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Distributed Model Predictive Control of linear discrete-time systems with local and global constraints☆

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ABSTRACT

This paper proposes a Distributed Model Predictive Control (DMPC) approach for a family of discrete-time linear systems with local (uncoupled) and global (coupled) constraints. The proposed approach is based on the dual problem of a MPC optimization problem involving all systems. This dual problem is then distributedly solved, based on the Alternating Direction Multiplier Method (ADMM) with several known simplifications. When the network of systems is large or sparsely connected, the computation of the optimal control using ADMM can be expensive. The proposed approach mitigates this problem by allowing early termination of the ADMM process. This is made possible via a finite-time consensus algorithm that determines the satisfaction of the termination condition and by appropriate tightening of the coupled constraints. Under reasonable assumptions, the approach is guaranteed to converge to a small neighborhood of the optimal so long as the network is connected. Recursive feasibility and exponential stability of the closed-loop system are shown. The performance of the proposed approach is demonstrated by a numerical example.

1. Introduction

This paper considers the Distributed Model Predictive Control (DMPC) of M discrete-time linear dynamical systems, each of which is of the form

$$x^i(t+1) = A^i x^i(t) + B^i u^i(t), \tag{1}$$

$$x^i(t) \in X^i, \qquad u^i(t) \in U^i, \quad i = 1, \ldots, M \tag{2}$$

and all of them have to satisfy a coupled/global constraint of the form

$$\sum_{i=1}^{M} \left(\Psi_x^i x^i(t) + \Psi_u^i u^i(t)\right) \leq \mathbf{1}_p, \quad \text{for all } t \tag{3}$$

where x^i, u^i are the states and controls of the ith system, respectively, and $X^i \subset \mathbb{R}^{n_i}$, $U^i \subset \mathbb{R}^{m_i}$ are the corresponding constraint sets; the matrices $\Psi_x^i \in \mathbb{R}^{p\times n_i}$ and $\Psi_u^i \in \mathbb{R}^{p\times m_i}$ define the coupled constraints for all M systems and $\mathbf{1}_p$ is the p-vector of all ones.

The study of DMPC is an active area of research (Christofides, Scattolini, Peña, & Liu, 2013; Keviczky, Borrelli, & Balas, 2006; Maestre & Negenborn, 2014; Scattolini, 2009; Wang & Ong, 2010) and one popular area is when the systems are dynamically coupled (Conte, Voellmy, Zeilinger, Morari, & Jones, 2012; Farina & Scattolini, 2012; Riverso, Farina, & Ferrari-Trecate, 2013; Riverso & Ferrari-Trecate, 2012; Summers & Lygeros, 2012; Wang & Ong, 2016b). However, these approaches are not suitable for the problem above due to the complications arising from (3). To the best of our knowledge, DMPC approaches for (1)–(3) are somewhat limited. The method of Richards and How (2007) ensures the satisfaction of (3) using a sequential process: one system is optimized at a time while all others stay constant; this is followed sequentially by another system so that all M systems are optimized once in M time steps. Another approach is known as the cooperative MPC method (Trodden, 2014; Trodden & Richards, 2010, 2013). While specific details vary, the basic idea is that all systems within a cooperating set (possibly a singleton) are optimized jointly (or in parallel) while systems outside the cooperating set follow their predicted states and predicted controls. These methods optimize individual or groups of systems sequentially. However, the optimality of the overall system is unclear as they are not explicitly pursued. In addition, these approaches require direct communications among all systems that are coupled by (3) which, even for a

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system of moderate size, is a strong requirement (Halvgaard, Vandenberghe, Poulsen, Madsen, & Jorgensen, 2016; Low & Lapsley, 1999; Spudić, Conte, Baotić, & Morari, 2015).

A reasonable approach for (1)–(3) (Bertsekas, 1999; Bertsekas & Tsitsiklis, 1997) that achieves overall optimality is to solve the dual problem involving the Lagrangian function. In this case, the Lagrangian function is the sum of M separable functions except for the dual variable associated with (3). This dual variable is treated as a consensus variable in a distributed consensus optimization problem (DCOP). Typically, consensus of the dual variable is ensured (Chapter 6 of Bertsekas, 1999) using a central/master node.

