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Brief Paper

Output feedback control of linear systems with input, state and output delays by chains of predictors[☆]

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ABSTRACT

In this paper, we consider the control problem of linear systems with state time-delays when the control signal is affected by a possibly time-varying delay. The problem is solved by a chain of predictors under the assumption that a stabilizing control exists for the case of no input delay. This solution is then extended to the case of partial information when only delayed output is available. Both the input and the output delays may have arbitrary bounds. With respect to previous approaches to the same problem appeared recently in the literature the method proposed here has a simpler structure and in particular it does not need the computation of distributed terms. As a by-product we show that it is possible to derive an observer for a system with state and output delays from an observer for the same system without output delays.

1. Introduction

The control problem of dynamic systems with delays has received growing attention in recent years due to novel application areas such as control over networks. The reader may refer to the excellent surveys in Gu and Niculescu (2003), Krstic (2010a), Richard (2003) and to the recent books (Fridman, 2014; Karafyllis, Malisoff, Mazenc, & Pepe, 2016). In this paper, we focus on linear systems, for which stabilization in the presence of input delays is an important problem in the applications. In order to overcome the limitation of the original Smith predictor (Smith, 1959) to systems that are open-loop stable, the standard approach is based on predictor feedback by means of finite spectrum assignment (Manitius & Olbrot, 1979) and model reduction approaches (Artstein, 1982). The prediction approach has received renewed interest in recent years, for both linear and nonlinear systems with state and input delays (Bekiaris-Liberis & Krstic, 2010; Jankovic, 2009, 2010; Kharitonov, 2014; Krstic, 2009). For linear systems with input delay only, some recent approaches aim at avoiding the computation of distributed terms, which is computationally challenging and may be the source of instabilities. This is the case of the truncated predictor feedback (Yoon & Lin, 2013; Zhou, 2014c; Zhou, Lin, & Duan, 2012) and the closed-loop predictor approaches (Cacace, Germani, & Manes, 2013; Zhou, 2014b). The truncated predictor feedback is also applicable to some classes of systems with state delays, whereas the closed-loop approach can be applied to nonlinear systems that are feedback linearizable (Cacace, Conte, Germani, & Pepe, 2016).

This work considers linear systems with input, state and output delays, a framework recently studied in Yoon and Lin (2015) by means of recursive predictors involving distributed terms. Recent approaches for state and input delays only include Zhou (2014a, 2015) and Zhou, Liu, and Mazenc (2017), where ingenious approaches based on nested predictors, integrators and observers have been employed to overcome large input delays. The approach developed in this paper is a variant of the closed-loop predictor approach of Cacace et al. (2013), that in its general formulation can be recast as a special case of observer-based predictor, an approach that has been pursued in several recent papers (Krstic, 2010b; Léchappé, Moulay, & Plestan, 2016; Mirkin & Raskin, 2003; Zhou et al., 2017). The main advantage is that the resulting predictor is a chain of DDEs without distributed terms that are easy to implement. Moreover, the exponential rate of convergence to the origin of the prediction error is easy to determine, the method can be extended to the case of incomplete information with large and time-varying output delays, and optimality of the control is preserved when the state is accessible. As in Yoon and Lin (2015), Zhou (2014a), Zhou (2015) and Zhou et al. (2017), we start from the assumption that a controller is available for the case of no input delays.

The remainder of the paper is organized as follows. In Section 2 we provide some background material and formalize the problem we want to solve. The case of state predictor feedback is considered

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in Section 3 and the case of delayed output predictor feedback is considered in Section 4. Some properties of the proposed approaches are discussed in Section 5 and validated in Section 6 through a benchmark example.

Notation. $\mathcal{C}(A;B)$ denotes the set of continuous functions that map A into B with the uniform convergence norm $\|\cdot\|_\infty$. For $x:\mathbb{R}\to\mathbb{R}^n$ $x_{[a,b]}$ denotes the restriction of x to $[a,b]$. $\mu(A)$ denotes the spectral abscissa of A, *i.e.* all the eigenvalues of A have real parts $\le \mu(A)$. If $\mu(A)<0$ then A is said to be Hurwitz.

