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Brief paper

# Output-feedback protocols without controller interaction for consensus of homogeneous multi-agent systems: A unified robust control view ☆

Xianwei Li, Yeng Chai Soh, Lihua Xie

*School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798, Singapore*

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ABSTRACT

This paper investigates the consensus problem of homogeneous linear multi-agent systems using output feedback. An observer-type protocol is formulated, which only requires the relative output information of neighbours and does *not* require information exchange between controllers. It is shown that the protocol bridges some existing ones of the same nature. A robust control approach is presented for consensus protocol design, which requires one to solve a Riccati equation and a linear matrix inequality. Dual results are also discussed. By virtue of low- and high-gain techniques, it is shown that certain solvability conditions are to be satisfied to achieve consensus for agents that are not exponentially unstable or are minimum-phase, leading to a unified point of view for some existing results. A numerical example is provided to illustrate the advantages of the proposed design method.



## 1. Introduction

In recent years, coordination control of multi-agent systems undoubtedly has been one of the most active research themes in the systems and control community because of its wide variety of applications in formation control, flocking, oscillator synchronization, space rendezvous and so on (Kopeikin, Ponda, Johnson, & How, 2013; Olfati-Saber, Fax, & Murray, 2007). A fundamental task of controlling a network of multi-agent agents is to design a protocol such that all agents work cooperatively and finally reach an agreement, which is the consensus problem. A protocol determines how agents interact with each other and exchange information over the network. There have been a lot of studies on the design of various consensus protocols for multi-agent systems, see Li, Duan, Chen, and Huang (2010), Ma and Zhang (2010), Qin and Yu (2014), Ren and Beard (2008), Tang, Gao, and Kurths (2014), Tuna (2008) and You and Xie (2011). For static protocols with relative state feedback, Riccati equation based methods (Tuna, 2008; Zhang, Lewis, & Das, 2011) or linear matrix inequality based methods (Li et al., 2010) have been derived for protocol design.

It is known that state feedback sometimes is not practical because full state information is not always available, which motivates the development of output feedback based controller design methods. For this reason, considerable attention has been paid to the design of consensus protocols without agent state information but with agent output information instead, see Hengster-Movric and Lewis (2013), Hengster-Movric, Lewis, and Sebek (2015), Hong, Chen, and Bushnell (2008), Li et al. (2010), Scardovi and Sepulchre (2009), Trentelman, Takaba, and Monshizadeh (2013), Wieland, Sepulchre, and Allgöwer (2011), Zhang et al. (2011) and Zhou and Lin (2014). In particular, the distributed observer-type protocols presented in Li et al. (2010), Trentelman et al. (2013) and Zhang et al. (2011) still satisfy the fascinating separation principle such that the two gain matrices in the protocols can be designed separately.

It should be pointed out that most of the aforementioned observer-type protocols require extra information exchange between the controller/observer of each agent and that of its neighbours. Note that the information exchange we mention here would be completely different from the output information collection from neighbours: output information in fact indicates some measured physical quantities, while the information exchanged between controllers/observers is artificial. This feature may incur

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*E-mail addresses:* lixianwei1985@gmail.com (X. Li), eycsoh@ntu.edu.sg (Y.C. Soh), elixie@ntu.edu.sg (L. Xie).

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technical difficulties and/or costs to implement these observer-type protocols. Therefore, the study of protocols without any extra information exchange except the output information is of practical significance and has been one of the recent hotspots (Grip, Saberi, & Stoorvogel, 2015; Seo, Shim, & Back, 2009; Zhao, Wen, Duan, Xu, & Chen, 2013; Zhou & Lin, 2014). A consensus problem of this nature for homogeneous linear multi-agent systems was investigated in Seo et al. (2009), where the low-gain feedback control technique was shown to be useful for designing dynamic output feedback protocols without controller information exchange. Other low-gain-type approaches were also derived in Zhao et al. (2013) and Zhou and Lin (2014) but different techniques were employed. A common assumption for these low-gain approaches is that each agent is not exponentially unstable. On the other hand, the authors in Grip et al. (2015) proposed a high-gain method for designing dynamic output feedback protocols for homogeneous and heterogeneous multi-agent systems that satisfy some kinds of minimum-phase assumptions. Regarding these results, what draws our interest is the fact that the aforementioned methods actually discuss quite similar protocols, but are established in the low-gain and high-gain frameworks that appear to be sharply distinct from each other. Therefore we are naturally led to the following questions: Is it possible to formulate these protocols in a general framework? And how to design and interpret such protocols from a unified point of view? In this paper we attempt to provide an answer to these questions.

In this paper, we will investigate the output feedback based consensus problem of generic homogeneous linear multi-agent systems. Specifically, we focus on designing an observer-type dynamic protocol that only makes use of the relative output information of neighbouring nodes but does not involve any extra information exchange between controllers/observers. A unified robust control method will be presented for designing such dynamic protocols (Section 2.2). Furthermore, we will discuss the low-gain and high-gain aspects of the protocol design method (Sections 2.3 and 2.4), respectively, so as to uncover the conditions under which a required output feedback based protocol must exist and can be found by the presented design method. Dual results will also be addressed (Section 2.5). Preliminary results in this paper was presented in Li, Soh, and Xie (2016). The main contributions of the paper are stated as follows.

(1) A general dynamic observer-type protocol is composed, which involves an additional parameter to bridge the commonly used output feedback based protocols that deal with similar problems. Moreover, the meaning and benefit of this parameter are clarified in a robust control framework.

