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Brief paper

Adaptive asymptotic control of multivariable systems based on a one-parameter estimation approach[☆]

Ci Chen[a,b,c], Changyun Wen[c], Zhi Liu[a], Kan Xie[a,b], Yun Zhang[a], C.L. Philip Chen[d]

[a] School of Automation, Guangdong University of Technology, Guangzhou, PR China
[b] Guangdong Key Laboratory of IoT Information Technology, Guangzhou, PR China
[c] School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore
[d] Faculty of Science and Technology, University of Macau, Macau

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ABSTRACT

Multivariable adaptive control is an important and challenging research area, where it usually requires designing adaptive laws to estimate a number of unknown parameters. Hence, this process may excessively consume limitedly available computational resources and large amount of computational time, which may degrade system performances, particularly for the case of online updating of parameter estimates. Thus reducing the number of parameters to be estimated is a promising way to solve the problem. The state-of-the-art result in the area is to update just one adaptive parameter. However, the tracking errors are only ensured bounded and thus the resulting closed-loop system is not asymptotically stable. Hence, a question that arises is whether such a result of non-zero tracking errors is the price paid for reducing the number of updating parameters. Up to now, there is still no answer to this question. In this paper, we address such an issue and realize asymptotic stability with online estimation of just one parameter. To achieve such a goal, a novel dynamic loop gain function based approach is proposed to incorporate with the backstepping control design procedure, which enables us to solve the algebraic loop problem caused by all the existing traditional Nussbaum function based approaches and thus establish system stability. A bound for the tracking error is explicitly established in terms of L_2 norm, which helps improve the transient performance by selecting the control parameters. Moreover, guaranteed by the Barbalat's Lemma, the tracking error is further ensured to approach zero asymptotically. Finally, simulation examples are conducted to testify the effectiveness of the proposed approach.

1. Introduction

Driven by many theoretical and practical challenges as reviewed in Tao (2014), several promising adaptive control schemes (Ioannou & Sun, 2012; Narendra & Annaswamy, 2012; Tao, 2003; Tao, Chen, Tang, & Joshi, 2013) including model reference based adaptive controls (De Mathelin & Bodson, 1995; Hsu & Costa, 1989; Tao & Ioannou, 1989; Weller & Goodwin, 1994) and backstepping design based adaptive controls (Costa, Hsu, Imai, & Tao, 2002; Ling & Tao, 1997) have been proposed for linear time-invariant multivariable systems. Considerable researches have been led to improve the multivariable control quality by studying the high-frequency gain matrix (HFGM) and by reducing the computational burden (Elliott & Wolovich, 1984). By utilizing the positive definiteness assumption for the HFGM, Ling and Tao (1997) successfully presented the backstepping control based solutions for multivariable systems. Costa et al. (2002) and Imai, Costa, Hsu, Tao, and Kokotovic (2004) creatively proposed a HFGM factorization based multivariable control to relax the positive definiteness assumption in Ling and Tao (1997). Furthermore, Wang and Lin (2010) proposed an adaptive output-feedback control to eliminate the explosion of complexity problem in multivariable backstepping design. However, in Costa et al. (2002), Imai et al. (2004) and Wang and Lin (2010), it is assumed that the prior knowledge of leading principal minors of the HFGM is available for the control design. To remove such an assumption, the Hurwitz HFGM condition has been artfully proposed for multivariable systems in Cunha, Hsu, Costa, and Lizarralde (2003) and Oliveira, Peixoto, and Hsu (2010). Although remarkable progresses have been made to handle the HFGM, the number of parameters to be updated with these

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E-mail addresses: gdutcc@gmail.com (C. Chen), ecywen@ntu.edu.sg (C. Wen), lz@gdut.edu.cn (Z. Liu), kanxiegdut@gmail.com (K. Xie), yz@gdut.edu.cn (Y. Zhang), philip.chen@ieee.org (C.L.P. Chen).

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schemes gets larger as the order or input–output dimension of the multivariable system becomes higher. It implies that the number of the required adaptive parameters could be usually large for multivariable adaptive control, which dramatically limits the algorithm's practicality (Elliott & Wolovich, 1984). As a result, a question on how to reduce the computational burden for multivariable systems attracts much attention.

As for the reduction of the computational burden, some pioneering works have been proposed. In Polycarpou and Ioannou (1996), an effective robust backstepping scheme was proposed where unknown system nonlinearities were assumed to satisfy a structural condition of known functions multiplied by unknown parameters. Yao and Tomizuka (2001) further proposed an adaptive robust control to systematize nonlinear systems in semi-strict feedback forms. It is noted that such methods still include a large number of updated parameters. Most recently, a great achievement has been made by Wang et al. in Wang and Lin (2012), where only one adaptive parameter is constructed based on the estimation of a norm. Hence, it significantly improves the computational efficiency. However, such an improvement is gained at the cost of sacrificing the control accuracy. More specifically, the tracking error obtained in Wang and Lin (2012) can no longer approach zero, but only be bounded near the origin due to the emergence of nonzero constants from Young's inequality. Therefore, even though the computational burden is reduced, a question whether the bounded error result is the price to pay for reducing the computational burden remains to be answered.

