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Brief paper

Output and error feedback regulator designs for linear infinite-dimensional systems☆

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ABSTRACT

This manuscript addresses the output regulation problem for linear distributed parameter systems (DPSs) with bounded input and unbounded output operators. We introduce novel methods for the design of the output feedback and error feedback regulators. In the output feedback regulator design, the measurements available for the regulator do not belong to the set of controlled outputs. The proposed output feedback regulator with the injection of the measurement $y_m(t)$ and reference $y_r(t)$ can realize both the plant and the exosystem states estimation, disturbance rejection and reference signal tracking, simultaneously. Moreover, new design approach provides an alternative choice for seeking the output injection gain in a traditional error feedback regulator design. The regulator parameters are easily configured to solve the output regulation problems, and to ensure the stability of the closed-loop systems. The results are demonstrated via computer simulation in two types of representative systems: the parabolic partial differential equation (PDE) system and the first order hyperbolic PDE system.

1. Introduction

An important control problem is output regulation, i.e., to realize the asymptotic tracking of given reference signals in the presence of disturbances. Under the assumption that these exogenous signals can be generated by an exosystem, a systematic solution to the output regulation problem can be derived. The geometric theory of output regulation was first introduced in Francis (1977) for finite-dimensional systems and then the output regulation theory was documented and developed for lumped-parameter systems in Knobloch, Isidori, and Flockerzi (1993) for linear systems and in Byrnes, Priscoli, and Isidori (1997) and Isidori and Byrnes (1990) for nonlinear systems. Essentially, there are different approaches for solving the output regulation problem. In the first one, Davison (1976) developed a robust controller with an exosystem, driven by the output tracking error. The second method originates from the work of Johnson (1971), where the controller is designed based on an observer with an exosystem. The observer is able to estimate the state of the plant and the exosystem.

In chemical, biochemical or/and mechanical processes, the dynamics of linear systems may exhibit both temporal and spatial effects, so that the setting of the linear distributed parameter systems, known as infinite-dimensional systems, has to be taken into account. Many contributions have been made to generalize the output regulation theory in finite-dimensional systems to infinite-dimensional systems. In Kobayashi (1983) and Pohjolainen (1982), a PI-controller was designed for stable distributed parameter systems with constant disturbance. Later, the work of Pohjolainen (1982) was extended to the systems with infinite-dimensional exosystems in Hämäläinen and Pohjolainen (2000), Immonen (2007) and Paunonen and Pohjolainen (2010) and to well-posed systems in Rebarber and Weiss (2003). In Aksikas, Fuxman, Forbes, and Winkin (2009), the linear–quadratic (LQ) optimal regulator was introduced to realize the constant trajectory tracking of the first order hyperbolic PDE systems.

To realize the output regulation, the geometric theory in Francis (1977) was generalized to infinite-dimensional systems with bounded control and observation operators in Byrnes, Laukó, Gilliam, and Shubov (2000), and was later developed by Natarajan, Gilliam, and Weiss (2014) for the system with unbounded input and output operators. Recently, this theory was extended to non-spectral systems, e.g., the first order hyperbolic PDE systems by Xu and Dubljevic (2016a). Schumacher (1983) developed a finite-dimensional error feedback regulator with the aid of the Sylvester equation. In the similar line, on the basis of *Johnson's* approach, Deutscher (2011) designed a finite-dimensional output feedback regulator for Riesz-spectral systems. Motivated by these works, Xu, Pohjolainen, and Dubljevic (under review) developed finite-dimensional output feedback and error feedback regulators

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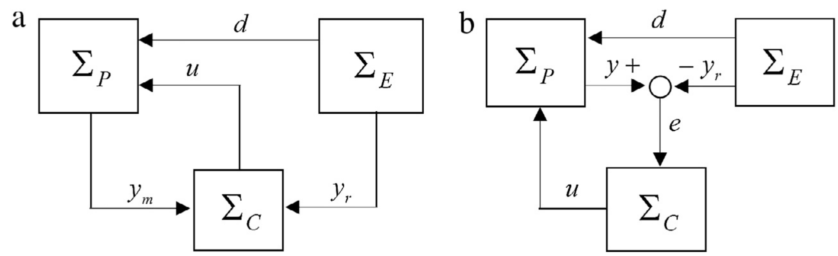