This work follows the above formulation resulting in a DCOP. However, the DCOP is solved using the Distributed ADMM algorithm where each system has a local copy of the dual variable. The choice of ADMM is motivated by its reported nice numerical properties (Boyd, Parikh, Chu, Peleato, & Eckstein, 2011), various simplifications (Chang, Hong, & Wang, 2015; Mota, Xavier, Aguiar, & Püschel, 2012, 2013; Wei & Ozdaglar, 2012) and its preference as a solver for distributed system in many applications (Boyd et al., 2011). The local copies of the dual variable need not reach consensus but only within some fixed bound of one another. Such an approach is used because the computational effort of the Distributed ADMM can be high and allowing premature termination of the ADMM algorithm provides for computational expediency. Measures to handle such premature termination are provided, together with recursive feasibility and stability of the closed-loop system. Under reasonable assumptions, the approach is guaranteed to converge to some small neighborhood of the overall optimal solution so long as the network is connected. The approach is iterative, similar to other MPC schemes (Necoara & Nedelcu, 2014; Patrinos & Bemporad, 2014; Rubagotti, Patrinos, & Bemporad, 2014) but for multiple systems with coupled constraints.

The rest of this paper is organized as follows. This section ends with a description of the notations used. Section 2 reviews some results of the standard stand-alone MPC for a single system and discusses the formulation of the overall MPC problem. Section 3 presents the proposed approach, including the discussion of the coupled constraint, its dual and the convergence of the distributed ADMM algorithm. The recursive feasibility and stability results are given in Section 5. The performance of the approach is illustrated by a numerical example in Section 6 with the conclusions given in Section 7. All proofs are given in the Appendices.

The notations used in this paper are as follows. Non-negative and positive integer sets are indicated by $\mathbb{Z}_0^+$ and $\mathbb{Z}^+$, respectively. Let $M, L \in \mathbb{Z}_0^+$ with $M \geq L$. Then, $\mathbb{Z}^M := \{1, 2, \ldots, M\}$ and $\mathbb{Z}_L^M := \{L, L+1, \ldots, M\}$. Similarly, $\mathbb{R}_0^+$ and $\mathbb{R}^+$ refer, respectively, to the sets of non-negative and positive real number. I_n is the $n \times n$ identity matrix, $\mathbf{1}_n$ is the n-column vector of all ones (subscript omitted when the dimension is clear) and $|S|$ is the cardinality of the index set S. Given $\sigma > 0$, $X \subset \mathbb{R}^n$ with $0 \in int(X)$ where $int(\cdot)$ is the interior of a set, $\sigma X = \{\sigma x : x \in X\}$. For a square matrix Q, $Q \succ (\succeq) 0$ means Q is positive definite (semi-definite). The ℓ_p-norm of $x \in \mathbb{R}^n$ is $\|x\|_p$ while $\|x\|_Q^2 = x^T Q x$ for $Q \succ 0$. Several representations of the states and controls are needed: $x^i(t)$, $u^i(t)$ refer to the state and control of the ith system at time t; x_k^i, u_k^i are the kth predicted state and predicted control of the ith system; $x = (x^1, x^2, \ldots, x^M)$, $u = (u^1, u^2, \ldots, u^M)$ are the collections of x^i and u^i over the M systems; boldface $\boldsymbol{x}^i = (x_1^i, x_2^i, \ldots, x_N^i)$, $\boldsymbol{u}^i = (u_0^i, u_1^i, \ldots, u_{N-1}^i)$ are, respectively, the collections of the N predicted states and predicted controls over the horizon (of length N) for the ith system; in situation where the reference to time is needed, x_k^i, u_k^i can be written as $x_{k|t}^i$ and $u_{k|t}^i$. Hence, $x_{0|t}^i = x^i(t)$ and $u_{0|t}^i = u^i(t)$. Additional notations are introduced as required in the text.

