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Brief paper

Fault reconstruction for stochastic hybrid systems with adaptive discontinuous observer and non-homogeneous differentiator[☆]

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ABSTRACT

This paper investigates the state estimation and fault reconstruction problems for continuous-time Markovian jump systems, where unknown additive sensor and actuator faults, and actuator degradation are considered simultaneously. First, an augmented descriptor system is proposed where the extended vector is composed of state vector, additive sensor fault and actuator fault vectors. Then, an adaptive sliding mode observer is presented where a switching term is injected to eliminate the effect of actuator degradation. The developed robust observer can achieve estimation of state, additive sensor and actuator fault vectors simultaneously. Based on the observer technique, two methods, namely *equivalent output error injection method* and *non-homogeneous differentiator method*, are employed to reconstruct the actuator degradation. Finally, a practical example with an F-404 aircraft engine system is exploited to illustrate the effectiveness of the proposed observer approaches, and make comparisons on these two fault reconstruction schemes.

1. Introduction

In engineering practice, full state information is not always measured due to possible physical constraints and expensive cost, and state estimation has become an active area of research which attracted considerable interest (Sahebsara, Chen, & Shah, 2007). The classical Luenberger observer theory can achieve asymptotic state estimation for given accurate mathematical models (Luenberger, 1966). However, in practice parameter uncertainties, input/output noise, external disturbances and sensor/actuator faults always exist inevitably (Casavola, Famularo, & Franze, 2005; Niu & Zhao, 2013; Patton & Hou, 1998; Tan & Habib, 2007; Zhang, Jiang, & Staroswiecki, 2010), which indeed refers to the main sources of estimation performance degradation (Jiang, Staroswiecki, & Cocquempot, 2006). These phenomena cause that classical observer techniques may not be feasible to provide accurate asymptotic estimate, which has thus motivated the development of a variety of improved observer approaches including disturbance observer (Polycarpou, 2001), adaptive observer (Boskovic & Mehra, 1999), high gain observer (Gao, Breikin, & Wang, 2007), learning observer (Yang, Jiang, & Staroswiecki, 2009) and sliding mode observer (SMO) (Yan & Edwards, 2007; Yu, Sun, & Karimi, 2012), etc.

Among these existing methodologies, SMO is one of the most effective schemes due to its robustness and insensitiveness to exogenous disturbances and component faults (Song, Niu, & Zou, 2016). The main idea of SMO is to realize fault reconstruction based on the concept of equivalent output error injection. In an SMO design, an appropriate sliding surface is first designed, then the trajectory of estimation error will be forced onto the sliding surface in finite time under the switching term of the observer, along which a sliding motion occurs subsequently (Yeu, Kim, & Kawaji, 2005; Yu & Liu, 2009). Sensor/actuator fault reconstruction is then achieved via the equivalent output injection signal. In the past decade, SMO has received considerable attention, and a few SMO methods have been reported in the literature with respect to various types of systems.

On another active research front, Markovian jump systems (MJSs) which involves both time-evolving and event-driven mechanisms, have attracted considerable research attention in recent years (Liu, Daniel W. C. Ho, & Shi, 2015; Zou, Lam, Niu, & Li, 2015). As aforementioned discussion, components faults and external disturbance are inevitably encountered in realistic systems including MJSs, which will cause performance degradation for state estimation process (Liu et al., 2015). It is natural that the SMO design problem for MJSs has gained increasing interest and several results

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have been reported in the existing literature. To mentioned a few, the SMO design and observer-based fault tolerant control problem is investigated for MJSs with sensor faults in Liu, Shi, Zhang, and Zhao (2011). The SMO-based fault tolerant control problem is addressed in Li, Gao, Shi, and Zhao (2014) for stochastic systems with Markovian jump parameters, where sensor/actuator faults and external disturbances are considered in a unified framework.

It should be pointed out that, although there have been some available results for SMO design of MJSs, the following two essential difficulties have not been fully considered and remain open: (i) most of the existing results have focused on the additive actuator faults only, whereas another type of faults, multiplicative actuator faults, which are always described as unknown actuator efficiency factor, have received much little research attention (Ha & Trinh, 2004; Xiong & Saif, 2003); (ii) when additive sensor faults, additive actuator faults, and actuator efficiency factor are considered simultaneously, the corresponding actuator fault reconstruction is a challenging problem, which has been largely untouched. Traditional SMO approaches may not be effective to cope with this design problem. Thus, it is of both theoretical importance and practical requirement to develop effective observer approaches to investigate this research issue. The purpose of this paper is, therefore, to provide a new SMO method to fully solve this problem.

