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Brief paper

Maximum delay bounds of linear systems under delay independent truncated predictor feedback[☆]

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ABSTRACT

In a predictor feedback law for a linear system with input delay, the future state is predicted as the state solution of the linear system. The zero input solution contains the transition matrix. The zero state solution gives rise to the distributed nature of the feedback law. In a 2007 IEEE TAC paper, it is established that, when the system is not exponentially unstable, low gain feedback can be designed such that the predictor feedback law, with the distributed term truncated, still achieves stabilization for an arbitrarily large delay. Furthermore, in the absence of purely imaginary poles, the transition matrix in the truncated predictor feedback (TPF) can be safely dropped, resulting in a delay independent TPF law, which is simply a delay independent linear state feedback. In this paper, we first construct an example to show that, in the presence of purely imaginary poles, the linear delay independent TPF in general cannot stabilize the system for an arbitrarily large delay. By using the Lyapunov–Krasovskii Stability Theorem, we derive a bound on the delay under which the delay independent truncated predictor feedback law achieves stabilization for a general system that may be exponentially unstable.

1. Introduction

Control problems, especially, the problems of stability analysis and stabilization, for time-delay systems have attracted much attention during the past several decades. Various problems for time-delay systems have been investigated and a great number of results have been reported in the literature (see, Cao, Lin, & Hu, 2002; Chen & Latchman, 1995; Fang & Lin, 2006; Fridman, 2001; Gao, Chen, & Lam, 2008; García, Aguirre, & Suárez, 2008; Gu, Han, Luo, & Niculescu, 2001; Gu, Kharitonov, & Chen, 2003; Krstic, 2010a,b; Lu & Huang, 2015; Mazenc, Mondie, & Francisco, 2004; Mazenc, Mondie, & Niculescu, 2003, 2004; Pepe, Jiang, & Fridman, 2008; Sipahi, Niculescu, Abdallah, Michiels, & Gu, 2011; Tarbouriech, Peres, Garcia, & Queinnec, 2002; Villafuerte, Mondie, & Garrido, 2013; Zhong, 2004 and Zhong, 2005, for a small sample).

In this paper, we consider the asymptotic stabilization problem for the following linear system with time-varying delay in the input,

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(\phi(t)), \\ x(\theta) = \psi(\theta), \ \theta \in [-D, 0], \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ are state and input, respectively. The time-varying delay function $\phi(t) : \mathbb{R}^+ \to \mathbb{R}$ is assumed to have the standard form of $\phi(t) = t - d(t)$, where $d(t) : \mathbb{R}^+ \to \mathbb{R}^+$ denotes time-varying delay which is bounded by a finite positive constant D, *i.e.*, $0 \le d(t) \le D$, $\forall t > 0$. Only the information on the bound D, but not the delay $d(t)$ itself, will be required in the stability analysis. We also assume that the pair (A, B) is stabilizable.

The predictor feedback is a popular control design for system (1) that has been studied extensively and widely adopted in practice since the classical Smith predictor method was introduced for stable linear plants in Smith (1959). The most common predictor-type controllers considered in the literature are based on the Artstein model reduction technique (Artstein, 1982; Mayne, 1968) and the finite spectrum assignment technique (Manitius & Olbrot, 1979). In the presence of a constant delay d, a predictor state feedback law takes the form of

$$u(t) = Fx(t+d) = Fe^{Ad}x(t) + F\int_{t-d}^{t} e^{A(t-\lambda)}Bu(\lambda)\mathrm{d}\lambda, \tag{2}$$

where the future state $x(t+d)$ is predicted as the sum of the zero input solution and the zero state solution of the linear system, and F is the feedback gain matrix. We note that the zero input solution is the product of the transition matrix and the current state. The zero state solution involves the integration of the past input and gives rise to the distributed nature of the feedback law. Under the feedback law (2), the closed-loop system is a delay free system

$$\dot{x}(t) = (A + BF)x(t).$$

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In spite of the simple form of the closed-loop system it results in, a predictor state feedback law is a distributed control law that is difficult to implement.

In a 2007 paper (Lin & Fang, 2007), it is shown that, when the open loop system is not exponentially unstable, a parameterized feedback gain matrix $F(\gamma)$ can be designed by the low gain feedback design technique (Lin, 1998) such that the finite dimensional feedback law $u(t) = F(\gamma)e^{Ad}x(t)$, that is the predictor feedback law (2) with the integral term truncated, would still asymptotically stabilize system (1) for an arbitrarily large delay d as long as the low gain parameter γ is tuned small enough. In the absence of purely imaginary poles, the transition matrix in the truncated predictor feedback (TPF) law can be dropped and the feedback law further simplifies to a delay independent linear state feedback law $u(t) = F(\gamma)x(t)$. Such a linear feedback law, parameterized in the low gain parameter, can be referred to as the delay independent truncated feedback law. A simple example was also constructed in Lin and Fang (2007) to show that such a result would not be true if the open-loop system is exponentially unstable.

The truncated predictor feedback design originally proposed in Lin and Fang (2007) uses the eigenstructure assignment based low gain feedback design method. The design was significantly simplified in Zhou, Lin, and Duan (2012), where a parametric Lyapunov equation based low gain feedback design was adopted. The parametric Lyapunov equation based truncated predictor feedback design was extended to general, possibly exponentially unstable, systems in Yoon and Lin (2013), where it is shown that, for the stability of the closed-loop system, the maximum allowable time-delay in the input is inversely proportional to the sum of the unstable poles of the open loop system. Among the different versions of truncated predictor feedback laws, the delay independent truncated predictor feedback does not require the exact knowledge of the delay and is thus robust to the uncertainty in, the delay. However, it was only established for systems that are not exponentially unstable and in the absence of purely imaginary poles.

