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Ultimate boundedness and regions of attraction of frequency droop controlled microgrids with secondary control loops☆

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ABSTRACT

In this paper we study theoretical properties of the frequency control problem in inverter-based microgrids with primary and decentralised/distributed secondary control loops. Stability of these microgrids has been the subject of a number of recent studies. Conventional approaches based on standard hierarchical control rely on time-scale separation between primary and secondary control loops to show local stability of equilibria. In this paper we show that (i) frequency regulation can be ensured without assuming time-scale separation and, (ii) ultimate boundedness of the trajectories starting inside a region of attraction is guaranteed under a condition on the power mismatch between demand and generation at each inverter bus. An estimate of the region of attraction is obtained from which an ultimate bound set for the state trajectories can be determined by recursive iterations of a nonlinear mapping. The derived results provide a certificate of the overall stability and performance of the controlled microgrid.

1. Introduction

The last decade has witnessed a gradual transition from large centralised energy grids towards small-scale distributed generation (DG) of power (Ustun, Ozansoy, & Zayegh, 2011), driven by the need to reduce the environmental impacts of coal-fired generation. DG sources are typically integrated in microgrids before being connected to the main energy grid. A microgrid is a small-scale power system consisting of DG units, loads and local storage, operating together with energy management, control and protection devices (Lasseter, 2001; Peng, Li, & Tolbert, 2009). Microgrids can operate while being connected to the main grid or in an islanded mode. The DG resources connected to the microgrid may generate either variable frequency AC power or DC power, and are interfaced with an AC grid via power electronic DC/AC inverters. In islanded mode, inverters act as ideal voltage sources through which control actions to ensure various tasks such as synchronisation, power balance and load sharing can be executed (Peças Lopes, Moreira, & Madureira, 2006).

Control strategies to provide stability in microgrids have been the focus of recent study. In Bidram and Davoudi (2012) and Guerrero, Vasquez, Matas, García de Vicuña, and Castilla (2011), hierarchical control for microgrids has been proposed in order to standardise their operation and functionalities following a model traditionally applied in power grids. In this hierarchical approach, three main control layers are defined to manage voltage and frequency stability and regulation, power flow and economic optimisation. This paper focuses on the primary and secondary control layers, which are commonly used to implement the automatic control mechanisms to achieve voltage and frequency stability and regulation (Ainsworth & Grijalva, 2013; Andreasson, Sandberg, Dimarogonas, & Johansson, 2012; Bidram & Davoudi, 2012; Bouattour, Simpson-Porco, Dörfler, & Bullo, 2013; Dörfler, Simpson-Porco, & Bullo, 2016; Guerrero et al., 2011; Heidari, Seron, & Braslavsky, 2014; Lu & Chu, 2014; Schiffer, Ortega, Astolfi, Raisch, & Sezi, 2014; Shafiee, Vasquez, & Guerrero, 2012; Simpson-Porco, Dörfler, & Bullo, 2013).

For systems with inductive lines, inverters are typically controlled to emulate the droop characteristic of synchronous generators. Conventionally, the frequency/active-power (or "$\omega - P$")

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droop control (Chandorkar, Divan, & Adapa, 1993) is adopted as the decentralised control strategy for the autonomous active power sharing at primary layer. Since the standard droop control is a purely proportional control strategy, the secondary control layer has the task of compensating for frequency steady-state errors induced by the primary control layer. The secondary control layer can be implemented in a centralised, decentralised or distributed fashion (Andreasson et al., 2012; Schiffer, Anta, Trung, Raisch, & Sezi, 2012; Shafiee et al., 2012). In this paper we consider microgrids with decentralised and distributed secondary control loop and study their impact on the stability of the system.

Stability and convergence properties of droop-controlled networks of inverters and loads have only very recently started to be analysed in detail (Ainsworth & Grijalva, 2013; Andreasson et al., 2012; Bouattour et al., 2013; Lu & Chu, 2014; Schiffer et al., 2014; Simpson-Porco et al., 2013). For example, in Ainsworth and Grijalva (2013), the authors proved that the frequency error in microgrids with decentralised secondary control loop can be made arbitrarily small, although not zero. To prove this result, they assumed time-scale separation between primary and secondary loops, which is a conventional method to study properties of microgrids. In Shafiee, Guerrero, and Vasquez (2014), a distributed secondary controller exploiting all-to-all communication between inverters is proposed to regulate frequency. In Simpson-Porco et al. (2013), the authors derive a necessary and sufficient condition for the existence of a unique and locally exponentially stable equilibrium state for a droop-controlled network. Further, they propose a distributed secondary-control scheme to dynamically regulate the network frequency to a nominal value while maintaining proportional power sharing among the inverters, and without assuming time-scale separation between primary and secondary control loops. This is in contrast with more conventional analyses which often rely on the time-scale separation assumption (Ainsworth & Grijalva, 2013; Bidram & Davoudi, 2012).

In this paper we analyse stability of an inverter-based microgrid with purely inductive lines under frequency (primary) droop control and with secondary control loops. It is well-known that the frequency in networks with purely inductive lines is mainly affected by the active-power balance (Kundur, 1994, Section 11.1). Of the various secondary control strategies in the frequency control literature, we consider microgrids with decentralised and distributed secondary control layers, where the distributed approach follows that in Simpson-Porco et al. (2013). In the decentralised approach, the control is implemented locally and there is no communication between inverters, whereas in the distributed approach, inverter controllers share information with a selection (not necessarily all) of the other inverters to stabilise the system. Under these control policies, we study frequency regulation and ultimate boundedness for the microgrid.

