Automatica 81 (2017) 322–334

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Output regulation by error dynamic feedback in hybrid systems with periodic state jumps[☆]

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ARTICLE INFO

Article history:
Received 17 March 2015
Received in revised form
8 December 2016
Accepted 14 March 2017
Available online 25 April 2017

Keywords:
Hybrid systems
Output regulation
Global asymptotic stability
Geometric approach

ABSTRACT

This work deals with output regulation in multivariable hybrid systems featuring a continuous-time linear dynamics periodically affected by instantaneous changes of the state. More precisely, given a hybrid linear plant and a hybrid linear exogenous system, with periodic state jumps, the problem consists in finding a hybrid feedback regulator, with the same characteristics, achieving global asymptotic stability of the closed-loop dynamics and asymptotic tracking of the reference generated by the exogenous system for all the initial states. Starting from a general, necessary and sufficient condition for the existence of a solution, the discussion leads to a more specific, sufficient condition which outlines the computational framework for a straightforward synthesis of the compensator. The internal model principle is shown to hold in a more general formulation than the original one, adapted to the hybrid systems considered. A numerical example is worked out with the aim of illustrating how to implement the devised technique. The geometric approach is the key methodology in attaining these results.

1. Introduction

Hybrid systems with state jumps are dynamical systems which exhibit a continuous-time behavior (the so-called flow dynamics) interrupted by state discontinuities (jump dynamics). These dynamical systems have drawn an increasing amount of research effort during the last decade, mainly because they are particularly effective in representing the peculiar way some real systems, occurring in various fields of science and engineering, operate (see, e.g., Goebel, Sanfelice, & Teel, 2009, 2012). Indeed, many classes of jump hybrid systems can be distinguished on the basis of several characteristics, such as the flow and jump dynamics being linear or nonlinear, the jumps being time driven or state driven, and so on. Thus, the control synthesis raises a number of typical issues and requires ad-hoc devised methodologies, depending on the features of the hybrid systems addressed. In particular, this work is focused on hybrid systems with a continuous-time linear dynamics subject to periodic state jumps and investigates the output regulation problem.

Output regulation is a classic problem of control theory and it essentially consists in finding a feedback regulator which, for a given plant and a given exogenous system, ensures stability of the closed-loop dynamics and asymptotic tracking of the reference generated by the exogenous system for all the initial states. A less basic formulation of this problem (including decoupling of a disturbance generated by the exogenous system and directly affecting the plant) has been studied for scalar hybrid systems with periodic state jumps in Marconi and Teel (2010, 2013) and for multivariable hybrid systems of the same class in Carnevale, Galeani, and Menini (2012a); Carnevale, Galeani, Menini, and Sassano (2016); Carnevale, Galeani, and Sassano (2013).

The works by Carnevale et al. (2012a, 2016, 2013) – which, referring to the multivariable case, are closer to this one – give a necessary and sufficient condition for the existence of a solution to the considered problem in terms of solvability of a set of differential linear matrix equations. This result is derived by elaborating further on the regulator equations that originally characterized solvability of the output regulation problem for linear time-invariant systems (Francis, 1977). In principle, the hybrid regulator can be obtained by solving the so-called hybrid regulator equations (Carnevale et al., 2016, Section III). However, as acknowledged by the same authors (Carnevale et al., 2016, Remark 2), this

[☆] The material in this paper was partially presented at the 14th annual European Control Conference, July 15–17, 2015, Linz, Austria. This paper was recommended for publication in revised form by Associate Editor Mario Sznaier under the direction of Editor Richard Middleton.

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http://dx.doi.org/10.1016/j.automatica.2017.03.037

method presents serious difficulties due to the infinite number of constraints implied in such equations. For this reason, the analysis is deepened so as to define a viable, valid in general, synthesis procedure, based on the solution of two finite-dimensional Francis equations, which are algebraic matrix equations (Carnevale et al., 2016, Section IV-C).

In this work, the output regulation problem for multivariable hybrid systems with periodic state jumps is considered from a different perspective. Namely, new solvability conditions are derived by employing the methodologies of the geometric approach (Basile & Marro, 1992; Wonham, 1985). More specifically, the results presented herein are obtained by exploiting the geometric interpretation of the output regulation problem that was first developed by Marro (1996) and that has lately inspired the solution of the same problem for more complex dynamical systems, such as linear parameter varying systems (Zattoni, 2008), linear time-delay systems (Conte, Perdon, & Zattoni, 2012), and linear switching systems (Zattoni, Perdon, & Conte, 2013). This methodology requires that some fundamental concepts of the geometric approach (e.g., invariance and controlled invariance) be generalized to hybrid systems with state jumps. Indeed, some of these notions have proven to be instrumental in analyzing the structure of the considered hybrid systems in Medina (2007); Medina and Lawrence (2009) and, to some extent, in Carnevale, Galeani, Menini, and Sassano (2014a). However, in this work, as in Perdon, Conte, and Zattoni (2015); Perdon, Zattoni, and Conte (2016), structural notions are used in combination with qualitative notions such as stability, thus allowing all the aspects of the problem to be handled in the geometric framework.

A main contribution of this work is establishing a new necessary and sufficient condition for problem solvability in strict geometric terms (Theorem 1). The geometric necessary and sufficient condition is perfectly consistent with the necessary and sufficient condition based on the hybrid regulator equations of Carnevale et al. (2016, Proposition 3), as will be explained in Remark 1. Meanwhile, the geometric condition has the merit of giving insight into the way the overall regulated system works, since, in particular, it points out the subspace of the admissible state motions. However, the necessary and sufficient geometric condition cannot be directly used as a design tool, since, referring to the overall compensated system, it involves the feedback regulator. Nonetheless, the geometric condition can be exploited to derive a more specific, sufficient solvability condition which provides particularly straightforward synthesis tools (although not applicable to the whole generality of solvable problems).

