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Robustification of sample-and-hold stabilizers for control-affine time-delay systems☆

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ABSTRACT

This paper deals with the stabilization in the sample-and-hold sense of nonlinear, control affine, retarded systems, affected by actuation disturbances and observation errors. Input-to-state stability redesign methods are used in order to design a new sampled-data controller. It is shown that stabilization in the sample-and-hold sense can be preserved by means of this new controller, regardless of the above disturbances and errors. It is assumed that both actuator disturbance and observation error are bounded, and the (arbitrary) bounds are known *a-priori*. It is moreover assumed that the observation errors do not affect or affect marginally the new control term obtained by input-to-state stability redesign. Simulations on a continuous stirred tank reactor with recycle validate the theoretical results.

1. Introduction

Sampled-data stabilization of linear (see Fridman, 2010; Liu & Fridman, 2012; Seuret, 2012; Seuret & Peet, 2013; Seuret & da Silva, 2012), bilinear (see Omran, Hetel, Richard, & Lamnabhi-Lagarrigue, 2014) and nonlinear systems (see Karafyllis & Krstic, 2012a, 2012b; Laila, Nesic, & Teel, 2002; Mazenc, Malisoff, & Dinh, 2013) has been studied in the literature by many approaches, such as: the time-varying delay approach (see Fridman, 2010, 2014; Fridman, Seuret, & Richard, 2004); the approximate system discretization approach (see Grune & Nesic, 2003; Laila et al., 2002; Nesic & Grune, 2005; Nesic & Teel, 2004; Nesic & Teel, 2001; Nesic, Teel, & Kokotovic, 1999; Postoyan, Ahmed-Ali, & Lamnabhi-Lagarrigue, 2009); the hybrid system approach (Carnevale, Teel, & Nesic, 2007; Dacic & Nesic, 2007; Goebel, Sanfelice, & Teel, 2009, 2012; Haddad, Chellaboina, & Nersesov, 2014; Naghshtabrizi, Hespanha, & Teel, 2006, 2008, 2010; Nesic & Teel, 2004; Nesic, Teel, & Carnevale, 2009); the stabilization in the sample-and-hold sense approach (see Clarke, 2010; Clarke, Ledyaev, Sontag, & Subbotin, 1997). In particular, the notion of stabilization in the sample-and-hold sense, introduced in 1997 in Clarke et al. (1997), has been widely studied for systems described by ordinary differential equations. The stabilization problem, in the continuous time, for nonlinear systems with delays in the state and/or in the input/output channels, has been studied by many researchers in the last years (see, for instance, Germani, Manes, & Pepe, 2003; Hua, X.Guan, & Shi, 2004; Liberis & Krstic, 2012, 2013; Liberis, Jankovic, & Krstic, 2012; Lien, 2004; Marquez-Martinez & Moog, 2004; Mazenc, Niculescu, & Bekaik, 2011; Oguchi & Richard, 2006; Oguchi, Watanabe, & Nakamizo, 2002; Pepe, 2013; Yang & Wang, 2012; Zhang & Cheng, 2005). Sampled-data controllers for nonlinear systems, under delayed measurements and delayed control law, have been considered in Karafyllis and Krstic (2012a, 2012b). More recently, the stabilization in the sample-and-hold sense has been extended to fully nonlinear time-delay systems (see Pepe, 2014, 2016).

It is well known that actuation disturbances and observation errors can deteriorate the performances of continuous-time controllers, and even cause instabilities (see, for instance, Malisoff & Sontag, 2004; Sontag, 1989a). The same, or even worse, kind of problems arises when the control law is applied by sampling and holding, and the deterioration of the performances may be significant specially with discontinuous feedbacks (see Ledyaev & Sontag, 1999; Sontag, 1999a, 1999b).

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In Ledyaev and Sontag (1999), Prieur (2005), Prieur and Trelat (2006) and Sontag (1999a, 1999b), the problems related to (suitably small/vanishing) actuation disturbances and observation errors, in sampled-data stabilization, have been extensively dealt with, for systems described by ordinary differential equations. In Pepe (2015), concerning the stabilization in the sample-and-hold sense of systems described by ordinary differential equations, an input-to-state stability redesign method is exploited, in order to attenuate the effects of bounded actuation disturbances and suitably bounded observation errors.

