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Brief paper

Adaptive near-optimal consensus of high-order nonlinear multi-agent systems with heterogeneity☆

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ABSTRACT

In this paper, the near-optimal distributed consensus of high-order nonlinear multi-agent systems consisting of heterogeneous agents is investigated. The consensus problem is formulated as a receding-horizon optimal control problem. Under the condition that the dynamics of all agents are fully known, a nominal near-optimal protocol is designed and proposed via making approximation of the performance index. For the situation with fully unknown system parameters, sliding-mode auxiliary systems, which are independent for different agents, are built to reconstruct the input–output properties of agents. Based on the sliding-mode auxiliary systems, an adaptive near-optimal protocol is finally presented to control high-order nonlinear multi-agent systems with fully unknown parameters. Theoretical analysis shows that the proposed protocols can simultaneously guarantee the asymptotic optimality of the performance index and the asymptotic consensus of multi-agent systems. An illustrative example about a third-order nonlinear multi-agent system consisting of 10 heterogeneous agents with fully unknown parameters further substantiates the efficacy and superiority of the proposed adaptive near-optimal consensus approach.

1. Introduction

In recent years, the consensus of multi-agent systems have been widely investigated due to the widespread applications. The consensus of integrator multi-agent systems was investigated in Ferrari-Trecate, Galbusera, Marciandi, and Scattolini (2009), Li, Du, and Lin (2011), Li and Zhang (2010), Olfati-Saber and Murray (2004), Rezaee and Abdollahi (2015) and Seyboth, Dimarogonas, and Johansson (2013). Some protocols for linear multi-agent systems were reported in Cheng, Hou, and Tan (2014), Li, Ren, Liu, and Xie (2013), Li and Yan (2015), Ma and Zhang (2010), Semsar-Kazerooni and Khorasani (2008) and Semsar-Kazerooni and Khorasani (2009). The consensus of nonlinear multi-agent systems is of greater significance since physical systems are more or less nonlinear. The consensus of nonlinear multi-agent systems with identical agents was investigated in Fan, Chen, and Zhang (2014), Liu, Xie, Ren, and Wang (2013), Song, Cao, and Yu (2010) and Yu, Ren, Zheng, Chen, and Lü (2013). In terms of homogeneous nonlinear multi-agent systems, only a few results have been reported (Hua, You, & Guan, 2016; Zhang, Liu, & Feng, 2015; Zhu & Chen, 2014). The investigations on the consensus of nonlinear multi-agent systems with unknown parameters are of great interests (Cao & Ren, 2014; Chen, Li, Ren, & Wen, 2014; Chen, Wen, Liu, & Liu, 2016; Chen, Wen, Liu, & Wang, 2014; Huang, Wen, Wang, & Song, 2015; Rezaee & Abdollahi, 2017; Zhu & Chen, 2014). For example, the adaptive consensus of high-order nonlinear multi-agent systems was investigated in Chen et al. (2016) and Rezaee and Abdollahi (2017). Note that the design of adaptive consensus to tackle unknown parameters is not trivial due to the distributed nature of consensus, and it is not straightforward to extend existing adaptive control methods, e.g., Liu, Gao, Tong, and Chen (2016), Liu, Gao, Tong, and Li (2016) and Liu and Tong (2016), for individual agents to the consensus of multi-agents. The distributed optimal consensus problem is difficult to solve since the solution of a global optimization problem generally requires centralized, i.e. global, information (Movric & Lewis, 2014). This problem becomes more difficult when it comes to nonlinear multi-agent systems. In Zhang, Zhang, Yang, and Luo (2015), the optimal consensus of a nonlinear multi-agent system with respect to local performance indices is investigated via game theory. Note that the optimal control of a nonlinear system often requires solving a partial differential equation called the Hamilton–Jacobi–Bellman equation for which

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an analytical solution is generally difficult to obtain. In Kamalapurkar, Dinh, Walters, and Dixon (2013), a distributed consensus protocol was proposed for a first-order nonlinear multi-agent system with uncertain nonlinear dynamics. It is remarkable that, in Chen, Ballance, and Gawthrop (2003), by making approximation to the performance index, an analytical near-optimal control law is obtained for a nonlinear system, which significantly reduces the computational cost compared with the dynamic programming approach for optimal control. Motivated by the above observations, we investigate the distributed optimal consensus of high-order nonlinear multi-agent systems with heterogeneous agents. The proposed adaptive protocol guarantees agents of high-order multi-agent systems asymptotically achieving consensus in the presence of fully unknown parameters. To the best of our knowledge, this is the first work of optimal consensus for parameter uncertain, nonlinear, non-identical agents with provable asymptotic optimality.