In this paper, we aim to investigate the observer design problem for continuous-time MJSs in the presence of unknown additive sensor and actuator faults as well as multiplicative actuator faults. The proposed design method is divided into the following three steps: (i) an augmentation strategy is performed on the original plant to construct a descriptor system, where the extended state vector is composed of the original state, sensor fault and exogenous disturbance vectors; (ii) based upon the descriptor system, a new type of adaptive SMO is presented to generate the asymptotic estimates of the augmented state vectors. In this design, an adaptive estimation of the unknown actuator efficiency factor is introduced in the observer to compensate the effect of multiplicative actuator faults. The presented observer technique can generate simultaneous estimates of the state, sensor fault and output disturbance vectors. (iii) based on the state estimation, *an equivalent output error injection method* is employed to achieve the actuator fault reconstruction. However, in this design there exist matrix conditions to be solved where a mode-independent matrix parameter H is involved (as seen in Lemma 3 and Remark 4). Hence, the proposed method may be conservative and may even cannot find solution for H. To overcome this difficulty, we present the second method to revisit the fault reconstruction problem. In the second method, the *non-homogeneous differentiator method* in Levant (2009) is employed to estimate the derivative of the output vector, based on which the actuator degradation reconstruction can be achieved successfully instead of using the equivalent output error injection. Moreover, in the second method, the aforementioned matrix parameter H in equality constraints can be relaxed as finding H_i for each $i \in \mathbb{S}$, thus reducing the design conservativeness. Finally, a simulation example with an F-404 aircraft engine model is given to show the validity of the developed observer approaches, where these two actuator fault reconstruction approaches are compared.

2. Problem formulation

Considering the following continuous-time MJS

$$\begin{cases}\dot{x}(t) = A(r_t)x(t) + B(r_t)u^F(t) + B_a(r_t)f_a(t),\\ y(t) = Cx(t) + D_u u^F(t) + D_a f_a(t) + F_s f_s(t),\end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u^F(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}^p$ denote the state vector, fault control input vector and measurement output vector, respectively, $f_s(t) \in \mathbb{R}^s$ and $f_a(t) \in \mathbb{R}^a$ denote the additive sensor and actuator fault vectors, respectively. Note that in system (1) we have considered actuator degradation and fault in output measurement $y(t)$. Such a model has physical meaning and has been investigated in Gao et al. (2007), where a high-gain-observer-based fault estimation approach was presented for gas turbine dynamic systems. $A(r_t) \in \mathbb{R}^{n\times n}$, $B(r_t) \in \mathbb{R}^{n\times m}$, $B_a(r_t) \in \mathbb{R}^{n\times a}$, $C \in \mathbb{R}^{p\times n}$, $D_u \in \mathbb{R}^{p\times m}$, $D_a \in \mathbb{R}^{p\times a}$ and $F_s \in \mathbb{R}^{p\times s}$ are system matrices, $\{r_t, t \geq 0\}$ is a right-continuous Markov chain on the probability space $(\Omega, \mathcal{F}, \mathcal{P})$ taking values in a finite state space $\mathbb{S} = \{1, 2, \ldots, N\}$. The mode transition probabilities $\Pi = (\pi_{ij})_{N\times N}$ of the Markov chain are given by

$$p_{ij} = \Pr\{r_{t+\Delta} = j | r_t = i\} = \begin{cases}\pi_{ij}\Delta + o(\Delta) & \text{if } i \neq j,\\ 1 + \pi_{ii}\Delta + o(\Delta) & \text{if } i = j,\end{cases}$$

where $\Delta > 0$ and $\lim_{\Delta\to 0} o(\Delta)/\Delta = 0$, π_{ij} is the switching rate from i to j and satisfies: $\pi_{ij} > 0$, $i \neq j$, and $\pi_{ii} = -\sum_{j\neq i}\pi_{ij} < 0$, $\forall i, j \in \mathbb{S}$. For each $i \in \mathbb{S}$, $A(r_t = i) = A_i \in \mathbb{R}^{n\times n}$, $B(r_t = i) = B_i \in \mathbb{R}^{n\times m}$ and $B_a(r_t = i) = B_{ai} \in \mathbb{R}^{n\times a}$ are constant matrices.

