



# Robust stabilization of delayed neural fields with partial measurement and actuation<sup>☆</sup>



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## ABSTRACT

Neural fields are integro-differential equations describing spatiotemporal activity of neuronal populations. When considering finite propagation speed of action potentials, neural fields are affected by space-dependent delays. In this paper, we provide conditions under which such dynamics can be robustly stabilized by a proportional feedback acting only on a portion of the neuronal population and by relying on measurements of this subpopulation only. To that aim, in line with recent works, we extend the concept of input-to-state stability (ISS) to generic nonlinear delayed spatiotemporal dynamics and provide a small-gain result relying on Lyapunov–Krasovskii functionals. Exploiting the robustness properties induced by ISS, we provide conditions under which a uniform control signal can be used for the whole controlled subpopulation and we analyze the robustness of the proposed strategy to measurement and actuation delays. These theoretical findings are compared to simulation results in a model of pathological oscillations generation in Parkinson's disease.

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## 1. Introduction

The use of spatiotemporal models to describe the activity of neuronal populations is considerably increasing. This success is due to technological advances, including electrode arrays and brain imaging, that allow recordings of unprecedented resolution in both time and space. The possibility to represent the spatiotemporal evolution of neural activity constitutes a crucial feature to deepen our understanding of cerebral functions or diseases that involve propagating waves or pattern formation, such as memory, epilepsy, or Parkinson's disease (Bressloff, 2012; Coombes et al., 2014).

Neural fields are nonlinear integro-differential equations designed to model the spatiotemporal evolution of neuronal populations. They offer a good compromise between physiological plausibility, richness of behaviors, and analytical tractability. Neural

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fields dynamics are derived based on statistical considerations by assuming that neural activity evolves on a continuous medium. They rely on the seminal works (Amari, 1977; Wilson & Cowan, 1973). They have been the subject of an intense research, with a wide range of applications to neuroscience: we refer the reader to Bressloff (2012) for a detailed survey on neural fields. From an analytical point of view, several works have been devoted to the existence and estimation of equilibrium patterns, local and global stability analysis, bifurcation analysis, and existence of periodic orbits: see e.g. Faugeras et al., (2009); Laing et al., (2002); Pinto & Ermentrout, (2001); Veltz & Faugeras, (2010).

A key source of complexity, and subsequent richness of producible dynamical behaviors, lie in the delays induced by the non-instantaneous communication between neurons. From a physiological perspective, these delays are needed to model the finite speed of signals along axons. They typically depend on the physical distance between the considered neurons. Delayed neural fields have been the subject of several mathematical studies, including Atay & Hutt, (2004, 2006); beim Graben & Hutt, (2014); Veltz & Faugeras, (2011).

A central question in the analytical study of neural fields stands in the stability of stationary solutions. This question is crucial for the understanding of the resulting brain function as modeled by the equilibrium pattern. In some applications, it is also motivated by the need for non-oscillating responses. In the case of Parkinson's disease, which is the original motivation for the present work,

such oscillations in specific frequency bands are correlated with motor symptoms. The mechanisms for this pathological oscillations onset are still a matter of debate, but are believed to be linked to the combination of propagation delays and exaggerated synaptic weights between neuronal populations (Nevado-Holgado et al., 2010; Pasillas-Lépine, 2013; Plenž & Kital, 1999). Following this hypothesis, a natural way to improve motor performance would be to attenuate these pathological oscillations, by relying on electrical (Benabid et al., 1991) or optical (Gradinaru et al., 2009) stimulation. This was investigated in Haidar et al. (2016) by relying on a model of the averaged activity, and then extended in Detorakis et al. (2015) to spatiotemporal dynamics.

Despite numerous theoretical developments on neural fields, with or without delays, the question of their stabilization by a control signal had not yet been addressed. The purpose of this note is to analyze the stability and robustness induced by proportional feedback control. In order to cope with implementation constraints, we consider the case when the stimulation signal impacts directly only a portion of the neuronal population, which we refer to as the “controlled population”. We also impose that the feedback relies only on measurements on this controlled population in order to limit the number of required sensing electrodes. Our main result states that global robust stabilization of delayed neural fields can always be achieved provided that the internal synaptic strength of the “uncontrolled” population is lower than the inverse of the Lipschitz constant of the activation function. This result in turn corrects a flaw in the (incomplete) proof provided in Detorakis et al. (2015). We also address two constraints of practical relevance, which were addressed only through simulations in Detorakis et al. (2015). The first one is the limited number of available stimulation points: we show that stabilization can still be achieved with a unique control signal provided that the synaptic weights are homogeneous enough. The second one is the unavoidable feedback delays needed to acquire and estimate the neural activity: we show that the considered proportional scheme is robust to such delays but that, not surprisingly, this robustness may decrease as the feedback gain increases.

