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Brief paper

Robust ranking and selection with optimal computing budget allocation☆

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ABSTRACT

In this paper, we consider the ranking and selection (R&S) problem with input uncertainty. It seeks to maximize the probability of correct selection (PCS) for the best design under a fixed simulation budget, where each design is measured by their worst-case performance. To simplify the complexity of PCS, we develop an approximated probability measure and derive an asymptotically optimal solution of the resulting problem. An efficient selection procedure is then designed within the optimal computing budget allocation (OCBA) framework. More importantly, we provide some useful insights on characterizing an efficient robust selection rule and how it can be achieved by adjusting the simulation budgets allocated to each scenario.

1. Introduction

Many problems in control engineering involve large-scale discrete-event dynamic systems. Since these systems are usually too complex to be described by succinct mathematical models, stochastic simulation has been a popular choice for analyzing these systems. However, due to slow convergence, simulation efficiency is a major concern, especially when the number of competing designs to be compared is large. This explains the increasing popularity of research in ranking and selection (R&S): techniques that determine the number of simulation replications for each design alternative such that the selection quality for the best design is optimized or guaranteed at a pre-specified level.

There are two primary approaches for R&S problems: indifference-zone (IZ) and optimal computing budget allocation (OCBA). The IZ approach aims to provide a guarantee for the selection quality, assuming that the mean performance of the best design is at least δ^* better than each alternative, where δ^* is the minimum difference worth detecting (Kim & Nelson, 2001). The OCBA approach allocates the samples sequentially in order to optimize the selection quality under a simulation budget constraint (Chen, Lin, Yücesan, & Chick, 2000; Gao & Chen, 2015, in press-b). The high efficiency of the OCBA method has been demonstrated via a variety of numerical experiments (Branke, Chick, & Schmidt, 2007; Chen, Gao, Chen, & Shi, 2014).

An implicit assumption for the abovementioned R&S procedures is that the true input distributions and their parameters are known, while in practice, they are typically unknown and have to be estimated from limited historical data. The finiteness of historical data leads to uncertainty in the estimated input distributions and their parameters, which might (severely) affect the quality of the selection in R&S procedures.

In view of the importance of this issue, Corlu and Biller (2013) developed a subset selection procedure that accounts for the input uncertainty. Fan, Hong, and Zhang (2013) presented a robust IZ-based R&S formulation that selects the best design with respect to the worst-case choices among a finite collection of possible input models, called *robust selection of the best* (RSB).

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This issue is also related to two streams of literature. The first is input uncertainty quantification, which quantifies the impact of input uncertainty on the simulation output (Barton, Nelson, & Xie, 2014; Song, Nelson, & Pegden, 2014). The other is robust optimization (RO) (Ben-Tal, den Hertog, Waegenaere, Melenberg, & Rennen, 2013; Delage & Ye, 2010). Different from simulation-based optimization in which targeted problems do not have nice structures to be exploited, RO often requires the optimization problems to be available explicitly in closed form.

In this research, we utilize the OCBA approach and develop a new and efficient procedure for R&S problems with input uncertainty. To the best of our knowledge, it is the first OCBA-based procedure for R&S problems with input uncertainty. The uncertainty set is assumed to contain a finite number of scenarios for the underlying input distributions and parameters. The selection problem is formulated as maximizing the probability of correctly selecting the best design under a fixed simulation budget, where the performance of a design is measured by its worst-case performance among all the possible scenarios in the uncertainty set. When all the scenarios of each design and their means and variances are known, we derive the asymptotic (as the simulation budget goes to infinity) optimal solution for the selection problem considered. A sequential selection algorithm, called R-OCBA (robust OCBA), is designed which heuristically implements the derived solution. The significantly higher efficiency of R-OCBA is demonstrated via numerical tests. A preliminary study of this problem has been presented without proof in Gao, Xiao, Zhou, and Chen (2016).

The rest of the paper is organized as follows. Section 2 formulates the selection problem with input uncertainty. Section 3 derives the asymptotic optimal solution for the problem formulated and develops a corresponding sequential selection procedure. Numerical experiments are provided in Section 4, followed by conclusions in Section 5.

2. Problem statement

In this section, we provide some notation and assumptions and formulate the selection problem with input uncertainty.

2.1. Preliminaries

Essentially, we want to solve the following selection problem

$$\min_{x\in\mathscr{X}} E_P[h(x,\xi)], \tag{1}$$

where the set of system designs (solutions) $\mathscr{X} = \{x_1, x_2, \ldots, x_k\}$ is non-empty and finite. The performance measure function $E_P[h(x,\xi)]$ has no analytical form and must be evaluated via simulation. $h(x,\xi)$ is some random estimate of the performance of the system given a design x. ξ represents the random noise of the system and follows an unknown distribution P.