Suppose that the fault control input $u^F(t)$ in (1) possesses q fault models, each of which is represented as $u_k^F(t)$ for $k = 1, 2, \ldots, q$. The general actuator fault model $u_k^F(t) \triangleq [u_{1k}^F(t), \ldots, u_{mk}^F(t)]^T$ is described as

$$u_{hk}^F(t) = \rho_h^k u_h(t) + \theta_h^k \sigma_{sh}(t), \tag{2}$$

where ρ_h^k $(h = 1, \ldots, m,\ k = 1, 2, \ldots, q)$ denotes the unknown actuator efficiency factor, h stands for the hth actuator, and k means that the hth actuator is in the kth fault mode; $\sigma_{sh}(t)$ represents the unparametrizable bounded time-varying actuator stuck fault for the hth actuator, and θ_h^k is defined as $\theta_h^k = \begin{cases}0 & \rho_h^k > 0,\\ 0 \text{ or } 1 & \rho_h^k = 0.\end{cases}$ For the fault model (2), it is reasonable to suppose

$$0 < \underline{\rho}_h^k \leqslant \rho_h^k \leqslant \overline{\rho}_h^k \leqslant 1,\quad 0 < \underline{\sigma}_{sh} \leqslant \sigma_{sh} \leqslant \overline{\sigma}_{sh} \leqslant 1,$$

where $\underline{\rho}_h^k$ and $\overline{\rho}_h^k$ denotes the known lower bound of ρ_h^k, and $\underline{\sigma}_{sh}$ and $\overline{\sigma}_{sh}$ denotes the unknown lower bound of σ_{sh}. We denote

$$\rho^k \triangleq \operatorname{diag}\{\rho_1^k, \rho_2^k, \ldots, \rho_m^k\},\quad \theta^k \triangleq \operatorname{diag}\{\theta_1^k, \theta_2^k, \ldots, \theta_m^k\},$$

then the set of ρ_h^k is defined by

$$\Delta_{\rho^k} \triangleq \{\rho^k : \rho^k = \operatorname{diag}\{\rho_1^k, \rho_2^k, \ldots, \rho_m^k\},\ \rho_h^k \in [\underline{\rho}_h^k, \overline{\rho}_h^k]\}.$$

For simplicity, (2) is expressed as the following form

$$u^F(t) = \rho u(t) + \theta\sigma_s(t), \tag{3}$$

Remark 1. In the aforementioned fault model, we have not considered the *fault-stuck* case, that is, $\underline{\rho}_h^k = 0$, $(h = 1, \ldots, m,\ k = 1, 2, \ldots, q)$. However, our design can cover the fault-stuck issue, and the reason is illustrated as follows. It is well known that, to design a fault control system successfully, the following actuator redundancy condition should satisfy (Liu et al., 2015): "For each $i \in \mathbb{S}$, $\operatorname{rank}(B_i) = \operatorname{rank}(B_i\rho) = l$ holds for all $\rho \in \Delta_{\rho^k}$, $k = 1, \ldots, q$. where $\operatorname{rank}(B_i) = l \leqslant m$ for each $i \in \mathbb{S}$."

It is obvious that if the fault-stuck case is considered here, that is, $\underline{\rho}_h^k = 0$ $(h = 1, \ldots, m,\ k = 1, 2, \ldots, q)$, then B_i should be constrained to satisfy $\operatorname{rank}(B_i) = l < m$ strictly. According to matrix theory, in this case B_i can be factorized as follows:

$$B_i = B_{1i}^0 B_{2i}^0, \tag{4}$$

where $B_{1i}^0 \in \mathbb{R}^{n\times l}$ is of full column rank, and $B_{2i}^0 \in \mathbb{R}^{l\times m}$ is of full row rank. In this setting, system (1) can be rewritten as

$$\begin{cases}\dot{x}(t) = A_i x(t) + B_{1i}^0(\breve{\rho}u(t) + \breve{\theta}\sigma_s(t)) + B_{ai}f_a(t),\\ y(t) = Cx(t) + D_u u^F(t) + D_a f_a(t) + F_s f_s(t)\end{cases} \tag{5}$$

where $\breve{\rho} = B_{2i}^0\rho \in \mathbb{R}$ and $\breve{\theta} = B_{2i}^0\theta$. As a result, the observer design work can be implemented based on system (5). However, for convenience, in the following discussion we will not considered *fault-stuck* case, and B_i is supposed to be of full column rank.