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Brief paper

Trajectory tracking for a class of contractive port Hamiltonian systems ☆

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ABSTRACT

In this paper, a class of contractive port Hamiltonian systems is characterized. Having wide range of applications, Port Hamiltonian systems match IDA-PBC (Interconnection and Damping Assignment Passivity Based Control) framework, as a powerful design technique. Through utilization of contraction properties of port Hamiltonian systems, an approach which stems from IDA-PBC is proposed for tracker design of such systems. In the line of showing the applicability and superiority of the proposed approach, it is applied to an electromechanical system, *i.e.*, magnetic levitation.

1. Introduction

It is more than two decades that some methods are developed which try to respect and use the nonlinearity for controller design. Nonlinearity cancellation with high gain feedbacks, despite its applications, has some drawbacks. For example, nonlinearity cancellation can destroy the structure of closed loop system, which in turn may reduce the robustness, raise the energy consumption and result in unnatural behaviors. Even more, nonlinear cancellation may obstruct controller design procedure for under-actuated and non-minimum phase systems. Passivity based control is a solution framework in order to respect and use the nonlinearity for controller design (Ortega & García-Canseco, 2004).

Passivity based controller design methods have a wide range of applications such as those in mechanical, electrical and electromechanical systems (Ortega, Perez, Nicklasson, & Sira-Ramirez, 1998). Passivity concept is closely related to the energy concept in physical systems; so, a physically based modeling approach may have some benefits for passivity based control methods; which are witnessed in port Hamiltonian framework (Duindam, Macchelli, Stramigioli, & Bruyninckx, 2009).

Port Hamiltonian framework stems from classical mechanics, combines it systematically with network modeling and can use these properties effectively in the controller design (Duindam et al., 2009). A port Hamiltonian system, simply consists of a Dirac structure which represents the network interconnection of energetic parts of the system, an energy function which is called Hamiltonian and models the dynamic behavior of energetic elements of the system, and a resistive structure which models the dynamic behavior of energy dissipative elements of the system.

It is evident that, in the presence of adequate dissipation, the energy of a system reaches its minimum. Therefore, if the minimum of energy function is placed at the desired equilibrium point, then damping injection is sufficient for controller design. Otherwise, beyond damping injection, the controller must shape the energy in order to put the minimum of energy at the desired equilibrium point. In some cases, in addition to energy shaping and damping injection, the interconnection of the system, represented by Dirac structure in port Hamiltonian systems, must be modified. The method, which can handle these modifications, is called IDA-PBC (Interconnection and Damping Assignment Passivity Based Control) (Ortega, van der Schaft, Maschke, & Escobar, 2002).

In recent years, a vast number of applications of IDA-PBC for controller design in the framework of port Hamiltonian systems have been reported in the literature, such as Acosta, Ortega, Astolfi, and Mahindrakar (2005) for under-actuated mechanical systems, Dörfler, Johnsen, and Allgöwer (2009) for process systems and Rodrìguez and Ortega (2003) for electromechanical systems.

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See Duindam et al. (2009) and Ortega and García-Canseco (2004) for further applications.

Despite the priority of IDA-PBC in regulation, there is no tracking version of IDA-PBC except (Fujimoto, Sakurama, & Sugie, 2003; Kotyczka, 2013). For tracking, it is common to stabilize the error dynamics. However, it is rare that error dynamics of nonlinear systems admit a port Hamiltonian structure. As a result, it is not straightforward to gain from IDA-PBC for stabilizing error dynamics. This is the main obstruction of extending IDA-PBC for tracking controller design. In papers like Fujimoto et al. (2003), a time dependent transformation called generalized canonical transformation is used to transform the error dynamics to a time variant form of port Hamiltonian systems. This transformation is obtained by solving a PDE equation and could be applied to only a class of port Hamiltonian systems. Actually, methods similar to the one posed in Fujimoto et al. (2003) are not extension of IDA-PBC, but are tracking design methods for port Hamiltonian systems. Methods such as those introduced in Kotyczka (2013) use IDA-PBC for stabilization of predefined locally linear error dynamics. However, using error dynamics is not accordant with the spirit of energy based methods such as IDA-PBC.

