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Brief paper

Étale backstepping for control Lyapunov function design on manifold[☆]

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ABSTRACT

Backstepping is one of the most popular nonlinear controller and differentiable control Lyapunov function (CLF) design techniques. However, for asymptotic stabilization of systems defined on noncontractible manifolds, there exists no differentiable CLF; the semiconcave CLF design problem based on the backstepping has not been discussed. In this paper, we propose a backstepping based controller design method for asymptotic stabilization of systems defined on noncontractible manifolds. In the method, we design a controller and a CLF on an étale space. Then, we obtain a semiconcave CLF on the original space by the minimum projection method. The effectiveness of the proposed method is confirmed by computer simulation.

1. Introduction

Global asymptotic stabilization of systems defined on noncontractible manifolds, such as attitude control of rigid body dynamics or obstacle-avoidance control of mobile robots, cannot be asymptotically stabilized at the desired point by any continuous time-invariant state feedback due to its topological obstructions (Bhat & Bernstein, 2000). Moreover, continuous periodic time-varying state feedback controllers also fail to stabilize at any desired equilibrium point (Bernuau, Perruquetti, & Moulay, 2013). Then, it is natural to consider a discontinuous state feedback controller.

Backstepping is one of the most popular nonlinear controller design techniques (Khalil, 2002; Kokotović & Arcak, 2001). Extensions to nonsmooth controllers are discussed in Freeman, and Kokotović (1996), Zhang and Shen (2013) and Zheng, Shen, He, and Yang (2010). Further, discontinuous controllers are considered in Kristiansen, Nicklasson, and Gravdahl (2009) and Tanner and Kyriakopoulos (2003). For controller design on manifolds, both discontinuous backstepping methods (Kristiansen et al., 2009; Tanner & Kyriakopoulos, 2003) produce discontinuous Lyapunov functions; the method cannot be recursively used.

This paper aims to develop discontinuous backstepping that generates a continuous Lyapunov function. Specifically, we design a locally semiconcave Lyapunov function in this paper. The function achieves many advantages for control systems analysis (Cannarsa & Sinestrari, 2004; Clarke, 2011; Nakamura, Tsuzuki, Fukui, & Nakamura, 2013; Rifford, 2002), although a semiconcave function is a nonsmooth function.

For locally semiconcave control Lyapunov function (CLF) design on manifolds, the authors proposed the *minimum projection method* in Nakamura, Fukui, Nakamura, and Nishitani (2010), Nakamura et al. (2013) and Nakamura, Yamashita, and Nishitani (2009). The minimum projection method generates a locally semiconcave CLF on a manifold by using a (possibly smooth) CLF on an étale space. In this paper, we propose a novel control strategy based on the backstepping on an étale space over a manifold and the minimum projection method.

The paper is organized as follows. Section 2 illustrates a motivating example and a problem in the previously proposed method. Section 3 is devoted to the introduction of fundamental mathematical backgrounds. The main results of the paper are collected in Section 4. In Section 5, we discuss backstepping on étale spaces to prove main theorems, and then the proofs of the main theorems are given in Section 6. Section 7 illustrates the advantages of the proposed method by using the motivating example. A brief conclusion is summarized in Section 8.

2. Motivating example

We illustrate problems in the backstepping proposed in Tanner and Kyriakopoulos (2003). Let us consider the following cascaded

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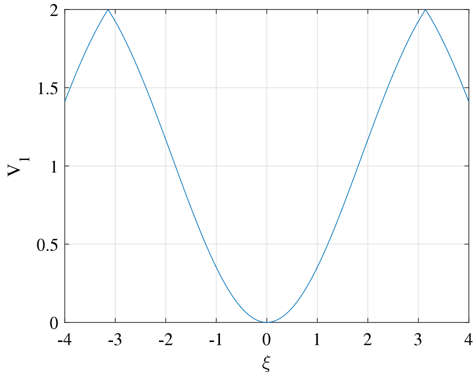

Fig. 1. Plot of V_1.

system:

$$\dot{\xi} = \eta \tag{1a}$$

$$\dot{\eta} = u, \tag{1b}$$

where $\xi \in \mathcal{X} = S^1$ parametrized by $(-\pi, \pi]$ and $\eta \in \mathbb{R}$ are state variables, and $u \in \mathbb{R}$ is the control input. We assume that $(0, 0) \in \mathcal{X} \times \mathbb{R}$ is the desired equilibrium point.

Since $\mathcal{X}$ is noncontractible, there is no smooth CLF for subsystem (1a). In contrast, as Kristiansen et al. found a locally Lipschitz CLF for the satellite attitude control problem (Kristiansen et al., 2009), we can easily construct one:

$$V_1(\xi) = -\frac{1}{2}\cos\xi - \cos\frac{\xi}{2} + \frac{3}{2}. \tag{2}$$

We can observe that V_1 is continuous but is not differentiable at $\xi = \pi$ (Fig. 1). Moreover, the following input $k_1(\xi)$ asymptotically stabilizes the origin of (1a):

$$\eta = k_1(\xi) = -\frac{1}{2}\sin\xi - \frac{1}{2}\sin\frac{\xi}{2}. \tag{3}$$

Note that there is no static feedback controller that globally and asymptotically stabilizes a single equilibrium point in the sense of Filippov; actually, controller (3) does not satisfy condition (3) in Tanner and Kyriakopoulos (2003).