2. Problem statement

We consider the class of linear time-invariant systems with input, state and output discrete delays where input and output delays may be time-varying,

$$\dot{x}(t) = \sum_{i=0}^{N} A_i x(t-r_i) + Bu(t-h(t)), \quad t \ge 0 \tag{1}$$

$$y(t) = Cx(t-\delta(t)). \tag{2}$$

$x(t)\in\mathbb{R}^n$, $u(t)\in\mathbb{R}^p$, and $y(t)\in\mathbb{R}^q$, are respectively the system variables, input and output signals. $r_0=0\le r_i\le\bar{r}$, $h(t)\le\bar{h}$, and $\delta(t)\le\bar{\delta}$ are respectively the state, input and output delays, that are supposed to be known together with their bounds. System (1) is initialized by $x_{[-\bar{r},0]}\in\mathcal{C}([-\bar{r},0];\mathbb{R}^n)$ and $u(t-h(t))=0$ whenever $t-h(t)<0$. When the input delay is time-varying, we assume as usual that the control signal generated by the controller is used only once at a known future time (Bekiaris-Liberis & Krstic, 2012; Yoon & Lin, 2013). Hence, $t-h(t)$ is bijective, $\forall t\exists!\tau: t-h(t)=\tau$ and t is known at time τ. This assumption is trivially satisfied when $h(t)=h$ and h is known. The output delay $\delta(t)$ is assumed to be continuous.

The problem is to asymptotically stabilize (1)–(2) with an output feedback control signal generated at time t by using only the available output $y(\tau)$, $\tau\le t$. We solve this problem under the assumption that a stabilizing state feedback control exists when no input delay is present.

Assumption 1. There exists a linear bounded operator $\mathcal{K}:\mathcal{C}([-\bar{r},0];\mathbb{R}^n)\to\mathbb{R}^p$ such that $u(t)=\mathcal{K}(x_{[t-\bar{r},t]})$ asymptotically stabilizes system (1) when $h(t)=0$.

Example. For $N=1$ the control $u(t)=K_0x(t)+K_1x(t-r_1)$ yields the closed-loop system

$$\dot{x}(t) = (A+BK_0)x(t) + (A_1+BK_1)x(t-r_1). \tag{3}$$

There are many approaches in the literature to design K_0 and K_1 such that (3) is asymptotically stable (Fridman, 2014; Gu, Chen, & Kharitonov, 2003; Yoon & Lin, 2015; Zhou, 2014a). Assumption 1 implies that when $h(t)\ne 0$ the stabilizing control can be computed when a prediction of $x_{[t+\bar{h}-\bar{r},t+\bar{h}]}$ is available. As discussed in Yoon and Lin (2015), Zhou (2014a) and Zhou (2015), the standard prediction scheme of Manitius and Olbrot (1979)

$$x(t+\bar{h}) = e^{A_0\bar{h}}x(t) + \int_t^{t+\bar{h}} e^{A_0(t+\bar{h}-\tau)}Bu(\tau-h(\tau))d\tau + \sum_{i=1}^{N}\int_t^{t+\bar{h}} e^{A_0(t+\bar{h}-\tau)}A_i x(\tau-r_i)d\tau \tag{4}$$

is causal and implementable only when $\bar{h}\le r_i$, $i=1,\ldots,N$. Moreover, (4) can be computed only when complete information on the state variables is available. In Kharitonov (2014), an extension of the prediction scheme of Manitius and Olbrot (1979) to the case of state delays when $\bar{h}>r_i$ is presented. We briefly compare it with our proposal in Remark 8. In Section 3, we present the solution of the prediction problem for $\bar{h}>r_i$ when the state is available. The solution for the case of incomplete and delayed information is presented in Section 4.

