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Robust observer design under measurement noise with gain adaptation and saturated estimates ☆

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ABSTRACT

We use incremental homogeneity, gain adaptation and incremental observability for proving new results on robust observer design for systems with noisy measurement and bounded trajectories. A state observer is designed by dominating the incrementally homogeneous nonlinearities of the observation error system with its linear approximation, while gain adaptation and incremental observability guarantee an asymptotic upper bound for the estimation error depending on the limsup of the norm of the measurement noise. A characteristic and innovative feature of this observer is the mixed low/high-gain structure in combination with saturated state estimates and dynamically tuned gains and saturation levels. The gain adaptation is implemented as the output of a stable filter using the squared norm of the measured output estimation error and the mismatch between each estimate and its saturated value.

1. Introduction

Homogeneity and homogeneous approximations have been investigated by many authors for the stability analysis of an equilibrium point: see e.g. the first contributions Massera (1956) and, more recently, Kawski (1989) and Rosier (1998). The homogeneity property has been exploited in the design of global state observers (Andrieu, Praly, & Astolfi, 2008; Qian, 2005; Qian & Lin, 2006; Yang & Lin, 2003): the idea is to design a state observer for the homogeneous approximation of the system and convergence to zero of the estimation error is preserved under any perturbation which does not change the homogeneous approximation. The class of systems for which an observer can be designed by domination techniques has been enlarged by adding dynamic gain adaptation (Andrieu, Praly, & Astolfi, 2009; Astolfi & Praly, 2006; Bullinger & Allgower, 1997; Khalil & Saberi, 1987; Lei, Wei, & Lin, 2005). The class of homogeneous systems has been enlarged by introducing (incremental) homogeneity in the upper bound in Battilotti (2014) and used together with gain adaptation and self-tuned saturations for designing global observers in Battilotti (2015a) for systems with bounded trajectories. Homogeneity in the upper bound gives enough a general framework for including triangular structures (feedback and feedforward systems), homogeneous and interlaced structures. Self-tuned saturations were previously used in Lei et al. (2005) in the observer design for feedback-linearizable systems with bounded trajectories. However, the gain adaptation is such that the dynamically adapted gain is non-decreasing along solutions. As known, this may lead to serious growth problems in the presence of measurement disturbance (Egardt, 1979, Example 4.2; Khalil & Saberi, 1987; Mareels, 1984; Peterson & Narendra, 1982). This problem has been addressed by several authors (Egardt, 1979; Ioannou & Kokotovic, 1984; Mareels, 1984; Peterson & Narendra, 1982), trying to reduce the adapted gain instead to let it grow with no bound, for example when the measured output estimation error is decreasing. In Vasilijevic and Khalil (2006) it is shown that measurement disturbance introduces an upper bound on the gain when good estimation performances are required. In this direction, we find the works of Ahrens and Khalil (2006), which relies on the knowledge of a bound for the nonlinearities of the system, and Boizot, Busvelle, and Gauthier (2010), which relies on the knowledge of a bound for the dynamic gain and the Lipschitz constant of the nonlinearities of the system. The effect of measurement disturbance on observer design has been studied, following Boizot et al. (2010), for a class of lower triangular systems with bounded trajectories and for a given class of observers in Sanfelice and Praly (2011), satisfying additional properties on the mismatch between the vector fields of the system and of the observer, by proving an upper bound (depending on the measurement noise) for the estimation error in the mean and an upper bound on the limsup of the

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estimation error in the mean. In the absence of measurement noise, this last bound can be made arbitrarily small by setting properly the parameters of the class of observers. This, however, does not discard a potential oscillatory behavior of the estimates (Mareels, Van Gils, Polderman, & Ilchmann, 1999).

In this paper, we prove new results on robust observer design in the presence of measurement disturbance for systems with bounded trajectories by using incremental homogeneity in the upper bound (Battilotti, 2014) and gain adaptation (Andrieu et al., 2008; Bullinger & Allgower, 1997; Khalil & Saberi, 1987; Lei et al., 2005) with saturated estimates and dynamically tuned saturation levels (Lei et al., 2005). A state observer is designed by dominating the incrementally homogeneous (in the upper bound) nonlinearities of the observation error system with its linear approximation. The gain adaptation and updating of the saturation levels is implemented through a stable filter which regulates its output by using a suitable function of the squared norm of the measured output estimation error. Our observer guarantees an upper bound on the limsup of the norm of the estimation error depending on the limsup of the norm of the measurement noise. As a particular case, if the measurement disturbance tends asymptotically to zero the estimation error itself tends to zero.

The paper is organized as follows. In Section 2 some notation is introduced. In Section 3 the class of system is described and the problem is formulated. In Section 4 an observer is presented together with the main result and the parameter observer design is discussed in Section 4.1. In Section 4.2 example and simulation are given and in Section 4.3 the main result is proved. In the Appendix the notion of incremental generalized homogeneity is shortly recalled together with some of its properties and related results.