Fig. 1. Block diagram of systems interconnection (plant Σ_P, exosystem Σ_E and regulators Σ_C) with disturbance d, measurement y_m, reference y_r, output y, input u and tracking error e. (a) Configuration of the output feedback regulator; (b) configuration of the error feedback regulator.

for non-spectral infinite-dimensional systems, e.g., the first order hyperbolic PDE systems.

Recently, in Paunonen (2016), three dynamic error feedback controllers were introduced for regular linear systems. In particular, one observer-based robust controller (Section VI) was designed based on $\mathcal{G}$-condition motivated by Hämäläinen and Pohjolainen (2010) where the controller has Internal Model Structure (IMS) and the controller operator $\mathcal{G}_1$ has a triangular form. Moreover, an *auxiliary operator* (not function-type) *Sylvester equation* needs to be solved. In this manuscript, a new form of the observer-based error feedback regulator is proposed and the solution of the auxiliary Sylvester equation is the function of the spatial variable, which simplifies and reduces complexity associated with the calculation of the auxiliary Sylvester equation.

In this manuscript, two types of regulators are proposed and designed, see Fig. 1. The main contribution is given as the observers design, i.e. the weighted regulator state $\left[(\tilde{H}r_m(t))^T \quad \hat{v}_r^T(t)\right]^T$ (or $\tilde{H}_e r_e(t)$) is used to obtain exponentially accurate estimates for the plant and exosystem states. To achieve the observer convergence, the observer error system is decoupled into the PDE-subsystem and the ODE-subsystem so that the ODE-subsystem and the PDE-subsystem can be stabilized separately by fixing free regulator parameters. This decomposition idea was applied in backstepping designs of the output regulator, see Deutscher (2015a) and in internal model regulator designs, see Hämäläinen & Pohjolainen (2010); Paunonen (2016). However, compared with Deutscher (2015a), the proposed regulators in this manuscript can address output regulation problems for coupled PDE systems with distributed or boundary control (with the aid of the approach in Natarajan et al. (2014)) inputs. Compared with, see Hämäläinen & Pohjolainen (2010) or Paunonen (2016), a novel output feedback regulator is provided in this manuscript and the auxiliary Sylvester equations introduced here are easier to solve (by introducing weights $\tilde{H}$ and $\tilde{H}_e$).

In more detail, the constructed output feedback regulator is driven by the measurement $y_m(t)$ and the reference $y_r(t)$. Therefore, the observability conditions are studied. Here, the measurement $y_m(t)$ does not belong to the set of the controlled output $y(t)$, while in the design of the error feedback regulator, the proposed approach yields an alternative and easy choice for finding the output injection gain for the traditional error feedback regulator design, see Byrnes et al. (2000) and Xu & Dubljevic (2016a). In contrast, the regulator parameters in this manuscript can be easily designed and configured. For infinite-dimensional systems, the proposed two regulator designs are both applicable and valid for Riesz-spectral systems, see Deutscher (2011) and non-spectral systems, see Aksikas et al. (2009). In particular, the free design parameters of the regulators are configured by applying the separation principle.

This paper is organized as follows: In Section 2, both the plant and the exosystem are introduced and some fundamental assumptions are stated. In addition, the key results for a full state feedback regulator design are recalled. Then, in Sections 3 and 4, two regulators are designed and the configuration of the parameters is demonstrated in detail. In order to guarantee the feasibility of the regulators, the solvability conditions of Sylvester equations and observability conditions are discussed. Finally, two types of representative systems: the parabolic PDE system (spectral system) and the first order coupled hyperbolic PDE system (non-spectral system) are studied to verify main results of this manuscript in Section 5. Section 6 contains the conclusion.