2. Preliminaries and problem formulation

This section reviews some well-known results in standard MPC and other related concepts. Consider a stand-alone system represented by one choice of $i \in \mathbb{Z}^M$ in (1) under a standard MPC setting of

$$\min_{\boldsymbol{u}^i} J^i(x^i, \boldsymbol{u}^i) := \sum_{\ell=0}^{N-1} (\|x_\ell^i\|_{Q^i}^2 + \|u_\ell^i\|_{R^i}^2) + \|x_N^i\|_{P^i}^2 \tag{4a}$$

$$\text{s.t. } \boldsymbol{u}^i \in \mathcal{U}_T^i(x^i) \tag{4b}$$

where N is the horizon length, $\boldsymbol{u}^i := \{u_0^i, u_1^i, \ldots, u_{N-1}^i\}$, $\boldsymbol{x}^i := \{x_0^i, x_1^i, \ldots, x_N^i\}$ are the predicted controls and predicted states, respectively, satisfying $x_{\ell+1}^i = A^i x_\ell^i + B^i u_\ell^i$ with $x_0^i = x^i$, $J(x^i, \boldsymbol{u}^i)$ is the standard quadratic costs parameterized by $(x^i, \boldsymbol{u}^i)$ defined by (4a) and

$$\mathcal{U}_T^i(x^i) := \{\boldsymbol{u}^i \in \mathbb{R}^{m_i N} : x_{\ell+1}^i = A^i x_\ell^i + B^i u_\ell^i, \quad x_0^i = x^i, x_\ell^i \in X^i, u_\ell^i \in U^i, x_N^i \in T_f^i, \ell \in \mathbb{Z}_0^{N-1}\} \tag{5}$$

where T_f^i is some appropriate terminal set satisfying

$$A_K^i x^i \in T_f^i, \quad K^i x^i \in U^i \text{ for all } x^i \in T_f^i \tag{6}$$

with $A_K^i := A^i + B^i K^i$ and K^i, P^i are the solutions to the Algebraic Riccati Equation (ARE) with weights $Q^i \succ 0$, $R^i \succ 0$. The overall MPC optimization problem over the M systems incorporating (3) at state $x = \{x^1, \ldots, x^M\}$ is

$$\mathbb{P}(x) : V(x) := \min_{\{\boldsymbol{u}^i, i \in \mathbb{Z}^M\}} \sum_{i=1}^M J^i(x^i, \boldsymbol{u}^i) \tag{7a}$$

$$s.t. \ \boldsymbol{u}^i \in \mathcal{U}_T^i(x^i), \quad \forall i \in \mathbb{Z}^M, \tag{7b}$$

$$\sum_{i=1}^M \Psi_x^i x_\ell^i + \Psi_u^i u_\ell^i \leq \mathbf{1}_p, \quad \forall \ell \in \mathbb{Z}_0^{N-1} \tag{7c}$$

where (7c) refers to the satisfaction of the coupled constraints at each predicted time step of the horizon. The conditions of (6) on T_f^i do not include the effect of the coupled constraint which is given by

$$\sum_{i=1}^M \bar{\Psi}^i x^i := \sum_{i=1}^M (\Psi_x^i + \Psi_u^i K^i) x^i \leq \mathbf{1}_p, \quad \forall x^i \in T_f^i. \tag{8}$$

2.1. Tightening the constraints

The formulation of (7) and the condition of (8) are appropriate when the Distributed ADMM algorithm achieves convergence at every time step. However, the online verification of the convergence of a distributed algorithm is numerically expensive. The approach adopted here is to allow premature termination of the Distributed ADMM algorithm up to a certain ϵ tolerance, enforced as a stopping criteria for the algorithm. Correspondingly, (7) and (8) need to be tightened to account for this tolerance. Specifically, the tightened constraints for (7c) and (8) are

$$\sum_{i=1}^M \Psi_x^i x_\ell^i + \Psi_u^i u_\ell^i \leq (1 - \epsilon M(\ell+1)) \mathbf{1}_p, \quad \forall \ell \in \mathbb{Z}_0^{N-1} \tag{9}$$

$$\sum_{i=1}^M \bar{\Psi}^i x^i \leq (1 - MN\epsilon) \mathbf{1}_p, \quad \forall x^i \in T_f^i. \tag{10}$$

Obviously, the choice of ϵ has to satisfy $0 < \epsilon < \frac{1}{MN}$ to ensure that $0 \in int(T_f^i)$ in (10). Note that the local constraints of (7b)