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Brief paper

Stochastic source seeking with forward and angular velocity regulation☆

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ABSTRACT

This paper studies a stochastic extremum seeking method to steer a nonholonomic vehicle to the unknown source of a spatial field in a plane. The key challenge lies in the lack of vehicle's position information and the distribution of the scalar field. Different from the existing stochastic strategy that keeps the forward velocity constant and controls only the angular velocity, we design a stochastic extremum seeking controller to regulate both forward and angular velocities simultaneously in this work. Thus, the vehicle decelerates near the source and stays within a small area as if it comes to a full stop, which solves the overshoot problem in the constant forward velocity case. We use the stochastic averaging theory to prove the local exponential convergence, both almost surely and in probability, to a small neighborhood near the source for elliptical level sets. Finally, simulations are included to illustrate the theoretical results.

1. Introduction

Source seeking is a problem of steering single or multiple autonomous agents to seek the source of an unknown scalar field, which may be thermal, electromagnetic, acoustic, or the concentration of a chemical agent. Source seeking is of interest in many areas, such as environmental studies, explosive detection, localizing the sources of hazardous chemicals leakage or pollutants, etc. There are a diversity of approaches to source seeking problem, such as bio-inspired methods (Li, Farrell, Pang, Arrieta, et al., 2006; Russell, 2004), mathematical programming methods (Khong, Tan, Manzie, & Nešić, 2014; Ogren, Fiorelli, & Leonard, 2004; Porat & Nehorai, 1996; Teel & Popović, 2001), and source-likelihood mapping methods (Jakuba, 2007; Pang & Farrell, 2006).

In this work, we consider steering a single nonholonomic vehicle to locate a static source which creates a continuous signal map in a plane. Recently there is a growing interest in the study of locating such a source without position information (Azuma, Sakar, & Pappas, 2012; Cochran & Krstic, 2009; Matveev, Teimoori, & Savkin, 2011). The lack of position information is taken account for vehicles operated in environments where their position information is unavailable or costly. Obviously, this constraint, along with the nonholonomic constraint of the vehicle kinematics, renders the guidance of the vehicle interesting and challenging.

Extremum seeking (ES) is a model free optimization method for dynamical systems with limited information (Krstić & Wang, 2000). It has been proved to an effective method for nonholonomic source seeking problems even without position information. In Cochran and Krstic (2009) and Zhang, Arnold, Ghods, Siranosian, and Krstic (2007) ES was applied to tune the forward or angular velocity of the vehicle to locate the source. In Ghods and Krstic (2010) Ghods and Krstic regulated both velocities to control the vehicle to stop near the source. While the above works focus on the 2D vehicles, Lin, Song, You, and Wu (2016) considered the more complicated 3D case. Different from the above perturbation-based ES methods, a novel perturbation-free regulator is proposed in Durr, Krstic, Scheinker, and Ebenbauer (2017) and Scheinker and Krstić (2014).

Motivated by the chemotactic behavior of bacteria (Berg, 2008), Liu and Krstic proposed a stochastic ES method in Liu and Krstic (2010a) and applied it to the source seeking problem in Liu and Krstic (2010b). The seeker can successfully locate the source but with an unpredictable, "nearly random" trajectory. This feature would be useful when the seeker itself is pursued by another hostile pursuer. In their work the forward velocity of the vehicle

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is constant, resulting in complicated asymptotic behaviors. Particularly, the vehicle cannot settle when it approaches close to the source. Instead it exhibits certain overshoots and finally revolves around the source. A small constant forward velocity may improve the asymptotic performance, but decreases the convergence rate.

In order to improve the asymptotic performance of the vehicle, we apply the stochastic ES to tune both forward and angular velocities simultaneously, which is different from Liu and Krstic (2010b). While the deterministic case has been discussed in Ghods and Krstic (2010), this work focuses on designing a stochastic excitation to modulate the velocities. Under a tunable forward velocity, the vehicle can slow down around the source and converge closer to the source. In addition, the undesired overshoots are eliminated due to a tunable forward velocity. We adopt the stochastic averaging theory to establish local exponential convergence, both almost surely and in probability, to a small neighborhood near the source, for signal fields with elliptical level sets. Note that in Cochran and Krstic (2009), Ghods and Krstic (2010) and Liu and Krstic (2010b) only the stability for circular level sets was proved.