In practice, the functional forms and the associated parameters of distribution P are estimated from historical data, which leads to uncertainty of P. To model this input uncertainty, we follow Fan et al. (2013) and assume that for all the designs in $\mathscr{X}$, the set of possible distributions of ξ is identical and contains a finite number of elements, denoted as $\mathscr{P} = \{P_1, P_2, \ldots, P_m\}$. We call $\mathscr{P}$ the *uncertainty set* and an element in $\mathscr{P}$ a *scenario*. Note that $\mathscr{P}$ incorporates the uncertainty from both the input distribution and its associated parameters and $h(x,\xi)$ is random given any $P_j \in \mathscr{P}$. In order to obtain a finite number of scenarios, we can first identify a number of appropriate input distributions from historical data and then discretize the possible ranges of the associated parameters to establish $\mathscr{P}$.

To facilitate our presentation, we call x_i design i and P_j scenario j with $i \in \{1,2,\ldots,k\}$ and $j \in \{1,2,\ldots,m\}$. We define the following notation:

$X_{i,j,t}$: output of the tth simulation replication for scenario j of design i, $i \in \{1,2,\ldots,k\}$ and $j \in \{1,2,\ldots,m\}$;
$\mu_{i,j}$: mean of $X_{i,j,t}$, i.e., $\mu_{i,j} = E[X_{i,j,t}]$;
$\sigma^2_{i,j}$: variance of $X_{i,j,t}$, i.e., $\sigma^2_{i,j} = Var[X_{i,j,t}]$;
n: total number of simulation replications (budget);
$n_{i,j}$: number of simulation replications allocated to scenario j of design i;
$\alpha_{i,j}$: proportion of the total simulation budget allocated to scenario j of design i;
$\bar{X}_{i,j}$: sample mean of scenario j of design i, i.e., $\bar{X}_{i,j} = \frac{1}{n_{i,j}}\sum_{t=1}^{n_{i,j}} X_{i,j,t}$;
$S^2_{i,j}$: sample variance of scenario j of design i, i.e., $S^2_{i,j} = \frac{1}{n_{i,j}-1}\sum_{t=1}^{n_{i,j}}(X_{i,j,t}-\bar{X}_{i,j})^2$;
b: the true best design, $b \in \{1,2,\ldots,k\}$;
$\delta_{ij,uv} = \mu_{i,j} - \mu_{u,v}$.

In this research, the performance of design i is measured by its worst-case performance among the m possible scenarios in $\mathscr{P}$, i.e., $\max_{j\in\{1,2,\ldots,m\}} \mu_{i,j}$, for all $i \in \{1,2,\ldots,k\}$. This is a common setting to account for input uncertainty (e.g., Ghaoui, Oks, & Oustry, 2003; Gülpınar & Rustem, 2007). Since making decisions based on the worst-case scenario can prevent potential high risk, this setting is preferred by conservative decision makers. We assume that for each design $i \in \{1,2,\ldots,k\}$, there exists scenario j_i such that $\mu_{i,j_i} > \mu_{i,j}$ for all $j \in \{1,2,\ldots,m\}$ and $j \neq j_i$, and there exists design $i \in \{1,2,\ldots,k\}$ such that $\mu_{i,j_i} < \mu_{i',j_{i'}}$ for all $i' \in \{1,2,\ldots,k\}$ and $i' \neq i$. This assumption ensures that the worst-case scenario of each design and the true best design are uniquely defined. To make the derivation more tractable, we further assume that the simulation output samples for each scenario are normally distributed for all the designs and are independent from replication to replication, as well as independent across different designs and scenarios. That is, $X_{i,j,t} \sim N(\mu_{i,j}, \sigma^2_{i,j})$ and $\bar{X}_{i,j} \sim N(\mu_{i,j}, \frac{\sigma^2_{i,j}}{n_{i,j}})$.

2.2. Problem formulation

Given a fixed simulation budget, the best design cannot be selected with certainty, and a common way to deal with this issue is to allocate the simulation budget to maximize the probability of correct selection (PCS), i.e., the probability of correctly selecting the best design. With the performance of each design measured by their worst-case performance, a correct selection occurs when the observed worst-case scenario of the true best design b is better than the observed worst-case scenarios of the other designs. Then,

$$\begin{aligned} PCS &= P\left\{\max_{j\in\{1,\ldots,m\}} \bar{X}_{b,j} < \min_{l\in\{1,\ldots,k\}, l\neq b} \max_{r\in\{1,\ldots,m\}} \bar{X}_{l,r}\right\} \\ &= P\left\{\bigcap_{l=1,l\neq b}^{k} \bigcup_{r=1}^{m} \bigcap_{j=1}^{m} (\bar{X}_{b,j} \le \bar{X}_{l,r})\right\}. \end{aligned} \tag{2}$$

The selection problem is formulated as,

$$\begin{aligned} \max_{n_{i,j}} \quad & PCS \\ s.t. \quad & \sum_{i=1}^{k}\sum_{j=1}^{m} n_{i,j} = n, \\ & n_{i,j} \ge 0,\ i=1,\ldots,k,\ j=1,\ldots,m. \end{aligned} \tag{3}$$

In this research, we ignore the minor technicalities associated with $n_{i,j}$'s not being integer. A major difficulty for solving (3) is that the objective function PCS is computationally intractable using the expression given in (2). To evaluate PCS in a relatively fast and inexpensive way, we present an approximation for PCS using a lower bound.