2. Notation

(N1) $\mathbb{R}^n$ (resp. $\mathbb{R}^{n\times n}$) is the set of n-dimensional real column vectors (resp. $n \times n$ matrices). $\mathbb{R}_\geqslant$ (resp. $\mathbb{R}^n_\geqslant$, $\mathbb{R}^{n\times n}_\geqslant$) denotes the set of real non-negative numbers (resp. vectors in $\mathbb{R}^n$, matrices in $\mathbb{R}^{n\times n}$, with real non-negative entries). $\mathbb{R}_>$ (resp. $\mathbb{R}^n_>$) denotes the set of real positive numbers (resp. vectors in $\mathbb{R}^n$ with real positive entries). $\lambda_{\min}(A)$ (resp. $\lambda_{\max}(A)$) denotes the minimum (resp. maximum) eigenvalue of $A \in \mathbb{R}^{n\times n}$.

(N2) For any matrix $V \in \mathbb{R}^{p\times n}$ we denote by V_{ij} the (i, j)-th entry of V and for any vector $v \in \mathbb{R}^n$ we denote by v_i the ith element of v. We retain a similar notation for functions. For any $v \in \mathbb{R}^n$ we denote by $\mathrm{diag}\{v\}$ the diagonal $n\times n$ matrix with diagonal elements $v_1, \ldots, v_n$. Also, $|a|$ denotes the absolute value of $a \in \mathbb{R}$, $\|a\|$ (resp. $\|a\|_P$) denotes the euclidean (resp. weighted by P) norm of $a \in \mathbb{R}^n$, $\|A\|$ denotes the norm of $A \in \mathbb{R}^{n\times n}$ induced from the euclidean norm $\|\cdot\|$ and $\langle\langle a\rangle\rangle$ the column vector of the absolute values of the elements of $a \in \mathbb{R}^n$, i.e. $\langle\langle a\rangle\rangle := (|a_1|\cdots|a_n|)^T$.

(N3) We denote by $\mathbf{C}^j(\mathscr{X}, \mathscr{Y})$, with $j \geqslant 0$, $\mathscr{X} \subset \mathbb{R}^n$ and $\mathscr{Y} \subset \mathbb{R}^p$, the set of j-times continuously differentiable functions $f : \mathscr{X} \to \mathscr{Y}$, $\mathbf{C}^0_0(\mathscr{X}, \mathscr{Y})$ the set of uniformly continuous functions $f : \mathscr{X} \to \mathscr{Y}$, by $\mathbf{L}^\infty(\mathbb{R}_\geqslant, \mathscr{Y})$ the set of functions $f \in \mathbf{C}^0(\mathbb{R}_\geqslant, \mathscr{Y})$ such that $\sup_{\theta\geqslant 0}\|f(\theta)\| < +\infty$ and by $\mathbf{L}^j(\mathbb{R}_\geqslant, \mathscr{Y})$, with $j \geqslant 1$, the set of $f \in \mathbf{C}^0(\mathbb{R}_\geqslant, \mathscr{Y})$ such that $\int_0^\infty \|f(\theta)\|^j d\theta < +\infty$. For each $d \in \mathbf{L}^\infty(\mathbb{R}_\geqslant, \mathscr{Y})$, we have the sup norm of d defined as $\|d\|_\infty := \sup_{t\geqslant 0}\|d(t)\|$. Moreover, $\mathcal{K}_0$ denotes the set of functions $f \in \mathbf{C}^0(\mathbb{R}_\geqslant, \mathbb{R}_\geqslant)$, strictly increasing with $f(0) \geqslant 0$ and $\mathcal{K}$ denotes the set of functions $f \in \mathcal{K}_0$ such that $f(0) = 0$.

(N4) A saturation function $\mathbf{sat}_h(\cdot)$ with levels $h \in \mathbb{R}^n_>$ is a function $\mathbf{sat}_h(x) := (\mathbf{sat}_{h_1}(x_1), \ldots, \mathbf{sat}_{h_n}(x_n))^T$ such that for each $i = 1, \ldots, n$ and $x_i \in \mathbb{R}$:

$$\mathbf{sat}_{h_i}(x_i) \begin{cases} x_i & |x_i| \leqslant h_i \\ \mathrm{sign}(x_i)h_i & \text{otherwise}. \end{cases} \tag{1}$$

