Automatica 85 (2017) 462–467

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Observability of singular systems with commensurate time-delays and neutral terms ☆

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ARTICLE INFO

Article history:
Received 23 March 2015
Received in revised form 3 April 2017
Accepted 1 August 2017

Keywords:
Singular systems
Descriptor systems
Systems with time-delays
Observability

ABSTRACT

This paper deals with the observability problem of a sort of singular systems with commensurate time-delays in the trajectories of the system, in the time derivative of the trajectories (neutral terms), and in the output system. By using a recursive algorithm, sufficient conditions (easy testable) are proposed for guaranteeing the backward and the algebraic observability of the system. This condition implies that the trajectories of the system can be reconstructed by using the actual and past values of the system output.

1. Introduction

The description of a variety of practical systems by means of singular systems, also called descriptive, implicit, or differential algebraic systems, has been shown to be useful since several decades ago as it is well explained in Campbell (1980). Such systems, as many others, may contain time-delay terms in the trajectory of the system, the input, and/or the system output. A compendium of new researching results for singular systems with time-delays has been recently published, Gu, Su, Shi, and Chu (2013). A variety of definitions and necessary and sufficient conditions of observability can be found in Cobb (1984), Hou and Müller (1999) and Yip and Sincovec (1981). Nevertheless, up to the authors' knowledge, there are few works dedicated to the study of the observability problem of singular systems with time delays, despite the increasing research on problems like solvability, stability, controllability (see, e.g. Cobb (2006)). In Perdon and Anderlucci (2006), an observer design is proposed for a general sort of discrete time singular systems with time-delays. For singular systems with one time-delay in the trajectories of the system, a condition guaranteeing the observability of the system is found in Wei (2013) (there, observability is interpreted as the reconstruction of the initial conditions). However such a condition seems to be difficult of checking. An observer design, based on a Lyapunov–Krasovsky functional, is proposed in Ezzine, Darouach, Souley Ali, and Messaoud (2013) for singular systems with a time-delay in the trajectories of the system (not in the system output nor in the time derivative of the trajectories of the system). Therefore, we may say that the observability problem of time-delay systems has not been tackled enough and certainly has not been completely solved.

The main motivation of this paper comes from the interest of tackling the observability problem of a general class of descriptor linear time-delay systems with neutral terms, namely systems whose dynamics is governed by equations as these ones,

$$J\dot{x}(t) = \sum_{i=1}^{k_f} F_i\dot{x}(t-ih) + \sum_{i=1}^{k_a} A_i x(t-ih)$$

$$y(t) = \sum_{i=1}^{k_c} C_i x(t-ih)$$

where the matrices J, F_i, A_i, and C_i are all constant and J could be a non square matrix, certainly it is assumed to be non invertible. The aim is to find out conditions under which the vector $x(t)$ may be reconstructed by using the trajectory of the system output $y(t)$. It is common to define the backward shift operator $\delta : x(t) \mapsto x(t-h)$ (see, e.g., Kamen (1991)), which allows for rewriting the

☆ F.J. Bejarano acknowledges the financial support by Proyecto SIP 20171673, Instituto Politécnico Nacional. Gang Zheng acknowledges the financial support by Ministry of Higher Education and Research Nord-Pas de Calais Regional Council, by FEDER through the Contrat de Projets Etat Region (CPER), and by ARCIR Project ESTIREZ Nord-Pas de Calais Regional Council. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Delin Chu under the direction of Editor Ian R. Petersen.

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http://dx.doi.org/10.1016/j.automatica.2017.08.001

above dynamic equations as

$$J\dot{x}(t) = F(\delta)\dot{x}(t) + A(\delta)x(t)$$
$$y = C(\delta)$$

where, by definition, $F(\delta) = \sum_{i=0}^{k_f} F_i\delta^i$, $A(\delta) = \sum_{i=0}^{k_a} A_i\delta^i$, and $C(\delta) = \sum_{i=0}^{k_c} C_i\delta^i$. The definition $E(\delta) = J - F(\delta)$ yields the following compact representation of the previous system equations:

$$E(\delta)\dot{x}(t) = A(\delta)x(t)$$
$$y(t) = C(\delta)x(t)$$

The above compact representation allows for studying the system considering the elements of the matrices appearing in the system over the polynomial ring.

The following notation will be used along the paper. $\mathbb{R}$ is the field of real numbers, and $\mathbb{N}$ is the ring of nonnegative integers. $\mathbb{R}[\delta]$ is the polynomial ring over the real field $\mathbb{R}$. I_n is the identity matrix of dimension n by n. Since hereinafter mostly matrices with terms in the polynomial ring $\mathbb{R}[\delta]$ will be used, instead of using the symbol (δ) in front of a matrix to indicate that the latter has terms in $\mathbb{R}[\delta]$, we will use the following notation. Given the ring $\mathfrak{R} = \mathbb{R}[\delta]$, $\mathfrak{R}^n$ denotes the module of column vectors with n terms in $\mathfrak{R}$ and $\mathfrak{R}^{1\times n}$ is the module of row vectors. $\mathfrak{R}^{r\times s}$ is meant for the set of matrices of dimension r by s, all of whose entries are in $\mathfrak{R}$. A square matrix M whose terms belong to $\mathfrak{R}$ is called unimodular if its determinant is a nonzero constant. A matrix $M \in \mathfrak{R}^{r\times s}$ is called left invertible if there exists a matrix $M^+ \in R^{s\times r}$ such that $M^+M = I_s$. For a matrix F (with terms in $\mathfrak{R}$), $\operatorname{rank} F$ denotes the rank of F over $\mathfrak{R}$. The degree of a polynomial $p(\delta) \in \mathbb{R}[\delta]$ is denoted by $\deg p(\delta)$. For a matrix F, with terms in $\mathfrak{R}$, $\deg F$ denotes the greatest degree of all entries of F. By $\operatorname{Inv}_s F$ we denote the set of invariant factors (or invariant polynomials) of the matrix F (Gohberg, Lancaster, & Rodman, 2009). The limit from below of a time valued function is denoted as $f(t_-)$.