2. Problem formulation

Consider a multi-agent system consisting of n nonlinear heterogeneous agents of order σ, of which the communication topology is described by an undirected connected graph with the Laplacian matrix denoted by L. The ith ($i = 1, 2, \ldots, n$) agent is described as $\dot{x}_i^{(0)} = x_i^{(1)}, \ldots, \dot{x}_i^{(\sigma-2)} = x_i^{(\sigma-1)}, \dot{x}_i^{(\sigma-1)} = f_i(\mathbf{x}_i) + g_i(\mathbf{x}_i)u_i$, where $x_i^{(j)} \in \mathbb{R}$ with $j = 1, 2, \ldots, \sigma$ denotes the jth time derivative of $x_i^{(0)} = x_i$; $\mathbf{x}_i = [x_i, x_i^{(1)}, \ldots, x_i^{(\sigma-1)}]^{\mathrm{T}} \in \mathbb{R}^\sigma$ and $u_i \in \mathbb{R}$ denote the state vector and input of the ith agent, respectively; $f_i(\cdot) : \mathbb{R}^\sigma \to \mathbb{R}$ and $g_i(\cdot) : \mathbb{R}^\sigma \to \mathbb{R}$ are continuously differentiable functions which may be different for each agent with $g_i(\mathbf{x}_i) > 0$ for all $i = 1, 2, \ldots, n$. Let $\mathbf{x}_{\mathrm{M}} = [x_1, x_2, \ldots, x_n]^{\mathrm{T}}$, $f_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}) = [f_1(\mathbf{x}_1), f_2(\mathbf{x}_2), \ldots, f_n(\mathbf{x}_n)]^{\mathrm{T}}$, $\mathbf{u} = [u_1, u_2, \ldots, u_n]^{\mathrm{T}}$ and $g_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}) = \mathrm{diag}([g_1(\mathbf{x}_1), g_2(\mathbf{x}_2), \ldots, g_n(\mathbf{x}_n)]^{\mathrm{T}})$. The multi-agent system is thus formulated as follows:

$$\begin{cases} \dot{\mathbf{x}}_{\mathrm{M}}^{(k)} = \mathbf{x}_{\mathrm{M}}^{(k+1)}, i = 0, 1, \ldots, \sigma - 2, \\ \dot{\mathbf{x}}_{\mathrm{M}}^{(\sigma-1)} = f_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}) + g_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}})\mathbf{u}. \end{cases} \tag{1}$$

To achieve the consensus of multi-agent system (1), we propose the following performance index:

$$J(t) = \int_0^T \mathbf{x}_{\mathrm{M}}^{\mathrm{T}}(t+\tau)L_0\mathbf{x}_{\mathrm{M}}(t+\tau)\mathrm{d}\tau + \int_0^T \dot{\mathbf{x}}_{\mathrm{M}}^{\mathrm{T}}(t+\tau)Q\dot{\mathbf{x}}_{\mathrm{M}}(t+\tau)\mathrm{d}\tau, \tag{2}$$

where $T > 0 \in \mathbb{R}$ denotes the predictive period; $L_0 \in \mathbb{R}^{n\times n}$ is a matrix to be determined, which satisfies two properties: (1) L_0 is symmetric; (2) only one of its eigenvalues is 0 with the corresponding eigenvector being $\mathbf{1} = [1, 1, \ldots, 1]^{\mathrm{T}} \in \mathbb{R}^n$ and the other eigenvalues are strictly bigger than 0. Besides, $Q = \zeta I$ with $\zeta > 0 \in \mathbb{R}$ and I being a $n \times n$ identity matrix.

Remark 1. The two integral terms of $J(t)$ correspond to the relative potential energy and the kinetic energy of the whole multi-agent system, respectively. When performance index $J(t)$ achieves its minimum, i.e., $J(t) = 0$, the relative potential energy and the kinetic energy become zero. It follows that all the agents of the multiagent system achieve static consensus. The results presented in this paper can also be extended to velocity consensus, acceleration consensus, etc., by modifying the two integral terms of $J(t)$.

