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Stabilization of MISO fractional systems with delays☆

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ABSTRACT

We consider multi-input single-output (MISO) fractional systems of commensurate fractional orders with different input or output delays. We derive explicit expressions of left and right coprime factorizations over H_∞ and of the associated Bézout factors of the transfer matrix of the systems. These factors allow the construction of the Youla–Kučera parametrization of the set of stabilizing controllers which guarantee the internal stability of the closed-loop systems.

1. Introduction

Fractional systems are systems described by differential equations involving non-integer order derivatives and/or integrals. Consequently, in the frequency domain, their transfer functions contain non-integer powers of the Laplace variable s. This kind of models has become more popular in many fields in the past two decades since it provides a better fit to data being then more succinct than a standard model. Refer, for example, to Miller and Ross (1993) for basic backgrounds on fractional calculus and to Freeborn (2013) and Sabatier, Agrawal, and Machado (2007) for its recent applications on modeling.

Delays are encountered almost everywhere due, for example, to distance of transmission and it is well-known that they have important effects on the stability of the systems (Richard, 2003).

While integer-order systems with delays have been intensively studied (Richard, 2003), the literature on fractional systems with delays is still quite small. Particularly, the stabilization problem of fractional systems with delays has rarely been addressed. Some available studies are classical (Özbay, Bonnet, & Fioravanti, 2012) and fractional PID controller design (Hamamci, 2007), fractional sliding mode control (Si-Ammour, Djennoune, & Bettayeb, 2009), factorization approach to control synthesis (Bonnet & Partington, 2002, 2007).

In the framework of fractional representation approach to synthesis problems (Vidyasagar, 1985), SISO fractional delay systems was considered in Bonnet and Partington (2002, 2007) and coprime factorizations together with the corresponding Bézout factors of the transfer function of these systems have been derived. In Curtain, Weiss, and Weiss (1996), coprime factors were presented for a large class of MIMO infinite-dimensional systems which include delay systems. The factors were determined from a state-space realization of the (regular) system which was given in terms of the semigroup of the system. Such realizations are not much considered for fractional systems.

For the particular class of MIMO (integer-order) systems with I/O delays, the problem of parametrization of stabilizing controllers was solved in Mirkin and Raskin (1999) and Moelja and Meinsma (2003). The idea was to reduce the problem to an equivalent finite-dimensional stabilization problem by involving an unstable finite-dimensional system and a stable infinite-dimensional system (FIR filter). In Mondié and Loiseau (2004), a procedure to compute right coprime factorizations over a Bézout domain was proposed for spectrally controllable MIMO (integer-order) systems with input delays. For MISO structure, a class of (integer-order) systems with multiple transmission delays was studied in Bonnet and Partington (2004) and coprime factorizations and associated Bézout factors over H_∞ were derived.

In this paper, we are interested in the stabilization problem of MISO fractional systems with different I/O delays which are not necessarily commensurate. This MISO structure appeared, for example, in communication systems (Quet, Ataşlar, İftar, Özbay, Kalyanaraman, & Kang, 2002). We would like to obtain the set of all stabilizing controllers by determining a doubly coprime factorization over H_∞ of the transfer matrix and the associated

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Bézout factors, which allow the construction of the Youla–Kučera parametrization (Vidyasagar, 1985). As in the finite-dimensional case, the Youla–Kučera parametrization gives the set of all H_∞-stabilizing controllers in terms of one free parameter. Note that in Quadrat (2006), a parametrization of the set of all stabilizing controllers is given in terms of two free parameters for MIMO systems once we already know a particular stabilizing controller. Our strategy here is to work directly on the Bézout identity in order to get explicit expressions of Bézout factors in terms of the matrix transfer function. Such explicit expressions could not be easily derived in Mirkin and Raskin (1999), Moelja and Meinsma (2003) and Mondié and Loiseau (2004) even in the case of standard delay systems. We hope that the explicit form will facilitate the use of these factors in controllers design while the use of the frequency domain representation of the systems agrees well with the modeling practice of fractional systems (Sabatier et al., 2007).

The paper is organized as follows. In Section 2, the class of systems of interest and some background are presented. The results are stated in Sections 3 and 4. We give in Section 3 explicit expressions of left coprime factorizations and associated Bézout factors over H_∞ of the transfer function of the systems under study. Right coprime factorizations and right Bézout factors are given in Section 4 for a large subclass of the class of systems considered. Examples are provided to illustrate the results. Finally, Section 5 gives conclusions and perspectives.