Instead of stabilization of error dynamics, tracker design may be treated through contraction technique in port-Hamiltonian systems. Contraction analysis studies the convergence of trajectories of a system to each other. Similar to Lyapunov theorem, for contraction analysis, it is needed to find a suitable function (metric) to deduce the convergence. It is a nice feature of port Hamiltonian systems that their contraction can be determined by checking some conditions on the corresponding Dirac structure, resistive structure and Hamiltonian function.

Based on contraction of port Hamiltonian systems, in the continuation of Yaghmaei and Yazdanpanah (2015), an extension of IDA-PBC for tracking controller design is proposed. IDA-PBC can be considered, simply, as a controller design technique which converts an open loop system to a desired closed loop one. Therefore, after characterizing desired closed loop port Hamiltonian systems suitable for tracking, IDA-PBC straightforwardly may be employed for tracking goal. This claim will be precisely stated and proved in the following sections.

The rest of the paper is organized as follows. The preliminaries on port Hamiltonian systems and original IDA-PBC are summarized in Section 2. Contractive port Hamiltonian systems, which can be considered as desired closed loop system for IDA-PBC, are characterized in Section 3. Section 4 introduces the extension of IDA-PBC for tracker design and includes some discussion on relation between regulation and tracking via IDA-PBC. The proposed method is applied to magnetic levitation for tracking design, in Section 5. The paper ends with a conclusion in Section 6.

In the subsequent sections, A^T (A^*) is used for the transpose (conjugate transpose) of matrix A, respectively. $A \succ B$ ($A \succeq B$) means that matrix $A-B$ is positive definite (positive semi-definite, respectively). $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ stand for the largest and smallest eigenvalues of matrix A, respectively. Similarly, $\sigma_{\max}(A)$ and $\sigma_{\min}(A)$ represent the largest and smallest singular values of matrix A, respectively. I is used for identity matrix, 0 for scalars, vectors and matrices with zero elements, and e_i for vectors whose ith element is 1 and the others are 0; dimensions of I, 0 and e_i are determined from the context. $\nabla H(x)$ is defined as $\begin{bmatrix}\frac{\partial H}{\partial x_1} & \frac{\partial H}{\partial x_2} & \cdots & \frac{\partial H}{\partial x_n}\end{bmatrix}^T$ for a continuously differentiable function of $H(x) : \mathbb{R}^n \to \mathbb{R}$ and $\nabla^2 H(x)$ is a matrix whose ijth element is $\frac{\partial^2 H}{\partial x_i \partial x_j}$. For any function $H(x,t) : \mathbb{I} \times \mathbb{R}^n \to \mathbb{R}$, where $\mathbb{I} \subset \mathbb{R}$, the term $\nabla H(x,t)$ is the derivative with respect to x, i.e., $\nabla H(x,t) = \begin{bmatrix}\frac{\partial H}{\partial x_1} & \frac{\partial H}{\partial x_2} & \cdots & \frac{\partial H}{\partial x_n}\end{bmatrix}^T$. Similarly, $\nabla^2 H(x,t)$ is defined with respect to x. Finally, throughout this paper, it is assumed that all functions are sufficiently smooth and, for the sake of simplicity, the arguments of a function are dropped whenever evident from the context.