Hence, the subsequent contribution of this work is a geometric sufficient condition for problem solvability, solely involving the problem data (Theorem 2). In fact, such sufficient condition is centered on the output-difference connection between the plant and the exogenous system (the so-called hybrid extended system) and requires the existence of a subspace with the property of being both controlled invariant for the flow dynamics and invariant for the jump dynamics (in addition to that of being contained in the kernel of the output map). Indeed, such sufficient condition may be rather conservative, a main reason being that it respectively demands controlled invariance and invariance under the linear maps of the flow and jump dynamics (i.e., hybrid controlled invariance) instead of a combination of the two. On the positive side, since such condition disregards the time period between two consecutive state jumps, it ensures the existence of a solution for any finite time period. Further, under such condition, the compensator synthesis is extremely simple. In fact, as will be shown in the proof of the theorem, the synthesis procedure amounts to the computation of a stabilizing friend for the resolving hybrid controlled invariant subspace and of a stabilizing output injection for the dynamics of the hybrid extended system.

In order to shed light on the conflict between conservativeness and constructiveness of the considered conditions, another sufficient condition for problem solvability is established (Theorem 3). Actually, Theorem 3 is focused on the dynamics obtained as the combination over one period of the flow and jump dynamics of the hybrid output-difference system and, as such, has a broader scope compared to Theorem 2. Namely, the condition of Theorem 3 implies that of Theorem 2, while the converse is not true in general, as will be made clear in Remark 5. Nevertheless, it is worth noting that the flow dynamics combined with the jump dynamics in the statement of Theorem 3 is assumed to be compensated by state feedback, so as to take into account the available control input in the way compatible with the compensation scheme considered in the general necessary and sufficient condition. Thus, the interplay between such unknown state feedback and the unknown controlled invariant subspace, which has the role of resolving subspace, makes the condition of Theorem 3 difficult to ascertain and nonconstructive.

As mentioned above, the compensator synthesis performed according to the proof of Theorem 2 presupposes that the resolving hybrid controlled invariant subspace be known. However, the geometric sufficient condition does not contain any hint on how to compute such subspace. Hence, in order to provide a complete synthesis tool for the whole set of problems whose solvability is ensured by Theorem 2, a necessary and sufficient constructive condition for a hybrid controlled invariant subspace to satisfy the requisites of Theorem 2 is established in Theorem 4.

The paper is organized as follows. In Section 2, the output regulation problem for multivariable hybrid linear systems with periodic state jumps is presented. In Section 3, a necessary and sufficient condition for problem solvability, referring to the overall compensated hybrid system, is stated in geometric terms. A geometric sufficient condition, focused on the hybrid extended system, is proven in Section 4. In the same section, a more extensive, yet nonconstructive, sufficient condition is also discussed. A necessary and sufficient condition for the existence of a subspace fulfilling the requirements of the constructive sufficient condition is shown in Section 5. A numerical example illustrating how to implement the devised synthesis procedure is worked out in Section 6. Section 7 contains the conclusions. Appendix discusses some results on the stabilization of a hybrid dynamics via a state feedback or via an output injection, each one acting on the flow dynamics only.

Notation: The symbols $\mathbb{Z}$, $\mathbb{Z}_0^+$, $\mathbb{Z}^+$, $\mathbb{R}$, $\mathbb{R}_0^+$, $\mathbb{R}^+$, and $\mathbb{C}$ stand for the sets of integer numbers, nonnegative integer numbers, positive integer numbers, real numbers, nonnegative real numbers, positive real numbers, and complex numbers, respectively. The symbol $\mathtt{i}$ stands for the imaginary unit and, given a complex number $\lambda = \lambda_a + \mathtt{i}\lambda_b$, $|\lambda|$ denotes its modulus and $\mathrm{Arg}\,(\lambda)$ its argument. Matrices and linear maps are denoted by slanted capital letters, like A. The image, the kernel, the inverse, and the transpose of A are denoted by $\mathrm{Im}\,A$, $\mathrm{Ker}\,A$, A^{-1}, and $A^\top$, respectively. Vector spaces and subspaces are denoted by calligraphic letters, like $\mathcal{V}$. The notation $\mathcal{W}/\mathcal{V}$ stands for the quotient space of a subspace $\mathcal{W} \subseteq \mathcal{X}$ over a subspace $\mathcal{V} \subseteq \mathcal{W}$. The expression $\mathcal{V} \oplus \mathcal{W} = \mathcal{X}$ stands for $\mathcal{V} + \mathcal{W} = \mathcal{X}$ and $\mathcal{V} \cap \mathcal{W} = \{0\}$. The symbol $A|_{\mathcal{J}}$ denotes the restriction of a linear map A to an A-invariant subspace $\mathcal{J}$, while $A|_{\mathcal{X}/\mathcal{J}}$ denotes the map induced by A on the quotient space $\mathcal{X}/\mathcal{J}$. The symbol $\|x\|$, where $x \in \mathbb{R}^n$, denotes the 2-norm of x, while $\|A\|$, where $A \in \mathbb{R}^{m\times n}$, denotes the norm induced on A by the vector 2-norm: i.e., $\|A\| = \sup_{x\in\mathbb{R}^n,\, x\neq 0} (\|Ax\|/\|x\|)$. Moreover, $\|A\|_{\mathcal{F}}$ denotes the Frobenius norm of A (i.e., the square root of the sum of the squares of all entries). The symbols I_n and $0_{m\times n}$ denote an identity matrix of dimension n and an $(m \times n)$-zero matrix (subscripts are omitted when the size is clear from the context). The symbol $\mathrm{diag}\,\{d_1, \dots, d_n\}$ denotes a diagonal matrix with the