2. A class of MISO fractional time-delay systems

We consider systems described by transfer matrices of the form

$$G(s) = \left[e^{-sh_1}R_1(s^\alpha), \dots, e^{-sh_n}R_n(s^\alpha)\right], \tag{1}$$

where $0 \le h_k \in \mathbb{R}$ for $k = 1, \dots, n$ are the delays; $\alpha \in \mathbb{R}$, $0 < \alpha < 1$; $R_k(s^\alpha) = \widetilde{q}_k(s^\alpha)/\widetilde{p}_k(s^\alpha)$, where $\widetilde{p}_k$ and $\widetilde{q}_k$ are polynomials of integer degree in s^α, $\widetilde{p}_k(s^\alpha)$ and $\widetilde{q}_k(s^\alpha)$ have no common roots, and $\deg\widetilde{p}_k \ge \deg\widetilde{q}_k$ for $k = 1, \dots, n$; d_k is the degree in s^α of $\widetilde{p}_k$; s is in the principle branch $\mathbb{C}\backslash\mathbb{R}_-$, that is $\arg(s) \in (-\pi, \pi)$, in order to guarantee a unique value of the transfer function involving s^α with $\alpha \in (0, 1)$.

Some notations used are $\mathbb{C}_+ = \{s \in \mathbb{C} \mid \mathrm{Re}(s) > 0\}$, $\mathbb{Z}_+ = \{p \in \mathbb{Z} \mid p > 0\}$, $\overline{\mathbb{Z}_+} = \{p \in \mathbb{Z} \mid p \ge 0\}$.

We are interested in H_∞-stability, i.e. a SISO system is stable if its transfer function $K(s)$ belongs to the H_∞ space of analytic and bounded functions in $\mathbb{C}_+$ with $\|K\|_{H_\infty} = \sup_{s\in\mathbb{C}_+}|K(s)|$. Let us denote $\mathbf{M}(H_\infty)$ the set of matrices whose components are in H_∞.

The following notion of coprimeness is considered.

A system G is said to have a right coprime factorization (r.c.f.) (N, M) over H_∞ if $G = NM^{-1}$, $\det M \neq 0$, $N, M \in \mathbf{M}(H_\infty)$ and there exist $X, Y \in \mathbf{M}(H_\infty)$ such that $XM + YN = I$. Then X, Y are called right Bézout factors.

A system G is said to have a left coprime factorization (l.c.f.) $(\widetilde{M}, \widetilde{N})$ over H_∞ if $G = \widetilde{M}^{-1}\widetilde{N}$, $\det\widetilde{M} \neq 0$, $\widetilde{M}, \widetilde{N} \in \mathbf{M}(H_\infty)$ and there exist $\widetilde{X}, \widetilde{Y} \in \mathbf{M}(H_\infty)$ such that $\widetilde{M}\widetilde{X} + \widetilde{N}\widetilde{Y} = I$. Then $\widetilde{X}, \widetilde{Y}$ are called left Bézout factors.

For $\widetilde{M}, \widetilde{N} \in \mathbf{M}(H_\infty)$, there exist $\widetilde{X}, \widetilde{Y} \in \mathbf{M}(H_\infty)$ such that $\widetilde{M}\widetilde{X} + \widetilde{N}\widetilde{Y} = I$ if and only if $\inf_{s\in\mathbb{C}_+}\sigma_m([\widetilde{M}, \widetilde{N}]^T) > 0$, where $\sigma_m(\cdot)$ is the smallest singular value of a matrix (Vidyasagar, 1985, Lemma 8.1.13 and Example 8.1.15).

If G has an r.c.f. (N, M) and an l.c.f. $(\widetilde{M}, \widetilde{N})$, then the set of all controllers guaranteeing the internal stability of the closed-loop system is given by the Youla–Kučera parametrization

$$\begin{aligned} C(G) &= \{(X - R\widetilde{N})^{-1}(Y + R\widetilde{M}) \mid R \in \mathbf{M}(H_\infty), \det(X - R\widetilde{N}) \neq 0\} \\ &= \{(\widetilde{Y} + MR)(\widetilde{X} - NR)^{-1} \mid R \in \mathbf{M}(H_\infty), \det(\widetilde{X} - NR) \neq 0\}, \end{aligned}$$

where X, Y and $\widetilde{X}, \widetilde{Y}$ are respectively the corresponding right and left Bézout factors (Vidyasagar, 1985). For $R = 0$, we obtain two particular stabilizing controllers $C = X^{-1}Y$ and $C = \widetilde{Y}\widetilde{X}^{-1}$.

Poles (resp. roots) in the closed right half-plane $\overline{\mathbb{C}_+}$ are referred to as unstable poles (resp. roots).