2. Interconnection and damping passivity based control

A most applied and not very special class of port Hamiltonian systems can be written as the following equations:

$$\begin{aligned} \dot{x} &= \big(J(x) - R(x)\big)\nabla H(x) + g(x)u, \;\; x \in D_0 \subset \mathbb{R}^n, \\ y &= g^T(x)\nabla H(x) \qquad u, y \in \mathbb{R}^m, \end{aligned} \tag{1}$$

$J(x)$ is an everywhere skew symmetric matrix (i.e., $J(x) + J^T(x) = 0 \;\forall x$). $J(x)$ is called interconnection matrix and is determined by the Dirac structure of the system. $R(x)$, which is called dissipation matrix, is determined by both dissipative elements and the Dirac structure and is positive semi-definite everywhere (i.e., $R(x) = R^T(x) \succeq 0 \;\; \forall x$). (u, y) is the control port and the product $u^T y$ is the power entering/outgoing into/from the system via the port (u, y). Finally, D_0 is the state space of the system, which is an open subset of $\mathbb{R}^n$.

Consider a closed loop system in the class of (1) (with no input) as:

$$\dot{x} = \big(J_d(x) - R_d(x)\big)\nabla H_d(x). \tag{2}$$

If $x^\star$ is the minimum of H_d, then H_d can be considered as a Lyapunov function for which $\dot{H}_d = -\nabla^T H_d R_d \nabla H_d \leq 0$. Therefore, if the largest invariant subset in the set $\{x \in \mathbb{R}^n \mid \nabla^T H_d R_d \nabla H_d = 0\}$ is equal to $\{x^\star\}$, then $x^\star$ is an asymptotically stable equilibrium point due to the LaSalle's invariance theorem. Control objective in this framework reduces to finding a controller that converts system (1) into (2) with the mentioned properties. The following theorem provides this powerful tool for the purpose of regulation of port Hamiltonian systems (Ortega et al., 2002).

Theorem 1 (*IDA-PBC*)**.** *Suppose that there exist $H_d : \mathbb{R}^n \to \mathbb{R}$, $J_d(x) = -J_d^T(x)$ and $R_d(x) = R_d^T(x) \succeq 0$ satisfying the following equation for the system (1):*

$$g_\perp(x)\Big(\big(J(x) - R(x)\big)\nabla H(x)\Big) = g_\perp(x)\Big(\big(J_d(x) - R_d(x)\big)\nabla H_d(x)\Big) \tag{3}$$

where $g_\perp$ is the full rank left annihilator of g. If $x^\star$(the desired equilibrium point) is a minimum of H_d, then there exists a controller such as:

$$\begin{aligned} u = {} & \big(g^T(x)g(x)\big)^{-1} g^T(x)\Big(\big(J_d(x) - R_d(x)\big)\nabla H_d(x) \\ & - \big(J(x) - R(x)\big)\nabla H(x)\Big) \end{aligned} \tag{4}$$

which can locally stabilize system (1) and the closed loop system becomes as (2). Furthermore, if $x^\star$ be the largest invariant subset of $\{x \in \mathbb{R}^n \mid \nabla^T H_d(x) R_d(x) \nabla H_d(x) = 0\}$, then $x^\star$ is asymptotically stable. Finally if $x^\star$ is a global minimum of H_d and H_d is radially unbounded, then the result holds globally. □

Eq. (3) is called *matching equation* and is the substantial part of this method. One can fix (assign) the closed loop dissipation matrix R_d and interconnection matrix J_d, and then obtains H_d by solving the mentioned equation. A sufficient condition for $x^\star$ to be a minimum for a twice continuously differentiable function $H_d(x)$ can be stated with the help of first and second derivative of $H_d(x)$ as:

$$\nabla H_d(x)|_{x^\star} = 0 \tag{5}$$

$$\nabla^2 H_d(x)\big|_{x^\star} \succ 0. \tag{6}$$

In this paper, we are seeking a method similar to Theorem 1 for non-constant desired trajectory $x^\star(t)$. It is clear that a system cannot track all trajectories in its state space. In other words, the desired trajectory must be feasible to be tracked. The trajectory $x^\star(t) : \mathbb{I} \subset \mathbb{R} \to \mathbb{R}^n$, where $\mathbb{I}$ is an open subset of $\mathbb{R}$, is a feasible