As far as sampled-data control laws for nonlinear time-delay systems are concerned, the problems related to arbitrarily large actuation disturbances and arbitrarily large observation errors have never been dealt with in the literature (see Pepe, 2014, 2016 and references therein). In particular, the problem of arbitrarily reducing the effect of an arbitrarily large actuator disturbance, as well as of an arbitrarily large observation error, has not been exhaustively addressed in the literature concerning nonlinear retarded systems. In this paper, we fill this gap.

We show, for the important class of nonlinear, control-affine, time-delay systems, that the input-to-state stability redesign method can be successfully used for sampled-data stabilization, in order to attenuate the effects of any bounded actuation disturbance and any bounded observation error, as long as this observation error does not affect, or affects marginally, the new added state feedback. Moreover, in order to take into account input saturation constraints (see, for instance, Fridman & Dambrine, 2009; Pepe & Ito, 2012 and Zhou, Lin, & Duan, 2010), here the new added state feedback sampled-data control term is suitably bounded, in the system state space. The bound mainly depends on the actuation disturbances and observation errors amplitudes. The results here provided improve significantly the robustness property of stabilizers in the sample-and-hold sense for nonlinear time-delay systems.

Here the standard LgV-type term to add to the control law is considered by the introduction of suitable control Lyapunov–Krasovskii functionals which are invariantly differentiable and smoothly separable. In order to fulfill input magnitude constraints, we incorporate saturation into the LgV-type control term. The LgV-type term can manage also observation errors, provided that their bounds are known *a-priori*, and as long as it is marginally affected by these errors. This fact can happen, for instance, when the LgV-term depends on state variables that can be measured better than other state variables, or when the variation of the LgV-term is sufficiently slow with respect to measurement errors.

We apply the theoretical results to a continuous stirred tank reactor with recycle time-delay (see Di Ciccio, Bottini, Pepe, & Foscolo, 2011). Sontag's universal formula is used in order to obtain a steepest descent feedback induced by a suitable control Lyapunov–Krasovskii functional (see Pepe, 2013, Sontag, 1989b). Then, the input-to-state stability redesign method is used to find the new term in the controller. It is shown that all the assumptions, introduced for the theory developed in this paper, hold for the continuous stirred tank reactor. Performed simulations show that the new robustified sampled-data state feedback can drastically reduce the effect of significant actuation disturbances and measurement errors affecting both the reactant concentration and the reactor temperature. In particular, the robustified controller forces the state variables into a neighborhood of the origin which is much smaller than the one obtained with the non-robustified controller. A preliminary version of this paper has been published in Di Ferdinando and Pepe (2016).

The paper is organized as follows: in Section 2, the problem under investigation is stated; in Section 3 a quick presentation of the main result is shown; in Section 4 the main assumptions, needed throughout the paper, are introduced and the new designed sampled-data control law is shown; in Section 5 theoretical convergence results are provided; in Section 6 the application of the provided methodology on a continuous stirred tank reactor with recycle is shown. For the sake of readability, the involved proof of the main result is reported in the Appendix.