Though (3) does not satisfy the assumptions of Theorem 5 in Tanner, and Kyriakopoulos (2003), we can design the following feedback controller for system (1a)–(1b) by Tanner's backstepping with the virtual input (3) (Tanner & Kyriakopoulos, 2003).

$$\begin{aligned} u &= \zeta(\xi, \eta) \\ &\quad + \left(K_z + \frac{V_1^\circ(\xi,\eta)[\eta - k_1(\xi)]}{(\eta - k_1(\xi))^2}\right)[k_1(\xi) - \eta], \end{aligned} \tag{4}$$

$$\zeta(\xi, \eta) = \left[-\frac{1}{2}\cos\xi - \frac{1}{4}\cos\frac{\xi}{2}\right]\eta, \tag{5}$$

$$V_1^\circ(\xi, \eta) = \begin{cases} \left(\frac{1}{2}\sin\xi + \frac{1}{2}\sin\frac{\xi}{2}\right)\eta & (x \neq \pi) \\ -\frac{1}{2}|\eta| & (x = \pi), \end{cases} \tag{6}$$

where $K_z > 0$ is an arbitrary constant. By the proof of Theorem 5 in Tanner and Kyriakopoulos (2003), the following function V_a is considered as a Lyapunov function:

$$V_a = -\frac{1}{2}\cos\xi - \cos\frac{\xi}{2} + \frac{3}{2} + \left(\eta + \frac{1}{2}\sin\xi + \frac{1}{2}\sin\frac{\xi}{2}\right)^2. \tag{7}$$

V_a is illustrated in Fig. 2. We show a computer simulation result with respect to the initial condition $(\xi(0), \eta(0)) = (3, 1)$ in Fig. 3.

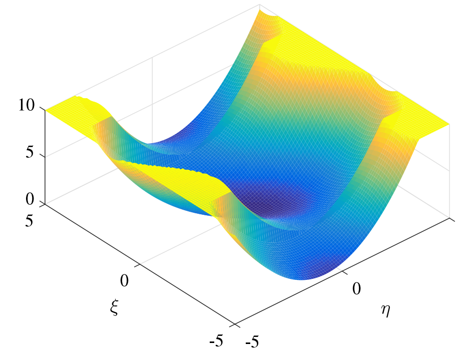

Fig. 2. Discontinuous function V_a.

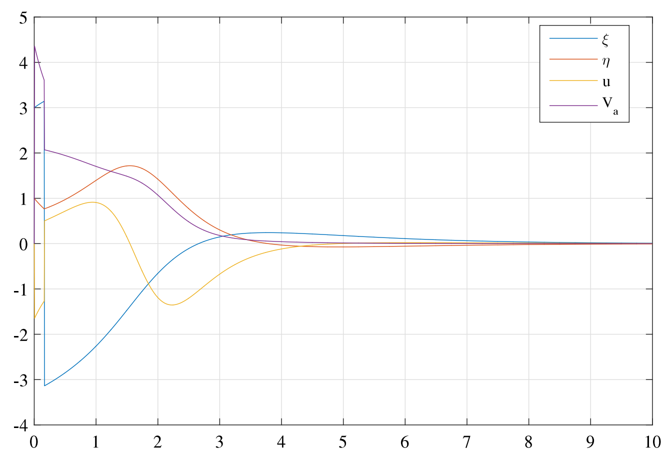

Fig. 3. History of Tanners' backstepping.

Note that V_a is discontinuous at $\xi = \pi$, and actually, we can find discontinuity of the time history of V_a at $t \simeq 0.16$ in Fig. 3. This implies that Tanner's nonsmooth backstepping cannot be used for asymptotic stabilization on manifolds. On the other hand, since Kristiansens' method heavily depends on the rigid body attitude control, the method cannot be applied to the motivating example.

The objective of the paper is to develop a new backstepping method that produces a continuous Lyapunov function.

3. Preliminaries

3.1. Control systems defined on differentiable manifolds

This paper considers a control system defined on a differentiable manifold $\mathcal{X}$. We suppose $\mathcal{X}$ is C^r differentiable for $r \in \mathbb{N} \cup \{\infty, \omega\} \setminus \{1, 2\}$, specifically $r \geq 3$ is assumed. Here, C^ω means real analytic. Let $\mathcal{X}$ denote an n-dimensional differentiable manifold, $T\mathcal{X}$ a tangent bundle of $\mathcal{X}$. Scalar multiplication and summation are defined on $T\mathcal{X}$ (Warner, 1983).

In this paper, we consider the following control affine nonlinear control system on $\mathcal{X}$:

$$\begin{aligned} \dot{x} &= f(x) + \sum_{i=1}^{m} g_i(x)u_i \\ &= f(x) + g(x)u, \end{aligned} \tag{8}$$

where $x \in \mathcal{X}$, $u \in \mathfrak{F}(\mathbb{R}, \mathbb{R}^m)$; $t \mapsto u(t) \in \mathbb{R}^m$, where $\mathfrak{F}(\mathbb{R}, \mathbb{R}^m)$ denotes a set of all mappings from $\mathbb{R}$ to $\mathbb{R}^m$. Moreover, mappings $f, g_i : \mathcal{X} \to T\mathcal{X}$ are assumed to be C^r differentiable.