denotes the n dimensional Euclidean space and we let $\mathbb{Z}$ and $\mathbb{Z}^+$ represent the set of nonnegative integers and positive integers respectively. The terms $\mathbb{Z}_q$ and $\mathbb{Z}_q^+$ stand for the integer sets $\{0, 1, 2, \dots, q\}$ and $\{1, 2, \dots, q\}$ respectively. The Minkowski sum and Pontryagin difference of two compact sets $\Theta_1 \subseteq \mathbb{R}^n,\ \Theta_2 \subseteq \mathbb{R}^n$ are $\Theta_1 \oplus \Theta_2 = \{\theta_1 + \theta_2 | \theta_1 \in \Theta_1,\ \theta_2 \in \Theta_2\}$ and $\Theta_1 \ominus \Theta_2 = \{\theta \in \mathbb{R}^n | \theta + \theta_2 \in \Theta_1,\ \theta_2 \in \Theta_2\}$. The convex hull of a set is given by $co\{\cdot\}$. The operator $\cup(\cdot)$ denotes the union of at least two sets and $\mathrm{eig}(A)$ stands for the set of eigenvalues of matrix A.

2. Problem formulation

Consider a switched system consisting of subsystems as

$$x(k+1) = A_{\sigma(k)}x(k) + w(k) \tag{1}$$

$$w(k) \in \mathbb{W},\ \forall k \in \mathbb{Z}^+ \tag{2}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $w(k) \in \mathbb{W}$ is the amplitude-bounded, persistent, additive disturbance and the set $\mathbb{W}$ is polygonal and contains zero in its interior. The term $\sigma(k)$ denotes the switching signal whose value is taken from a finite set $\mathbb{Z}_M^+$ and is assumed to be right-hand continuity everywhere. In this paper, the switching signal $\sigma(k)$ is considered to satisfy ADT whose definition is reproduced as follows:

Definition 1 (*Hespanha, 2004; Zhang & Gao, 2010; Zhao, Yu, Yang, & Li, 2014*). For a switching signal $\sigma(k)$ and any $\hat{k}^{[2]} \geq \hat{k}^{[1]} \geq 0$, let $N_\sigma(\hat{k}^{[2]}, \hat{k}^{[1]})$ be the switching times over the interval $[\hat{k}^{[1]}, \hat{k}^{[2]})$. If for any given N_0 and τ_a, it holds that

$$N_\sigma(\hat{k}^{[2]}, \hat{k}^{[1]}) \leq N_0 + \frac{\hat{k}^{[2]} - \hat{k}^{[1]}}{\tau_a}. \tag{3}$$

Hence, N_0 and τ_a are called the chatter bound and the ADT, respectively.

Referring to the robust positive invariant (RPI) set, we recall the following definitions:

Definition 2 (*Rawlings & Mayne, 2009*). A set $\Omega \subseteq \mathbb{R}^n$ is said to be a RPI set for system $x(k+1) = f(x(k), w(k)),\ w \in \mathbb{W}$ if $x(k) \in \Omega$ implies $x(t) \in \Omega$ for any $t > k$.

To be distinguished from the definitions of MPDT RPI set in Zhang et al. (2015b) and DT RPI set defined in Dehghan and Ong (2012a), the ADT RPI set and ADT GRPI set are defined as follows:

Definition 3. A convex set $\mathcal{O}_\infty \subseteq \mathbb{R}^n$ is said to be an ADT RPI set for system (1) with ADT τ_a if $x(k) \in \mathcal{O}_\infty$ implies that $x(t) \in \mathcal{O}_\infty,\ t > k$ for any admissible switching signal with ADT τ_a and $w(k) \in \mathbb{W}$.

Definition 4. A convex set $\Pi_\infty \subseteq \mathbb{R}^n$ is said to be an ADT GRPI set for system (1) with ADT τ_a if for any admissible switching signal with ADT τ_a and $w(k) \in \mathbb{W}$, once $x(k) \in \Pi_\infty$ holds for any $k \in \mathbb{Z}$, the trajectory of the system state will enter the set Π_∞ periodically, i.e. $x(k + \rho\mathbb{T}) \in \Pi_\infty$, where $\rho \in \mathbb{Z}^+$ and $\mathbb{T}$ is the period.