The following notations will be of intense use later.

Denote $p(s^\alpha)$ the lowest common denominator of $R_k(s^\alpha)$ for $k = 1, \dots, n$; d the degree in s^α of $p(s^\alpha)$. Then rational transfer functions $R_k(s^\alpha)$ can be rewritten as

$$R_k(s^\alpha) = \frac{q_k(s^\alpha)}{p(s^\alpha)},$$

where q_k are polynomials in s^α.

We can decompose

$$p(s^\alpha) = (s^\alpha)^{m_0}\left(\prod_{i=1}^{N}(s^\alpha - b_i)^{m_i}\right)\left(\prod_{j=1}^{N'}(s^\alpha - c_j)^{m'_j}\right),$$

where $b_i \in \mathcal{D} := \{\sigma \in \mathbb{C}\backslash\{0\} \mid -\pi\alpha/2 \le \mathrm{Arg}(\sigma) \le \pi\alpha/2\}$; $c_j \in \mathbb{C}\backslash\{\mathcal{D} \cup \{0\}\}$; $m_0 \in \overline{\mathbb{Z}_+}$, $m_i, m'_j \in \overline{\mathbb{Z}_+}$ for $i = 1, \dots, N$ and $j = 1, \dots, N'$. Hence $s_i = b_i^{1/\alpha}$ are the non-zero unstable roots in s of $p(s^\alpha)$.

Similarly, we write

$$\widetilde{p}_k(s^\alpha) = (s^\alpha)^{m_{0k}}\left(\prod_{i=1}^{N}(s^\alpha - b_i)^{m_{ik}}\right)\left(\prod_{j=1}^{N'}(s^\alpha - c_j)^{m'_{jk}}\right),$$

where $m_{0k}, m_{ik}, m'_{jk} \in \overline{\mathbb{Z}_+}$ for $i = 1, \dots, N$, $j = 1, \dots, N'$ and $k = 1, \dots, n$. It is obvious that $m_{0k} \le m_0$, $m_{ik} \le m_i$, and $m'_{jk} \le m'_j$.

3. Left coprime factorizations and Bézout factors

In this section, we present l.c.f.'s and Bézout factors for the transfer matrix (1).

3.1. Left coprime factorizations

Due to the dimension of the transfer matrix, finding an l.c.f. is straightforward. The next proposition was presented in Nguyen and Bonnet (2012) and is recalled here together with its proof for the paper to be self-contained.

Proposition 1 (Nguyen and Bonnet, 2012). *Let $G(s)$ be given by (1). Then $(\widetilde{M}(s), \widetilde{N}(s))$, where*

$$\begin{aligned} \widetilde{M}(s) &= \frac{p(s^\alpha)}{(s^\alpha+1)^d}, \\ \widetilde{N}(s) &= \frac{1}{(s^\alpha+1)^d}\left[e^{-sh_1}q_1(s^\alpha), \dots, e^{-sh_n}q_n(s^\alpha)\right], \end{aligned} \tag{2}$$

is an l.c.f. over H_∞ of $G(s)$.

Proof. It is obvious that $\widetilde{M}(s)^{-1}\widetilde{N}(s) = G(s)$.

We see that $\widetilde{M}(s) \in H_\infty$. Also, each component of $\widetilde{N}(s)$ is in H_∞, and then $\widetilde{N}(s) \in \mathbf{M}(H_\infty)$.

For all roots σ of $p(s^\alpha)$, there exists at least one $1 \le k \le n$ such that $q_k(\sigma) \neq 0$. Thus $\inf_{s\in\mathbb{C}_+}(\sum_{k=1}^{n}|\widetilde{N}_k| + |\widetilde{M}|) > 0$ which ensures that $(\widetilde{M}, \widetilde{N})$ is an l.c.f. over H_∞ of G.

3.2. Bézout factors

Now we propose left Bézout factors corresponding to the l.c.f. obtained above. From the left Bézout identity, we derive that $\widetilde{X} = \widetilde{M}^{-1}(1 - \widetilde{N}\widetilde{Y})$. The idea is to choose $\widetilde{Y} \in \mathbf{M}(H_\infty)$ such that $\widetilde{X} \in H_\infty$. To achieve that, we interpolate $(1 - \widetilde{N}\widetilde{Y})$ at unstable zeros of $\widetilde{M}$.

For some simple classes of systems (1), Bézout factors were derived in Nguyen and Bonnet (2012). In this paper, we consider the most general case of systems (1) where techniques developed in Bonnet and Partington (2004) and Nguyen and Bonnet