(2) An $H_\infty$ control approach is presented for protocol design. By virtue of the low-gain and high-gain techniques, it is shown that this approach must be feasible for agents that are either not exponentially unstable or minimum-phase. Since the presented approach can take advantage of both low-gain and high-gain techniques, it provides a unified point of view on how to design output feedback protocols without information exchange between controllers/observers. In addition, dual results are still discussed in the same framework.

*Notation:* $\mathbb{R}^{m\times n}$ and $\mathbb{C}^{m\times n}$ are the sets of all $m \times n$ real and complex matrices, respectively. $\bar{\mathbb{C}}^{+}$ and $\bar{\mathbb{C}}^{-}$ are the sets of complex numbers on the closed right and left half complex plane, respectively. $\mathbf{I}$ denotes an identity matrix with appropriate dimension. The notation $P > 0$ ($\geq 0$) means that matrix $P$ is positive definite (semi-definite). For a complex number, $\mathrm{Re}(\cdot)$ represents its real part. For two matrices $A$ and $B$, $A \otimes B$ is the Kronecker product. For a transfer function matrix $G(s) \in RH_\infty$ with $RH_\infty$ as the set of all stable rational transfer function matrices, $\|G\|_\infty$ denotes its $H_\infty$ norm.

Let $\mathscr{G}(\mathscr{V}, \mathscr{E})$ denote a directed graph consisting of a node set $\mathscr{V}$ and an edge set $\mathscr{E} \subseteq \mathscr{V}^2$. Suppose that $\mathscr{G}(\mathscr{V}, \mathscr{E})$ has $N$ nodes. We index the node set by $\mathscr{V} = \{1, 2, \ldots, N\}$. The ordered pair $(i, j) \in \mathscr{E}$ means that there is a link from node $i$ to node $j$, for which, node $i$ is said to be a neighbour of node $j$. Denote by $\mathscr{N}_i$ the neighbouring node set of node $i$. In the context of this paper, $(i, j) \in \mathscr{E}$ actually indicates that information can be transmitted from node $i$ to node $j$. In this paper, suppose that nodes are not self-connected, that is, $(i, i) \notin \mathscr{E}$ for $i = 1, 2, \ldots, N$. Define the adjacency matrix $A = [a_{ij}]_{N\times N}$ as $a_{ij} > 0$ for $(j, i) \in \mathscr{E}$ and $a_{ij} = 0$ otherwise. Further define the (normalized) Laplacian matrix $\mathscr{L} = \left[l_{ij}\right]_{N\times N}$ as $\mathscr{L}_{ii} = 1$ and $\mathscr{L}_{ij} = -a_{ij}/\left(\sum_{k\in\mathscr{N}_i} a_{ik}\right)$ for $i, j = 1, 2, \ldots, N$ and $i \neq j$. If the graph has a node from which every other node can be reached through any directed path, we say this graph has a directed spanning tree.

**Lemma 1** (*Fax & Murray, 2004*). *All eigenvalues of the Laplacian matrix $\mathscr{L}$ lie in a closed unit disk centred at the point $(1, 0)$ on the complex plane.*

## 2. Main results

### 2.1. Problem statement

Consider a distributed network of $N$ identical agents. The dynamics of each agent is given by a linear time-invariant system described by the following state-space equations:

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \qquad y_i(t) = Cx_i(t), \quad i = 1, 2, \ldots, N \tag{1}$$

where $x_i(t) \in \mathbb{R}^{n_x}$, $u_i(t) \in \mathbb{R}^{n_u}$, $y_i(t) \in \mathbb{R}^{n_y}$ are the local state, control input and measurement output, respectively, and $A, B, C$ are real, constant matrices with appropriate dimensions. The matrix pair $(A, B)$ is stabilizable and $(A, C)$ is detectable.

Let the communication topology that describes the information flow among agents be represented by a directed graph $\mathscr{G}(\mathscr{V}, \mathscr{E})$, and the associated adjacency matrix and Laplacian matrix by $[a_{ij}]_{N\times N}$ and $\mathscr{L} = [l_{ij}]_{N\times N}$, respectively. Suppose that each agent can collect the relative measurement output between itself and its neighbouring nodes. That is, the signal obtained by agent $i$ is given by

$$\tilde{y}_i(t) = \sum_{j\in\mathscr{N}_i} a_{ij}(y_i(t) - y_j(t)) = \sum_{j=1}^{N} l_{ij} y_j(t). \tag{2}$$

Throughout the paper, the following assumption is made for the communication graph $\mathscr{G}$.

**Assumption 1.** The graph $\mathscr{G}(\mathscr{V}, \mathscr{E})$ contains a directed spanning tree.

**Lemma 2** (*Ren & Beard, 2005*). *Zero is a simple eigenvalue of the Laplacian matrix $\mathscr{L}$ if and only if the associated graph has a spanning tree.*

To make use of $\tilde{y}_i(t)$ to attain consensus, in this paper, we are interested in a dynamic protocol of the following form:

$$\dot{\tilde{x}}_i(t) = (A + \theta BK + FC)\,\tilde{x}_i(t) - F\tilde{y}_i(t), \qquad u_i(t) = K\tilde{x}_i(t) \tag{3}$$

where $\tilde{x}_i(t) \in \mathbb{R}^{n_x}$, $\theta \in \mathbb{R}$, and $K, F$ are appropriately dimensioned real matrices. Scalar $\theta$ and matrices $K, F$ are protocol parameters to be determined.

**Remark 1.** Most of the existing results on observer-type output feedback protocols (e.g., Li et al., 2010; Trentelman et al., 2013; Zhang et al., 2011) require each observer to have access to the relative observer states of the neighbouring nodes. The protocol in (3) only requires the relative measurement $\tilde{y}_i(t)$ between a local node and its neighbours, which is simpler and easier to implement.