It should be mentioned that there are other methods to address stochastic source seeking problem without position information. In Manzie and Krstic (2009) Manzie and Krstic proposed a discrete-time stochastic ES. Azuma et al. (Azuma et al., 2012) adopted the stochastic approximation technique to solve this problem by sequentially generating source-oriented waypoints, which can work for a switching signal field. However, the controller is discontinuous and requires a clock to precisely decide when to take next action. Another representative method using a swarm of autonomous vehicles is proposed in Mesquita, Hespanha, and Åström (2008), which achieves high vehicle densities near the maximum. Compared with those methods, the advantage of the stochastic ES is that we can use a single vehicle under a simple continuous controller to locate the source. Moreover, the exponential convergence (in probability and almost surely) to a small attractor near the source can be established.

Overall, the advantages of the proposed stochastic ES method are as follows: (1) Only a single vehicle under a simple continuous controller is used to locate the source; (2) The forward velocity is tuned to achieve a better asymptotic behavior; (3) The local exponential convergence for elliptical signal map is established.

The rest of the paper is organized as follows. In Section 2 we describe the nonholonomic source seeking problem and propose the stochastic ES scheme. In Section 3 we prove the local exponential convergence for signal fields with elliptical level sets. We first derive an average system to approximate the original system, then we consider the local stability under a small bias forward velocity. After that we discuss the result for circular level sets as a special case. In Section 4 we include simulation results to illustrate the effectiveness of the control scheme.

2. Problem description and control scheme

In this section we firstly describe the vehicle model and formulate the source seeking problem. Then we propose a stochastic ES scheme to adjust the forward and angular velocities of the vehicle.

2.1. Problem description

Similar to Cochran and Krstic (2009), we consider an autonomous vehicle modeled as a 2D nonholonomic unicycle, see Fig. 1 for illustration. The heading angle is defined by θ, and the position of the vehicle center is defined by r_c. A sensor is mounted at the front end r_s, a distance R away from the vehicle center r_c. The vehicle has actuators which are used to impart the forward

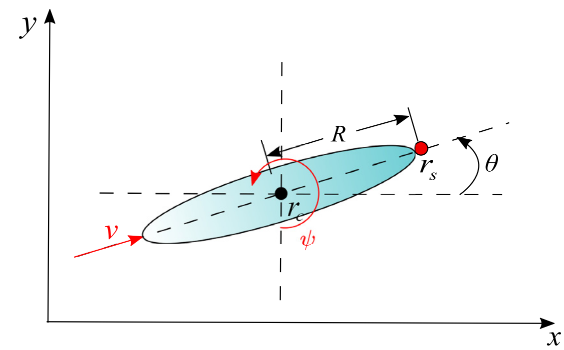

Fig. 1. Geometric interpretation of vehicle model.

velocity v and the angular velocity ψ. The kinematic equations of motion for the vehicle center and the sensor are

$$\dot{r}_c = ve^{j\theta}, \tag{1}$$

$$\dot{\theta} = \psi, \tag{2}$$

$$r_s = r_c + Re^{j\theta}, \tag{3}$$

where r_c and r_s are written as complex variables.

The task of vehicle is to seek a static source in a plane. We denote the signal strength at the location r by $J = f(r)$ and the unknown source location by r^*, both of which are constant versus time and non-random. We make the following assumption.

Assumption 1.

(1) The signal strength J can be measured by the sensor, but the position is unavailable.
(2) $f(r)$ decays away from the source and is twice continuously differentiable, satisfying that

$$\nabla f(r^*) = 0, \quad \nabla^2 f(r^*) \text{ is negative definite}, \tag{4}$$

where $\nabla f(r^*)$ and $\nabla^2 f(r^*)$ denote the gradient and Hessian of f at r^* respectively.

Note that the traditional gradient searching strategy is not suitable for the problem due to the lack of position information. By (4), we can approximate the signal distribution by a quadratic map when studying the local convergence. Without loss of generality, we assume the quadratic map takes the form

$$J = f^* - q_x(x_s - x^*)^2 - q_y(y_s - y^*)^2, \tag{5}$$

where $r_s = [x_s, y_s]^T$, $r^* = [x^*, y^*]^T$, and q_x and q_y are unknown positive constants.

2.2. Control scheme

We employ the stochastic ES method to tune the angular velocity ψ directly and the forward velocity v indirectly. The control scheme is depicted in Fig. 2. The control laws are given by

$$v = V_c + b\xi, \tag{6}$$

$$\psi = a\dot{\eta} + c\xi \sin(\eta), \tag{7}$$

$$\xi = \frac{s}{s+h}[J], \tag{8}$$

$$\eta = \frac{g\sqrt{\varepsilon}}{\varepsilon s + 1}[\dot{W}], \tag{9}$$

where the parameters a, g, ε, b, c, h and V_c are positive and will affect the performance of the approach, J is the sensor reading, and $W(t)$ is a standard Brownian motion defined in a complete