3. Nominal near-optimal design

In this section, we consider the situation where multi-agent system (1) is consisting of agents of fully known dynamics. Let $X_{\mathrm{M}} = [\mathbf{x}_{\mathrm{M}}, \mathbf{x}_{\mathrm{M}}^{(1)}, \ldots, \mathbf{x}_{\mathrm{M}}^{(\sigma-1)}, f_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}})]$. Then, $\mathbf{x}_{\mathrm{M}}(t+\tau)$ of multi-agent system (1) is approximated via time-scale Taylor expansion as $\mathbf{x}_{\mathrm{M}}(t+\tau) \approx X_{\mathrm{M}}(t)\mathbf{w}_1(\tau) + \tau^\sigma g_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}(t))\mathbf{u}(t)/\sigma!$, where $\mathbf{w}_1(\tau) = [1, \tau, \ldots, \tau^{\sigma-1}/(\sigma-1)!, \tau^\sigma/\sigma!]^{\mathrm{T}}$. Similarly, $\dot{\mathbf{x}}_{\mathrm{M}}(t+\tau) \approx X_{\mathrm{M}}(t)\mathbf{w}_2(\tau) + \tau^{\sigma-1}g_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}(t))\mathbf{u}(t)/(\sigma-1)!$, where $\mathbf{w}_2(\tau) = [0, 1, \tau, \ldots, \tau^{\sigma-1}/(\sigma-1)!]^{\mathrm{T}}$. In addition, $\mathbf{u}(t+\tau) \approx \mathbf{u}(t)$. Then, $J(t)$ in (2) is approximated as

$$\begin{aligned} J(t) \approx{}& \hat{J}(t) \\ ={}& \int_0^T (X_{\mathrm{M}}(t)\mathbf{w}_1(\tau) + \frac{\tau^\sigma}{\sigma!}g_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}(t))\mathbf{u}(t))^{\mathrm{T}}L_0 \\ &\times (X_{\mathrm{M}}(t)\mathbf{w}_1(\tau) + \frac{\tau^\sigma}{\sigma!}g_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}(t))\mathbf{u}(t))\mathrm{d}\tau \\ &+ \int_0^T (X_{\mathrm{M}}(t)\mathbf{w}_2(\tau) + \frac{\tau^{\sigma-1}}{(\sigma-1)!}g_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}(t))\mathbf{u}(t))^{\mathrm{T}} \\ &\times Q(X_{\mathrm{M}}(t)\mathbf{w}_2(\tau) + \frac{\tau^{\sigma-1}}{(\sigma-1)!}g_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}(t))\mathbf{u}(t))\mathrm{d}\tau. \end{aligned} \tag{3}$$

Let $\mathbf{v}_1 = \int_0^T \tau^\sigma \mathbf{w}_1(\tau)/\sigma!\mathrm{d}\tau$, $\mathbf{v}_2 = \int_0^T \tau^{\sigma-1}\mathbf{w}_2(\tau)/(\sigma-1)!\mathrm{d}\tau$, $\kappa_1 = \int_0^T \tau^{2\sigma}/(\sigma!)^2\mathrm{d}\tau = T^{2\sigma+1}/((2\sigma+1)(\sigma!)^2)$, and $\kappa_2 = \int_0^T \tau^{2\sigma-2}/((\sigma-1)!)^2\mathrm{d}\tau = T^{2\sigma-1}/((2\sigma-1)((\sigma-1)!)^2)$. Then, from (3), $\hat{J}(t) = \int_0^T \mathbf{w}_1^{\mathrm{T}}(\tau)X_{\mathrm{M}}^{\mathrm{T}}(t)L_0 \times X_{\mathrm{M}}(t)\mathbf{w}_1(\tau) + \mathbf{w}_2^{\mathrm{T}}(\tau)X_{\mathrm{M}}^{\mathrm{T}}(t)QX_{\mathrm{M}}(t)\mathbf{w}_2(\tau)\mathrm{d}\tau + 2\mathbf{v}_1^{\mathrm{T}}X_{\mathrm{M}}^{\mathrm{T}}(t)L_0g_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}(t))\mathbf{u}(t) + \kappa_1\mathbf{u}^{\mathrm{T}}(t)g_{\mathrm{M}}^{\mathrm{T}}(\mathbf{x}_{\mathrm{M}}(t))L_0 \times g_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}(t))\mathbf{u}(t) + 2\mathbf{v}_2^{\mathrm{T}}X_{\mathrm{M}}^{\mathrm{T}}(t)Qg_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}(t))\mathbf{u}(t) + \kappa_2\mathbf{u}^{\mathrm{T}}(t) \times g_{\mathrm{M}}^{\mathrm{T}}(\mathbf{x}_{\mathrm{M}}(t))Qg_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}(t))\mathbf{u}(t)$. In light of the fact that the decision variable is input $\mathbf{u}(t)$, minimizing $\hat{J}(t)$ is then equivalent to minimizing $\hat{J}_{\mathrm{e}}(t) = 2\mathbf{v}_1^{\mathrm{T}}X_{\mathrm{M}}^{\mathrm{T}}(t)L_0g_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}(t)) \times \mathbf{u}(t) + \kappa_1\mathbf{u}^{\mathrm{T}}(t)g_{\mathrm{M}}^{\mathrm{T}}(\mathbf{x}_{\mathrm{M}}(t))L_0g_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}(t))\mathbf{u}(t) + 2\mathbf{v}_2^{\mathrm{T}}X_{\mathrm{M}}^{\mathrm{T}}(t) \times Qg_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}(t))\mathbf{u}(t) + \kappa_2\mathbf{u}^{\mathrm{T}}(t)g_{\mathrm{M}}^{\mathrm{T}}(\mathbf{x}_{\mathrm{M}}(t))Qg_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}(t))\mathbf{u}(t)$. Note that $\kappa_1 > 0$, $\kappa_2 > 0$, Q is a positive-definite diagonal matrix, L_0 is a positive semi-definite symmetric matrix and $g_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}(t))$ is a diagonal matrix of positive elements. It follows that $\kappa_1 g_{\mathrm{M}}^{\mathrm{T}}(\mathbf{x}_{\mathrm{M}}(t))L_0g_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}(t)) + \kappa_2 g_{\mathrm{M}}^{\mathrm{T}}(\mathbf{x}_{\mathrm{M}}(t))Qg_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}(t))$ is positive-definite, i.e., $\hat{J}_{\mathrm{e}}(t)$ is a convex quadratic performance index. Then, a near-optimal protocol can be obtained by solving $\partial\hat{J}_{\mathrm{e}}(t)/\partial\mathbf{u} = 0$, i.e., $2g_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}(t))L_0X_{\mathrm{M}}(t)\mathbf{v}_1 + 2\kappa_1 g_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}(t))L_0g_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}(t))\mathbf{u}(t) + 2g_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}(t))QX_{\mathrm{M}}(t)\mathbf{v}_2 + 2\kappa_2 g_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}(t))Qg_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}(t))\mathbf{u}(t) = 0$. It follows that $L_0X_{\mathrm{M}}(t)\mathbf{v}_1 + \kappa_1L_0g_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}(t))\mathbf{u}(t) + QX_{\mathrm{M}}(t)\mathbf{v}_2 + \kappa_2Qg_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}(t))\mathbf{u}(t) = 0$. Substituting the expressions of $\mathbf{v}_1$ and $\mathbf{v}_2$ into the above equation yields