(N5) For any vectors $x \in \mathbb{R}^n$, $\mathfrak{r} \in \mathbb{R}^n_>$ and $\epsilon \in \mathbb{R}_>$, we define

$$\epsilon^{\mathfrak{r}} := (\epsilon^{\mathfrak{r}_1}, \ldots, \epsilon^{\mathfrak{r}_n})^T, \qquad \epsilon^{\mathfrak{r}} \diamond x := (\epsilon^{\mathfrak{r}_1}x_1, \ldots, \epsilon^{\mathfrak{r}_n}x_n)^T \tag{2}$$

viz. $\epsilon^{\mathfrak{r}} \diamond x$ is the dilation of a vector x with weights $\mathfrak{r}$. Note that for any $x, y \in \mathbb{R}^n$, $\mathfrak{r}_1, \mathfrak{r}_2 \in \mathbb{R}^n_>$ and $\epsilon \in \mathbb{R}_>$

$$\epsilon^{\mathfrak{r}_1} \diamond \epsilon^{\mathfrak{r}_2} \diamond x = \epsilon^{\mathfrak{r}_2} \diamond \epsilon^{\mathfrak{r}_1} \diamond x = \epsilon^{\mathfrak{r}_1+\mathfrak{r}_2} \diamond x, \tag{3}$$

$$(\epsilon^{\mathfrak{r}_1} \diamond x)^T(\epsilon^{\mathfrak{r}_2} \diamond y) = (\epsilon^{\mathfrak{r}_2} \diamond x)^T(\epsilon^{\mathfrak{r}_1} \diamond y) = (\epsilon^{\mathfrak{r}_1+\mathfrak{r}_2} \diamond x)^T y = x^T(\epsilon^{\mathfrak{r}_1+\mathfrak{r}_2} \diamond y) \tag{4}$$

(N6) for any vectors $x, y \in \mathbb{R}^n$ we write $x \preceq y$ if and only if $x_i \leqslant y_i$ for all $i = 1, \ldots, n$. We retain the same notation for matrices $A, B \in \mathbb{R}^{n\times n}$: $A \preceq B$ if and only if $A_{ij} \leqslant B_{ij}$ for all $i, j = 1, \ldots, n$. On the other hand $A \geqslant B$ (resp. $A > B$) for matrices $A, B \in \mathbb{R}^{n\times n}$ if and only if $A - B$ is positive semidefinite (resp. positive definite).

3. Main assumptions and problem statement

Consider the system

$$\dot{x} = f(x) := [A + BF + HC]x + \phi(x), \tag{5}$$

$$y = h(x, d) := Cx + \psi(x) + d \tag{6}$$

with state $x \in \mathbb{R}^n$, measurement $y \in \mathbb{R}$ and disturbance $d \in \mathbb{R}$. The triple (A, B, C) is in prime form:

$$A = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ldots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}, \tag{7}$$

$$C = \begin{pmatrix} 1 & 0 & \cdots & 0 & 0 \end{pmatrix} \tag{8}$$

with $F \in \mathbb{R}^{1\times n}$ and $H \in \mathbb{R}^{n\times 1}$. Moreover, ϕ and ψ are locally Lipschitz continuous with $\phi(0) = 0$, $\psi(0) = 0$, $\frac{\partial\phi}{\partial x}(0) = 0$ and $\frac{\partial\psi}{\partial x}(0) = 0$ so that $\dot{x} = [A + BF + HC]x$, $y = Cx + d$, represents the linear approximation of (5)–(6) around the origin. Motivations for considering $\dot{x} = [A + BF + HC]x$, $y = Cx + d$ as the linear approximation of (5)–(6) around the origin rely in the fact that any linear single-output system is equivalent under coordinate transformations to $\dot{x}_1 = (A+BF_1+H_1C)x_1+BF_2x_2$, $\dot{x}_2 = H_2Cx_1+Gx_2$, $y = Cx_1$ where (A, B, C) is in prime form and $\dot{x}_2 = Gx_2$ is the zero-dynamics. Therefore, for simplicity and to focus on main results we are neglecting in (5)–(6) the zero dynamics of its linear approximation around the origin. We can also assume without loss of generality that $B^TH = 0$.

We consider in (5)–(6) the class $\mathcal{D}(\Delta)$ of disturbances $d \in \mathbf{L}^\infty(\mathbb{R}_\geqslant, \mathbb{R})$ such that $\|d\|_\infty \leqslant \Delta$ and uniformly continuous on their domain. The problem is to give an estimate of the state of (5) using only the noisy measurement (6). Our assumptions on the class of systems (5)–(6) are the following ones (see the Appendix for few recalls on incremental homogeneity in the upper bound which we will abbreviate as i.h.u.b. throughout the paper):

(H0) (incremental homogeneity)

(i) $C^T\psi$ and $A^T(\phi + HC)$ are incrementally homogeneous in the upper bound (i.h.u.b.) with quadruples $(\mathfrak{r}, \mathfrak{r} - \mathfrak{g}, \mathfrak{g}, C^T\psi_U)$ and, respectively, $(\mathfrak{r}, \mathfrak{r}-\mathfrak{g}, \mathfrak{g}, A^T(\phi_U+H_UC))$, with $\phi_U(0, 0) = 0$ and $\psi_U(0, 0) = 0$ for some $H_U \in \mathbb{R}^{n\times 1}$,