In order to establish these results, we rely on a spatiotemporal extension of input-to-state stability (ISS, (Sontag, 1989, 2008)). This extension imposes that the state’s  $L_2$ -norm over the spatial domain be bounded by a vanishing function of the spatial  $L_2$ -norm of initial conditions plus a term “proportional” to the spatial  $L_2$ -norm of exogenous inputs. A further extension is required in order to allow for delayed dynamics. After having extended the classical Lyapunov sufficient condition for ISS (Sontag, 1989), we present a small-gain theorem applicable to such class of systems (Section 3). This extension is in line with recent works addressing ISS for infinite-dimensional systems (Dashkovskiy & Mironchenko, 2013; Karafyllis & Jiang, 2007; Mironchenko & Ito, 2016; Prieur & Mazenc, 2012), including retarded functional differential equations (Karafyllis et al., 2008; Mazenc et al., 2008; Pepe & Jiang, 2006; Teel, 1998). Focusing on delayed neural fields under partial proportional feedback, we then show that the uncontrolled population is ISS with respect to the state of the controlled population and possible exogenous signals, provided that the spatial  $L_2$ -norm of its internal synaptic weights is below a certain bound. On the other hand, we show that the controlled population can always be made ISS by picking a sufficiently large feedback gain. ISS of the overall closed-loop system is then established based on small-gain arguments. Both stabilization by a single stimulation signal and robustness to feedback delays are addressed by exploiting the robustness induced by the ISS property (Section 4). Simulations are then presented in Section 5 to confirm our theoretical expectancies and to evaluate the performance of the considered control laws (proportional, uniform, and involving feedback delays).

**Notation.** Given  $x = (x_1, \dots, x_n)^T \in \mathbb{R}^n$ ,  $|x|$  denotes its Euclidean norm:  $|x| := \sqrt{x_1^2 + \dots + x_n^2}$ . Given two sets  $\Omega_1$  and  $\Omega_2$ ,  $C(\Omega_1, \Omega_2)$  (resp.  $C^1(\Omega_1, \Omega_2)$ ) denotes the set of all continuous (resp. continuously differentiable) functions from  $\Omega_1$  to  $\Omega_2$ .  $L_2(\Omega_1, \Omega_2)$  denotes the set of all square integrable functions from  $\Omega_1$  to  $\Omega_2$ , meaning all functions  $f : \Omega_1 \rightarrow \Omega_2$  such that  $\int_{\Omega_1} |f(s)|^2 ds < \infty$ . Given a set  $\Omega \subset \mathbb{R}^q$ ,  $\#_\Omega$  denotes its Lebesgue measure. Given  $f : \Omega \rightarrow \mathbb{R}^n$ , with  $\Omega = \Omega_1 \times \dots \times \Omega_q$  where  $\Omega_i \subset \mathbb{R}$  for each  $i \in \{1, \dots, q\}$ , we compactly write  $\int_\Omega f(r) dr$  to denote the multiple integral  $\int_{\Omega_1} \dots \int_{\Omega_q} f(r) dr_q \dots dr_1$ , with  $r = (r_1, \dots, r_q)^T$ . We define  $\mathcal{F}^n := L_2(\Omega, \mathbb{R}^n)$  and  $\mathcal{C}^n := C([-d; 0], \mathcal{F}^n)$  for some constant  $d > 0$ .  $\mathcal{F}^n$  is a Banach space for the  $L_2$ -norm  $\|\cdot\|_{\mathcal{F}^n}$  defined as  $\|x\|_{\mathcal{F}^n} := \sqrt{\int_\Omega |x(s)|^2 ds}$  for each  $x \in \mathcal{F}^n$ . Similarly,  $\mathcal{C}^n$  is a Banach space for the norm  $\|\cdot\|_{\mathcal{C}^n}$  defined as  $\|x\|_{\mathcal{C}^n} := \sup_{t \in [-d; 0]} \|x(t)\|_{\mathcal{F}^n}$  for all  $x \in \mathcal{C}^n$ . We also denote by  $\mathcal{U}^n$  the set of all measurable locally bounded functions from  $\mathbb{R}_{\geq 0}$  to  $\mathcal{F}^n$ . When the context is sufficiently clear, we will simply refer to  $\mathcal{F}^n$ ,  $\mathcal{C}^n$  and  $\mathcal{U}^n$  as  $\mathcal{F}$ ,  $\mathcal{C}$  and  $\mathcal{U}$  respectively. Given  $x \in \mathcal{C}$  and  $t \in [-d; 0]$ , we indicate by  $[x(t)](r)$  the value taken by the function  $x(t) \in \mathcal{F}$  at position  $r \in \Omega$ . A function  $\alpha : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$  is said to be of class  $\mathcal{K}$  if it is continuous, zero at zero and increasing. It is said to be of class  $\mathcal{K}_\infty$  if it satisfies additionally  $\lim_{s \rightarrow +\infty} \alpha(s) = +\infty$ . A function  $\beta : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$  is said to be of class  $\mathcal{K}\mathcal{L}$  if  $s \mapsto \beta(s, t)$  is of class  $\mathcal{K}$  for any fixed  $t \in \mathbb{R}_{\geq 0}$  and, for any fixed  $s \in \mathbb{R}_{\geq 0}$ ,  $t \mapsto \beta(s, t)$  is continuous and non-increasing and tends to zero as its argument tends to  $+\infty$ .