Assume that X and Y are Hilbert spaces and $\mathcal{A} : X \mapsto Y$ is a linear operator, then $D(\mathcal{A})$ denotes the domain of $\mathcal{A}$. $\mathcal{L}(X, Y)$ denotes the space of all linear, bounded operators from X to Y. (If $X = \mathbb{C}^{n_X}$ and $Y = \mathbb{C}^{n_Y}$, then $\mathcal{L}(X, Y) = \mathbb{C}^{n_X \times n_Y}$.) If $X = Y$, then we write $\mathcal{L}(X)$. If $\mathcal{A} : X \to X$, then $\sigma(\mathcal{A})$ is the spectrum of $\mathcal{A}$ (the set of eigenvalues, if $\mathcal{A} \in \mathbb{C}^{n_X \times n_X}$), $\rho(\mathcal{A}) = \mathbb{C} \setminus \sigma(\mathcal{A})$ is the resolvent set and $R(\lambda; \mathcal{A}) = (\lambda I - \mathcal{A})^{-1} \in \mathcal{L}(X)$ denotes the resolvent operator for $\lambda \in \rho(\mathcal{A})$. The inner product is denoted by $\langle \cdot, \cdot \rangle$. $L^2(0, 1)^m$ with a non-negative integer m is a Hilbert space of an m-dimensional vector of the real functions that are a square integrable over $[0, 1]$. $H^k(0, 1)$ with a non-negative integer k, denotes a Hilbert space defined as the Sobolev space of order k, i.e. $H^k(0, 1)^m = \left\{ h(\cdot) \in L^2(0, 1)^m : \left(\frac{d^p h}{dz^p}\right) \in L^2(0, 1)^m, p = 1, 2, \ldots, k \right\}$. In particular, $H^0(0, 1) = L^2(0, 1)$. If the plant is a finite-dimensional system, the assumption: $\mathcal{A}$ generates a C_0-semigroup $T_{\mathcal{A}}(t)$ is always satisfied, and the semigroup is the matrix exponential function, i.e., $T_{\mathcal{A}}(t) = e^{\mathcal{A}t}, t \geq 0$ (Curtain & Zwart, 1995). $e^{\mathcal{A}t}$ is exponentially stable if and only if $\sigma(\mathcal{A}) \subset \mathbb{C}^-$, i.e., the matrix $\mathcal{A}$ is Hurwitz.

2. Problem formulation

The plant — We are concerned with the following infinite-dimensional linear system Σ_P:

$$\dot{x}(t) = \mathcal{A}x(t) + \mathcal{B}u(t) + \mathcal{G}d(t), t > 0, x(0) = x_0 \in X \tag{1}$$

$$y(t) = \mathcal{C}x(t), t \geq 0 \tag{2}$$

$$y_m(t) = \mathcal{C}_m x(t), t \geq 0 \tag{3}$$

where

$x \in X$ is the state of the system,
X is a complex Hilbert state space,
$u \in U$ is an input,
$y \in Y$ is the controlled output, and
$y_m \in Y_m$ is the measured output.

U, Y and Y_m are complex Hilbert control and output spaces, respectively. $\mathcal{A} : D(\mathcal{A}) \subset X \to X$ is the infinitesimal generator of a C_0-semigroup $T_{\mathcal{A}}(t)$ on X, $\mathcal{B} \in \mathcal{L}(U, X)$. The output operators $\mathcal{C}, \mathcal{C}_m \in \mathcal{L}(X_1, Y)$ are $\mathcal{A}$—admissible (see (Xu & Jerbi, 1995) and (Xu et al., under review)), where the space $X_1 = D(\mathcal{A})$ is equipped with the norm $\|x\|_1 = \|(\beta I - \mathcal{A})x\|$ and $\beta \in \rho(\mathcal{A})$. $d(t) \in U_d$ is disturbance and U_d is a complex Hilbert space. $\mathcal{G} \in \mathcal{L}(U_d, X)$ denotes disturbance location operator. According to Proposition 4.9 of Tucsnak and Weiss (2014), the system (1)–(3) is well-posed