Notation N denotes the set of nonnegative integer numbers, R denotes the set of real numbers, $R^\star$ denotes the extended real line $[-\infty, +\infty]$, R^+ denotes the set of nonnegative reals $[0, +\infty)$. The symbol $|\cdot|$ stands for the Euclidean norm of a real vector, or the induced Euclidean norm of a matrix. For a given positive integer n and a given positive real h, the symbol $\mathcal{B}_h^n$ denotes the subset $\{x \in R^n : |x| \le h\}$. The essential supremum norm of an essentially bounded function is indicated with the symbol $\|\cdot\|_\infty$. For a positive integer n, for a positive real Δ (maximum involved time-delay): $\mathcal{C}$ and $W^{1,\infty}$ denote the space of the continuous functions mapping $[-\Delta, 0]$ into R^n and the space of the absolutely continuous functions, with essentially bounded derivative, mapping $[-\Delta, 0]$ into R^n, respectively; $\mathcal{Q}$ denotes the space of bounded, right-continuous functions, with possibly a finite number of points with jump-type discontinuity, mapping $[-\Delta, 0)$ into R^n. For $\phi \in \mathcal{C}$, $\phi_{[-\Delta,0)}$ is the function in $\mathcal{Q}$ defined, for $\tau \in [-\Delta, 0)$, as $\phi_{[-\Delta,0)}(\tau) = \phi(\tau)$. For a positive real p, for $\phi \in \mathcal{C}$, $\mathcal{C}_p(\phi) = \left\{\psi \in \mathcal{C} : \|\psi - \phi\|_\infty \le p\right\}$. The symbol $\mathcal{C}_p$ denotes $\mathcal{C}_p(0)$. For a continuous function $x : [-\Delta, c) \to R^n$, with $0 < c \le +\infty$, for any real $t \in [0, c)$, x_t is the function in $\mathcal{C}$ defined as $x_t(\tau) = x(t+\tau)$, $\tau \in [-\Delta, 0]$. We recall that a continuous function $\gamma : R^+ \to R^+$ is: of class $\mathcal{P}_0$ if $\gamma(0) = 0$; of class $\mathcal{P}$ if it is of class $\mathcal{P}_0$ and $\gamma(s) > 0$, $s > 0$; of class $\mathcal{K}$ if it is of class $\mathcal{P}$ and strictly increasing; of class $\mathcal{K}_\infty$ if it is of class $\mathcal{K}$ and unbounded. The symbol I_d denotes the identity function in R^+. The symbol $\circ$ denotes composition (of functions). For a given positive integer n, for a symmetric, positive definite matrix $P \in R^{n\times n}$, $\lambda_{max}(P)$ and $\lambda_{min}(P)$ denote the maximum and the minimum eigenvalue of P, respectively. Throughout the paper, ODE stands for ordinary differential equation, RFDE stands for retarded functional differential equation, ISS stands for input-to-state stable or input-to-state stability, CLKF stands for control Lyapunov–Krasovskii functional and CSTR stands for continuous stirred tank reactor.

2. Problem statement

Let us consider a time-delay, control-affine nonlinear system, described by the following equations (see Hale & Lunel, 1993; Kolmanovskiii & Myshkis, 1999, Pepe, 2009):

$$\begin{aligned} &\dot{x}(t) = f(x_t) + g(x_t)\,u(t), \quad t \ge 0, \ a.e. \\ &x(\tau) = x_0(\tau), \quad \tau \in [-\Delta, 0], \quad x_0 \in \mathcal{C}, \end{aligned} \tag{1}$$

where: $x(t) \in R^n$, n is a positive integer; Δ is a positive real, the maximum involved time-delay; $x_t \in \mathcal{C}$; f is a map from $\mathcal{C}$ to R^n, Lipschitz on bounded sets; g is a map from $\mathcal{C}$ to $R^{n\times m}$, Lipschitz on bounded sets; m is a positive integer; $u(t) \in R^m$ is the input signal, Lebesgue measurable and locally essentially bounded. Eq. (1) admits a locally absolutely continuous solution in a maximal time interval $[0, b)$, with $0 < b \le +\infty$ (see Hale & Lunel, 1993).

In the following, the notion of semi-global practical stability, which is typical in the nonlinear sampled-data systems literature (see for instance Carnevale et al., 2007; Grune & Nesic, 2003; Karafyllis & Krstic, 2012; Nesic & Teel, 2004; Postoyan et al., 2009), is used. It is shown in Pepe (2014), that a steepest descent feedback $k : \mathcal{C} \to R^m$ (continuous or not), locally bounded, suitably induced by a CLKF, is a stabilizer in the sample-and-hold sense for the system (1). It means that: for any large ball and small ball of the origin, there exist a sampling period and a positive time T, such that, if the state feedback k is sampled and held by this (or smaller) sampling period, the trajectories starting in the large ball are kept uniformly bounded, are driven into the small ball in time T, and are kept in thereafter (see Definition 5.2 in Pepe (2014) for more details).