In this paper, we address such an issue and propose a one-parameter estimation approach to realize the asymptotic control for multivariable systems. Compared with Wang and Lin (2012), the proposed approach allows us to break the bounded error restriction and to boost the computational efficiency, simultaneously. To achieve asymptotic control, the argument of a Nussbaum function, rather than the norm of the unknown system dynamics in Wang and Lin (2012), is chosen as the only one adaptive parameter. Typically, a Nussbaum function is applied to handle the unknown control direction problem. Here, we explore its new application. However, if we directly employ the existing approaches based on traditional Nussbaum functions (Chen, Li, Ren, & Wen, 2014; Chen, Liu, Xie, Liu, & Chen, 2016; Chen, Liu, Zhang, Chen, & Xie, 2016; Ding, 2014; Ge & Wang, 2003), an algebraic loop problem will be encountered. Thus, we develop a novel dynamic loop gain function based approach and utilize it in each step of the backstepping based controller design in order to circumvent the algebraic loop problem. As an extension of the above approach, a useful tool is further proposed to help stability analysis for many practical systems including the situation that actuator nonlinearities are involved in the dynamical process. Then appropriate control law and adaptive law are proposed to design adaptive controllers. With such controllers, all the signals in the closed-loop system are ensured bounded and the bound for tracking errors is established in terms of L_2 norm. Moreover, by employing the *Barbalat's Lemma*, it is shown that the tracking error asymptotically approaches zero. Even for the case that the adaptation law is removed from the proposed control scheme, i.e., without any adaptive parameter, it is shown that the boundedness of all the signals in the closed-loop system is still guaranteed.

The remaining parts are summarized as follows. In Section 2, the control problem is formulated and all the required assumptions are provided. The proposed adaptive control scheme for the multivariable system is presented in Section 3, where how to choose the one adaptive parameter is detailed and why the proposed dynamic loop gain function is able to circumvent the common algebraic loop problem is explained. The main results are presented in Section 4, where how the proposed approach guarantees the stability of the closed-loop system, the transient performance and the asymptotic control are detailed. A simulation example is provided in Section 5 and the conclusion is drawn in Section 6.

2. Problem formulation

The investigated multivariable system is modeled as Tao (2003)

$$y = G(s)u, \tag{1}$$

where $y \in \mathbb{R}^r$ stands for the system output, $G(s) \in \mathbb{R}^{r\times r}$ stands for a strictly proper rational transfer function matrix, and $u \in \mathbb{R}^r$ stands for the system controller. Here, $G(s)$ in (1) is detailed as

$$G(s) = D^{-1}(s)N(s) = C_g(sI - A_g)^{-1}B_g, \tag{2}$$

where $D(s) = s^{\upsilon}I_r + A_{\upsilon-1}s^{\upsilon-1} + \cdots + A_1 s + A_0$, $N(s) = B_m s^m + B_{m-1}s^{m-1} + \cdots + B_1 s + B_0$, $C_g = [I_r, 0, \ldots, 0, 0] \in \mathbb{R}^{r\times r\upsilon}$, $A_g = \begin{bmatrix} -A_{\upsilon-1} & I_r & 0 & \cdots & 0 \\ -A_{\upsilon-2} & 0 & I_r & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \cdots \\ -A_1 & 0 & 0 & \cdots & I_r \\ -A_0 & 0 & 0 & \cdots & 0 \end{bmatrix} \in \mathbb{R}^{r\upsilon\times r\upsilon}$, and $B_g = [0, 0, \ldots, 0, B_p^T]^T \in \mathbb{R}^{r\upsilon\times r}$ with υ standing for the observability index of $G(s)$, $B_p = [B_m^T, \ldots, B_1^T, B_0^T]^T \in \mathbb{R}^{r(m+1)\times r}$, $A_i \in \mathbb{R}^{r\times r}$ for $i = 0, \ldots, \upsilon - 1$ and $B_j \in \mathbb{R}^{r\times r}$ for $j = 0, \ldots, m$, being constant parameter matrices, and $I_r \in \mathbb{R}^{r\times r}$ being an identity matrix. Considering (2), one thus changes (1) to

$$\begin{aligned} \dot{x} &= Ax + A_p y + B_g u, \\ y &= G_g x = x_1, \end{aligned} \tag{3}$$

where $x = [x_1^T, \ldots, x_\upsilon^T]$ with $x_i \in \mathbb{R}^r$ for $i = 1, \ldots, \upsilon$, $A \in \mathbb{R}^{r\upsilon\times r\upsilon}$ is the matrix A_g with the first r columns replaced by zeros, and $A_p \in \mathbb{R}^{r\upsilon\times r}$ stands for the first r columns of A_g. The objective is to construct an adaptive output-feedback control such that the system output $y(t)$ asymptotically converges to a desired signal $y_d(t) \in \mathbb{R}^r$.

Following Wang and Lin (2012), one has:

Assumption 1. $G(s)$ has full rank. No external matched disturbances are considered and the relative degree of the squared system is unitary.

Assumption 2. All zeros of $\det(N(s))$ are stable.

Assumption 3. The constants r, υ, m and $\rho = \upsilon - m$ are known for the control design with $\rho > 1$.

Assumption 4. The matrices $A_0, A_1, \ldots, A_{\upsilon-1}$, $B_0, B_1, \ldots, B_{m-1}$ and B_m are unknown.

Assumption 5. The matrix B_m is nonsingular and S is known such that $-B_m S$ satisfies the Hurwitz condition, that is, every eigenvalue of the matrix has strictly negative real part.

Assumption 6. $y_d(t)$ and its first ρ derivatives are bounded and known.

Remark 1. It is worth stating that this paper aims to improve the bounded error control (Wang & Lin, 2012) to the asymptotic control, but not at the expense of introducing extra assumptions or constructing extra parameter estimations. All the assumptions given above are the same as the ones in Wang and Lin (2012).