The three main purposes of this paper are listed as follows: (i) To establish the existence of the minimal ADT RPI (ADT mRPI) set for system (1). (ii) To develop a technique for computing the minimal ADT GRPI (ADT mGRPI) set explicitly for system (1). (iii) To exploit some characteristics of the obtained ADT mGRPI set. For facilitation, we make the following assumption:

Assumption 1. (i) The spectral radius of all matrices $A_i,\ i \in \mathbb{Z}_M^+$ is less than one and (ii) Two values $\tau_a \geq 1$ and $N_0 > 0$ have been identified such that switched system (1) under the ADT switching signal with $\tau_a,\ N_0$ and $w(k) \equiv 0$ is asymptotically stable.

Remark 1. A lot of stability and stabilization results of ADT switched system have been published in the literature such as Zhao et al. (2012). Therefore, it is not difficult to stabilize system (1) with ADT and to obtain the admissible ADT τ_a when $w(k) \equiv 0$.

3. Analysis of the ADT switching signal

In this section, the ADT switching signal is analyzed in detail and a connotative characteristic of the ADT switching signal is highlighted for future use. Firstly, the definition of PADT is proposed as follows:

Definition 5. Consider a time sequence $\Gamma : k_1,\ k_2,\ \cdots,\ k_s,\ k_{s+1},\ \cdots$, where the value of $h_s = k_{s+1} - k_s,\ \forall s \in \mathbb{Z}^+$ is taken from a finite integer set. A switching signal $\sigma(k)$ is called a PADT switching signal with respect to Γ under chatter bound N_0 and PADT τ_a if $\sigma(k)$ meets the requirement of ADT switching signal with chatter bound N_0 and ADT τ_a in each time interval $[k_s, k_{s+1}],\ s \in \mathbb{Z}^+$, which is called the stage s.

Lemma 1. *Given constants N_0 and τ_a, for any given class of h_s, let $\mathcal{K}_{ADT}(N_0, \tau_a)$ and $\mathcal{K}_{PADT}(N_0, \tau_a)$ denote the sets of admissible ADT switching sequences and PADT switching sequences, respectively. We have $\mathcal{K}_{ADT}(N_0, \tau_a) \subseteq \mathcal{K}_{PADT}(N_0, \tau_a)$.*

Proof. Firstly, note that $\mathcal{K}_{ADT}(N_0, \tau_a) \subseteq \mathcal{K}_{PADT}(N_0, \tau_a)$ holds if and only if a switching sequence $\mathcal{K} \in \mathcal{K}_{ADT}(N_0, \tau_a)$ implies $\mathcal{K} \in \mathcal{K}_{PADT}(N_0, \tau_a)$. Let the term $\mathcal{K}$ be an arbitrary ADT switching sequence, the terms $N_\sigma^a(\hat{k}^{[2]}, \hat{k}^{[1]})$ and $N_\sigma^p(\hat{k}^{[2]}, \hat{k}^{[1]})$ be the admissible total number of switching times of ADT switching signal and the PADT switching signal during time interval $[\hat{k}^{[1]}, \hat{k}^{[2]})$, respectively.

From Definition 1, for any $k_s \leq \hat{k}^{[1]} \leq \hat{k}^{[2]} \leq k_{s+1},\ s \in \mathbb{Z}^+$, it holds that $N_\sigma^a(\hat{k}^{[2]}, \hat{k}^{[1]}) \leq N_0 + \frac{\hat{k}^{[2]} - \hat{k}^{[1]}}{\tau_a}$ which meets the requirements of Definition 5. For any $0 \leq \hat{k}^{[1]} \leq k_s \leq \hat{k}^{[2]},\ s \in \mathbb{Z}^+,\ s \geq 2$, let $\bar{k}^{[1]} = \min\{k_{\bar{s}} : \hat{k}^{[1]} \leq k_{\bar{s}},\ \bar{s} \in \mathbb{Z}^+\}$ and