This paper improves on existing work by (i) providing a new modelling framework to study stability properties of microgrids, and (ii) deriving regional stability characteristics rather than focusing on local results around equilibrium points, as is the case with most of the literature to-date.

Our first contribution is a structured nonlinear model for a microgrid with embedded primary and secondary control layers. By performing a suitable change of coordinates, we show that the stability analysis for the controlled system can be decoupled into a linear system stability problem, and that of characterising ultimate boundedness of the trajectories of a perturbed nonlinear subsystem around steady-state solutions. Our second and main contribution is then to establish stability properties of the original nonlinear system by exploiting this model separation. Frequency regulation is proved through this structured model. Moreover, the linear analysis shows that frequency regulation is ensured without the need for time-scale separation. For the perturbed nonlinear subsystem, we show that ultimate boundedness of the trajectories starting inside a region of the state space is guaranteed under a condition on the power injection errors for the inverters. The ultimate bounds for the trajectories can be computed by iterating a well-specified nonlinear map, which provides key certificates for the overall performance of the controlled microgrid.

Preliminary results on the problem considered in this paper were presented in Heidari et al. (2014); Heidari, Seron, and Braslavsky (2015), where we analysed frequency control of microgrids with decentralised secondary loops and homogeneous droop coefficients. A particular case of a network consisting of two inverters was analysed in Heidari, Seron, and Braslavsky (2015). The present paper extends these preliminary results to a more general control scheme for microgrids with an arbitrary number of inverters, and further analyses the distributed secondary control approach. The current study includes the estimation of the region of attraction to the ultimate bound set on the system trajectories, that is, trajectories starting within the determined estimate of the region of attraction will ultimately lie inside a bounded region of the state space, guaranteeing practical stability of the microgrid.

The rest of the paper is organised as follows. Section 2 introduces notations and preliminary results, followed by the formulation of a structured nonlinear model for the microgrid system in Section 3. The main results, characterising ultimate bound sets and their regions of attraction, are stated and discussed in Section 4. Section 5 provides a robustness analysis of the ultimate bound sets with respect to changes in load conditions in the network. The results are illustrated by means of a numerical example in Section 6.1. The example in Section 6.2 further demonstrates scalability of the proposed technique. Some concluding remarks are presented in Section 7. The proofs of most of the results in the paper are given in the Appendix.

2. Preliminaries

We introduce the paper mathematical notation and some definitions, and adapt from Haimovich and Seron (2013) a key result on ultimate boundedness of nonlinear dynamical systems.

Mathematical Notations: For a matrix M, $M_{(i,:)}$, $M_{(:,j)}$, $M_{(i:j,:)}$ and $M_{(i,j)}$ denote its ith row, jth column, rows i to j, and ijth entry, respectively. Let $\mathbf{1}_n$ and $\mathbf{0}_n$ be the n-dimensional vectors of unit and zero entries. $\{x_i\}_{i=1,\ldots,n}$ is a column vector with entries x_i, $i = 1, \ldots, n$, and $\{y_{ij}\}_{i,j=1,\ldots,n}$ is a matrix with entries y_{ij}, $i, j = 1, \ldots, n$. A diagonal matrix with entries $d_i \in \mathbb{R}$, $i = 1, \ldots, n$, in the diagonal, is denoted by $\mathrm{diag}(d_1, \ldots, d_n)$. $\mathbb{R}^n_{+0}$ denotes the set of real n-vectors with nonnegative components. Inequalities and absolute values are taken componentwise. A nonnegative vector function $\Psi : \mathbb{R}^n_{+0} \to \mathbb{R}^n_{+0}$ is said to be componentwise nondecreasing (CND) if whenever $\xi_1, \xi_2 \in \mathbb{R}^n_{+0}$ and $\xi_1 \le \xi_2$, then $\Psi(\xi_1) \le \Psi(\xi_2)$.

We now recall from Haimovich and Seron (2013) a key result on ultimate boundedness of a linear system subject to non-vanishing, state-dependent nonlinear perturbations defined by the equation

$$\dot{x}(t) = -\Lambda x(t) + H\psi(x(t)), \tag{1}$$

where $x \in \mathbb{R}^r$, $\psi \in \mathbb{R}^s$, $\Lambda \in \mathbb{R}^{r\times r}$, $H \in \mathbb{R}^{r\times s}$ and the magnitude of the nonlinear function $\psi(\cdot)$ is bounded as

$$|\psi(x(t))| \le \Psi(|x(t)|), \quad \forall t \ge 0, \tag{2}$$

where $\Psi(\cdot)$ is a CND function.

Lemma 1 (*Ultimate Boundedness Haimovich & Seron, 2013, Theorem 3*)**.** *Consider the system (1) with a perturbation bound of the form (2), where $-\Lambda$ is a diagonal Hurwitz matrix and the bounding function Ψ is CND. Define the nonlinear mapping $T : \mathbb{R}^r_{+0} \to \mathbb{R}^r_{+0}$ as*

$$T(x) = \Lambda^{-1}|H|\Psi(x). \tag{3}$$