## 2. Delayed neural fields

We start by introducing the spatiotemporal model under concern. Delayed neural fields are integro-differential equations of the form:

$$\tau_i(r) \frac{\partial z_i}{\partial t}(r, t) = -z_i(r, t) + S_i \left( \sum_{j=1}^n \int_\Omega w_{ij}(r, r') z_j(r', t - d_j(r, r')) dr' + I_i^{\text{ext}}(r, t) \right), \quad (1)$$

for  $i \in \{1, \dots, n\}$ .  $\Omega$  denotes a set of  $\mathbb{R}^q$ ,  $q \in \{1, 2, 3\}$ , representing the physical support of the populations; throughout this paper, we will assume that  $\Omega$  is compact.  $r, r' \in \Omega$  are the space variables, whereas  $t \in \mathbb{R}_{\geq 0}$  is the time variable.  $z_i(r, t) \in \mathbb{R}$  represents the neuronal activity of population  $i$ , at position  $r \in \Omega$ , and at time  $t \in \mathbb{R}_{\geq 0}$ .  $\tau_i(r) > 0$  is the time decay constant of the activity of population  $i$  at position  $r \in \Omega$ . The kernel  $w_{ij} : \Omega \times \Omega \rightarrow \mathbb{R}$  is a bounded function describing the synaptic strength between location  $r'$  in population  $j$  and location  $r$  in population  $i$ .  $I_i^{\text{ext}} : \Omega \times \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}$  is a bounded function describing the external input of population  $i$ , arising either from the influence of exogenous cerebral structures or from an artificial stimulation device. The function  $d_j : \Omega \times \Omega \rightarrow [0; \bar{d}]$ ,  $\bar{d} \geq 0$ , is a continuous function representing the axonal, dendritic and synaptic delays between a pre-synaptic neuron at position  $r'$  in population  $j$  and a post-synaptic neuron at position  $r$ .  $S_i : \mathbb{R} \rightarrow \mathbb{R}$  is a nondecreasing globally Lipschitz function, known as the activation function of the neural population  $i$ .

The compactness assumption on  $\Omega$  is realistic as neuronal populations are typically circumscribed to a limited physical domain. Although  $\tau_i$  is assumed to be uniform in most existing works on neural fields, we allow it to depend on the position  $r$  for the sake of generality. Similarly, in most neuroscience works, the synaptic kernels  $w_{ij}$  are assumed to depend only on the distance between  $r$  and  $r'$  (i.e.  $w_{ij}(r, r') = w_{ij}(|r - r'|)$ ). Typical shapes include Gaussian functions or “Mexican hats”. Here we allow for more

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