3. Predictor-based feedback with complete information

We first consider systems of the form

$$\dot{x}(t) = A_0x(t) + A_1x(t-r) + Bu(t-h), \quad t\ge 0. \tag{5}$$

The extension to multiple state delays and time-varying input delay is straightforward and will be described later. The assumptions are that $x(t)$ is available, and a stabilizing input for $h=0$ exists (Assumption 1). Our aim is to predict $x(t+h)$ to generate the control when $h>r$.

Lemma 2. *For a given $A\in\mathbb{R}^{n\times n}$, let $\bar{L}$ be any matrix such that $\bar{A}=A-\bar{L}$ is Hurwitz. Consider the delay differential equation*

$$\dot{\xi}(t) = A\xi(t) - \bar{L}e^{\bar{A}d}\xi(t-d), \quad t>0, \tag{6}$$

initialized by any $\xi_{[-d,0]}\in\mathcal{C}([-d,0];\mathbb{R}^n)$. If, for some $\alpha\in[0,-\mu(\bar{A})]$,

$$\Gamma(\bar{L},\alpha,d) = \int_0^d \|\bar{L}e^{\bar{A}\theta}\|e^{\alpha\theta}d\theta < 1, \tag{7}$$

then $\|\xi(t)\|\le c_\xi e^{-\alpha t}$ for some $c_\xi>0$.

The proof of Lemma 2 is in Appendix.

Lemma 3. *If $\bar{A}=A-\bar{L}$ is Hurwitz and $\Gamma(\bar{L},0,d)<1$, the solution $\xi(t)$ of (6) is asymptotically stable.*

Remark 4. It is always possible to choose d that satisfies (7). $\Gamma(\bar{L},\alpha,d)$ is nonnegative, monotone and continuous with respect to d, and $\Gamma(\bar{L},\alpha,0)=0$. If, for a given $\bar{L}$, α and $\forall\xi$, $\Gamma(\bar{L},\alpha,\xi)<1$, any $d\ge 0$ satisfies (7). Otherwise, any $d<\min\{\xi:\Gamma(\bar{L},\alpha,\xi)>1\}$ satisfies (7). Consequently, Lemma 3 allows to determine an interval $[0,d]$ of delays for which the system is exponentially stable with a given rate α. Conversely, the rate of exponential stability given the delay d can be computed as $\alpha=\sup\{a\le-\mu(\bar{A}):\Gamma(\bar{L},a,d)<1\}$.

Theorem 5. *Given system (5), $\bar{L}$ such that $\bar{A}=A_0-\bar{L}$ is Hurwitz, and $\alpha\in[0,-\mu(\bar{A})]$, let $\bar{w}=\sup_w\{w:\Gamma(\bar{L},\alpha,w)<1\}$ and $d=\min\{r,\bar{w}\}$. Then*

$$\dot{\hat{x}}_1(t) = A_0\hat{x}_1(t) + A_1x(t+d-r) + Bu(t+d-h) + \bar{L}e^{\bar{A}d}\left(x(t)-\hat{x}_1(t-d)\right) \tag{8}$$

is an exponential estimate of $x(t+d)$ with rate α, that is, $\|x(t+d)-\hat{x}_1(t)\|\le c_1e^{-\alpha t}$ for some $c_1>0$ and for any $\hat{x}_{1,[-d,0]}\in\mathcal{C}([-d,0];\mathbb{R}^n)$.

Proof. In the first place, notice that (8) can be implemented because $d\le r\le h$ implies that both $x(t+d-r)$, $u(t+d-h)$ are available at time t. $\Gamma(\bar{L},\alpha,\delta)$ monotonically increases with δ, thus $\bar{w}$ is well defined (it may be ∞ when the inequality is always satisfied), and so it is d. The prediction error $\epsilon_1(t)=x(t+d)-\hat{x}_1(t)$ satisfies

$$\dot{\epsilon}_1(t) = A_0\epsilon_1(t) - L\epsilon_1(t-d), \tag{9}$$

with $L=\bar{L}e^{\bar{A}d}$ and $\epsilon_{1,[-d,0]}=x_{[0,d]}-\hat{x}_{1,[-d,0]}$, and it is exponentially stable with rate α by Lemma 2.