$$\begin{aligned} &Q\kappa_4\sum_{j=1}^{\sigma-1}\frac{T^{j-\sigma}(2\sigma-1)(\sigma-1)!}{(\sigma+j-1)(j-1)!}\mathbf{x}_{\mathrm{M}}^{(j)} + (L_0\kappa_3 + Q\kappa_4) \\ &\times(f_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}) + g_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}(t))\mathbf{u}(t)) = \sum_{j=0}^{\sigma-1}\frac{-T^{j+2}L_0\mathbf{x}_{\mathrm{M}}^{(j)}}{(\sigma+j+1)\sigma j!}, \end{aligned} \tag{4}$$

where $\kappa_3 = \kappa_1(\sigma-1)!/T^{\sigma-1} = T^{\sigma+2}/(2\sigma+1)/\sigma/\sigma!$ and $\kappa_4 = \kappa_2(\sigma-1)!/T^{\sigma-1} = T^\sigma/(2\sigma-1)/(\sigma-1)!$. Let $(L_0\kappa_3 + Q\kappa_4)^{-1}L_0 = L$. Define α_j and β_j as follows:

$$\alpha_j = \frac{T^{j+2}}{(\sigma+j+1)\sigma j!} - \frac{T^{j+2}(2\sigma-1)}{\sigma^2(2\sigma+1)(\sigma+j-1)(j-1)!},$$

$$\beta_j = \frac{T^{j-\sigma}(2\sigma-1)(\sigma-1)!}{(\sigma+j-1)(j-1)!}.$$

Then, by solving Eq. (4), a nominal near-optimal protocol is obtained as follows:

$$\begin{aligned} \mathbf{u}(t) ={}& g_{\mathrm{M}}^{-1}(\mathbf{x}_{\mathrm{M}}(t))\left(-\frac{T^2L\mathbf{x}_{\mathrm{M}}(t)}{\sigma(\sigma+1)} - L\sum_{j=1}^{\sigma-1}\alpha_j\mathbf{x}_{\mathrm{M}}^{(j)}(t)\right. \\ &\left. - f_{\mathrm{M}}(\mathbf{x}_{\mathrm{M}}(t)) - \sum_{j=1}^{\sigma-1}\beta_j\mathbf{x}_{\mathrm{M}}^{(j)}(t)\right). \end{aligned} \tag{5}$$