2. Formulation of the problem

Hence, we will consider the sort of systems that can be represented by the following equations:

$$E\dot{x}(t) = Ax(t) \quad (1a)$$
$$y(t) = Cx(t) \quad (1b)$$

where, $x(t) \in \mathbb{R}^n$ and $y(t) \in \mathbb{R}^p$. The dimension of the matrices is as follows: $E \in \mathfrak{R}^{\bar{n}\times n}$, $A \in \mathfrak{R}^{\bar{n}\times n}$, $C \in \mathfrak{R}^{p\times n}$ ($\mathfrak{R} = \mathbb{R}[\delta]$). According to the notation defined at the introduction, we use δ as the shift backward operator, i.e., $\delta : x(t) \mapsto x(t-h)$, where h is a real positive number. We assume that there exists a solution of (1a) (which might be not unique) and that every solution of (1a) is piecewise differentiable.

The following definitions are taking as the starting point for the observability analysis that will be done further.

Definition 1. The system (1) is called backward observable (**BO**) on $[t_1, t_2]$ if, and only if, for each $\tau \in [t_1, t_2]$ there exist $\bar{t}_1$ and $\bar{t}_2 \leq \tau$ such that $y(t) = 0$ for all $t \in [\bar{t}_1, \bar{t}_2]$ implies $x(\tau_-) = 0$.

The previous definition is somewhat different to definitions given in Delfour and Mitter (1972) and Lee and Olbrot (1981). The main difference has to do with the fact that backward observability considers only the previous values of the system output. In that sense, backward observability is related with the final observability given in Lee and Olbrot (1981). In fact, final observability implies backward observability.

Definition 2. The system (1) is algebraically observable (**AO**) if it exists a time t_1 such that $x(t)$ can be expressed for all $t \geq t_1$ by a formula of the type

$$x(t) = \beta_0 y(t) + \beta_1\dot{y}(t) + \cdots + \beta_l y^{(l)}(t), \quad (2)$$

for some nonnegative integer l, where $\beta_i \in \mathfrak{R}^{n\times p}$ $(i = 0, 1, \ldots, l)$, provided that the system output $y(t)$ is a smooth function.

The backward observability is related with the map between the trajectories of the system and the system output, whereas a explicit relationship is given by the algebraic observability. Furthermore, by (2), it is clear that AO implies BO. However, as we will see in the next example, in general, BO does not imply AO.

Example 1. Let us see a system that is backward observable, but is not algebraically observable.

$$\dot{x}(t) = x(t-h)$$
$$y(t) = x(t-h)$$

There, if $y(\xi) = x(\xi - h) = 0$ on the interval $[0, \gamma h]$ $(\gamma \geq 2)$ then, $x(\xi) = 0$ on $[-h, (\gamma-1)h]$ and $x(\xi)$ is constant on $[0, \gamma h]$. Therefore, $x(\xi) = 0$ on $[(\gamma-1)h, \gamma h]$. Therefore, we can say that $x(t)$ is BO on $[\gamma h, \gamma h + \bar{t}]$ for any $\bar{t} > 0$.

However, as it is possible to verify, $x(t)$ cannot be expressed as in (2). Indeed, for the initial condition $x(t) = \begin{cases} 0, & t \in [-h, 0) \\ 1, & t = 0 \end{cases}$, we have

$$x(t) = \begin{cases} 1, & t \in [0, h] \\ t - h + 1, & t \in [h, 2h] \\ \dfrac{t^2}{2} + (1-h)t + 1 - h, & t \in [2h, 3h] \end{cases}$$

By the previous equation we can see that it is not possible to have an expression for $x(t)$ like that of (2).

3. Like Silverman–Molinari algorithm

The technique to be used is based on the approach followed in Bejarano, Perruquetti, Floquet, and Zheng (2013) and Bejarano and Zheng (2014). The condition guaranteeing the observability will be checked by means of a matrix denoted by N_{k^*} that will be defined further. As for, let us select a unimodular matrix S_0 so that the following equation is obtained:

$$S_0 \begin{bmatrix} -I_{\bar{n}} & | & E \end{bmatrix} = \left[\begin{array}{c|c} J_0 & R_0 \\ H_0 & 0 \end{array}\right] \text{ such that } R_0 \in \mathfrak{R}^{\beta_0\times n} \quad (3)$$

where $\beta_0 = \operatorname{rank}(E)$.

Now, let us define $\Delta_0 = C$. For the kth step $(k \geq 1)$ the matrices Δ_k, N_k and H_k are generated by using the following general procedure. The matrix Δ_k is defined as follows:

$$\Delta_k = H_{k-1}A \quad (4)$$

The matrix N_k is formed by the concatenation of matrices Δ_0 to Δ_k $(k \geq 1)$, that is,

$$N_k = \begin{bmatrix} \Delta_0 \\ \Delta_1 \\ \vdots \\ \Delta_k \end{bmatrix} \quad (5)$$

For the construction of the matrix H_k, we require to select a unimodular matrix S_k so that

$$S_k \left[\begin{array}{c|c} -I_{\bar{n}} & E \\ \hline 0 & N_k \end{array}\right] = \left[\begin{array}{c|c} J_k & R_k \\ \hline H_k & 0 \end{array}\right] \text{ such that } R_k \in \mathfrak{R}^{\beta_k\times n} \quad (6)$$

where $\beta_k = \operatorname{rank}\begin{bmatrix} E \\ N_k \end{bmatrix}$.