In this paper, we will examine the properties of such delay independent truncated predictor feedback for general systems, which may have purely imaginary or exponentially unstable poles. In particular, we will first construct an example to show that, in the presence of purely imaginary poles, the linear delay independent truncated predictor feedback in general does not have the ability to stabilize the system for an arbitrarily large delay. We then derive, by applying the Lyapunov–Krasovskii Stability Theorem, a bound on the delay under which the delay independent truncated predictor feedback law achieves stabilization for a general system. The expression of this bound indicates that, when all the closed right-half plane poles are at the origin, stabilization of the system would be achieved for an arbitrarily large delay as long as the low gain parameter is chosen to be sufficiently small. This observation coincides with the results in both Lin and Fang (2007) and Zhou, Lin, and Duan (2009). Moreover, it will be shown that, for an arbitrarily given delay, the upper bound of the low gain parameter that guarantees stability is less conservative than that of result in Zhou, Lin, & Duan (2009).

The remainder of this paper is organized as follows. Section 2 gives the problem statement and presents some necessary preliminaries for establishing our main results. In Section 3, we first construct an example to show that, in the presence of purely imaginary poles, and with no poles in the open right-half plane, the delay independent truncated predictor feedback fails to stabilize the system when the delay is sufficiently large. Then, through Lyapunov–Krasovskii stability analysis, a bound is derived on the delay under which the delay independent truncated predictor feedback law still achieves stabilization. Section 4 contains numerical examples which illustrate the theoretical results derived in the paper. Section 5 concludes the paper.

2. Problem statement and preliminaries

The truncated predictor feedback law as constructed in Lin and Fang (2007) is given as

$$u(t) = F(\gamma)e^{Ad}x(t), \ \ \gamma > 0, \tag{3}$$

where $F(\gamma)$ is a parameterized feedback gain constructed by the eigenstructure assignment based low gain feedback design technique (Lin, 1998). It was established in Lin and Fang (2007) that, when all the eigenvalues of A are in the closed left-half plane, the truncated predictor feedback law (3) would asymptotically stabilize system (1) for an arbitrarily large delay d as long as the low gain parameter γ is tuned small enough. It is further shown in Lin and Fang (2007) that in the absence of purely imaginary poles, the system can be stabilized for an arbitrarily large delay by linear delay independent truncated predictor feedback law with the tuning parameter γ,

$$u(t) = F(\gamma)x(t), \ \ \gamma > 0. \tag{4}$$

Alternative construction of the truncated predictor feedback law was later given in Zhou, Lin, & Duan (2012) by utilizing the Lyapunov equation based low gain design technique (Zhou, Duan, & Lin, 2008). That is, for a controllable pair (A, B), the parameterized feedback gain matrix $F(\gamma)$ in (3) is constructed as,

$$F(\gamma) = -B^{\mathrm{T}}P(\gamma), \tag{5}$$

where the positive definite matrix $P(\gamma)$ is the solution to the following parametric algebraic Riccati equation,

$$A^{\mathrm{T}}P + PA - PBB^{\mathrm{T}}P = -\gamma P,$$

with

$$\gamma > -2\min\{\mathrm{Re}(\lambda(A))\}. \tag{6}$$

Note that $P(\gamma)$ can be obtained from $P(\gamma) = W^{-1}(\gamma)$, where $W(\gamma)$ is the unique positive definite solution to the Lyapunov equation

$$W\left(A + \frac{\gamma}{2}I\right)^{\mathrm{T}} + \left(A + \frac{\gamma}{2}I\right)W = BB^{\mathrm{T}}. \tag{7}$$

More recently, the Lyapunov equation based truncated predictor feedback law has been extended to general systems that may be exponentially unstable (Yoon & Lin, 2013), where it is shown that, for the stability of the closed-loop system, the maximum allowable time-delay in the input is inversely proportional to the sum of the unstable poles in the open loop system.

In this paper, we will first show that, when system (1) is not exponentially unstable but has purely imaginary poles, the delay independent truncated predictor feedback law (4) is in general not able to achieve asymptotic stabilization for a large enough delay. We will then derive bound on the delay under which the delay independent truncated predictor feedback law would achieve stabilization for a general system that may be exponentially unstable.

To achieve our objectives, we need some technical preliminaries. We first recall some properties of the solution $P(\gamma)$ of the algebraic Riccati equation (6) from Zhou, Lin, & Duan (2009). By post-multiplying $P^{-1}(\gamma)$ and performing trace operations on both sides of Eq. (6), we get

$$\mathrm{tr}(B^{\mathrm{T}}PB) = 2\mathrm{tr}(A) + n\gamma. \tag{8}$$

Furthermore, we can also verify that

$$\begin{aligned} PBB^{\mathrm{T}}P &= P^{\frac{1}{2}}\left(P^{\frac{1}{2}}BB^{\mathrm{T}}P^{\frac{1}{2}}\right)P^{\frac{1}{2}} \leq \mathrm{tr}(P^{\frac{1}{2}}BB^{\mathrm{T}}P^{\frac{1}{2}})P \\ &= \mathrm{tr}(B^{\mathrm{T}}PB)P = (2\mathrm{tr}(A) + n\gamma)\,P. \end{aligned} \tag{9}$$

We also have the following further properties of $P(\gamma)$.