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Brief paper

Sliding mode control for semi-Markovian jump systems via output feedback☆

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ABSTRACT

This paper focuses on the output-feedback sliding mode controller design problem for uncertain continuous-time semi-Markovian jump systems (MJSs) in a descriptor system setup. The transition rates (TRs) of semi-MJSs rely on the random sojourn-time, which is different from the constant TRs in the conventional MJSs. By carefully exploiting the dynamical properties of the original system, combining with the switching functions, a descriptor system is firstly formulated to describe the holonomic dynamics of the sliding mode. Then, with the construction of a semi-Markovian Lyapunov function and the full utilization of the characteristics of cumulative distribution functions, a sufficient condition on the sliding surface synthesis is presented, which also guarantees the stochastic stability (SS) of sliding mode dynamical system. Furthermore, a sliding mode controller is synthesized to drive the underlying closed-loop system onto the sliding surface in finite time, locally for a given sliding region. Finally, an illustrative example is carried out to validate the effectiveness of the developed approach.

1. Introduction

Jump systems (JSs), as a class of hybrid stochastic systems, have attracted substantial attention from theoretical research to industrial applications (Feng, Loparo, Ji, & Chizeck, 1992; Mariton, 1990). The motivation behind this fact is owing to the powerful modeling ability to reliability of systems, evolution of populations, and medical treatments, etc. Basically, JSs can be described by a linear differential equation, whose parameters are randomly jumping (switching) in a finite set. Especially, the parameter-switching phenomenon is characterized by a stochastic process (SP) (Basin & Rodriguez-Ramirez, 2014; Lam, Shu, Xu, & Boukas, 2007), and the SP generally relies on the duration h between two consecutive transitions, which is also termed as *sojourn-time*. It is worth pointing out that the sojourn-time h is a random variable following the probability distribution F. In some circumstances, F is an exponential distribution, and the TRs $\lambda_{ij}(h) = \lambda_{ij}$ are hence constant as the memoryless property of the exponential distribution, which renders that the switchings are only related with the latest state. Such situations are ubiquitously witnessed in MJSs (He & Liu, 2010a,b; Li, Lam, Gao, & Xiong, 2016; Li & Wu, 2016; Xu, Lam, & Mao, 2007; Zhang, Zheng, & Xu, 2013).

In practice, however, it is difficult to guarantee the rigorous restriction on the memoryless characteristics of the sojourn-time distribution. More generally, the TRs $\lambda_{ij}(h)$ are sojourn-time-dependent. In this case, the underlying continuous SP with sojourn-time obeying *nonexponential distribution* is often addressed as a semi-Markov process. Correspondingly, a JS that evolves with a semi-Markov process is referred to a semi-MJS (Campo, Mookerjee, & Bar-Shalom, 1991; Hou, Luo, Shi, & Nguang, 2006; Huang, Shi, & Zhang, 2014; Shmerling & Hochberg, 2008). It can be readily inspected that the traditional MJSs are a particular case of semi-MJSs that can be employed to model and describe a broader class of practical stochastic systems. Additionally, it is known that most analysis and design conditions of MJSs are analytically tractable (Foucher, Mathieu, Saint-Pierre, Durand, & Daurès, 2005; Janssen & Manca, 2006; Ouhbi & Limnios, 2002; Stone, 1973). Nevertheless, the sojourn-time-dependent characteristics of TRs

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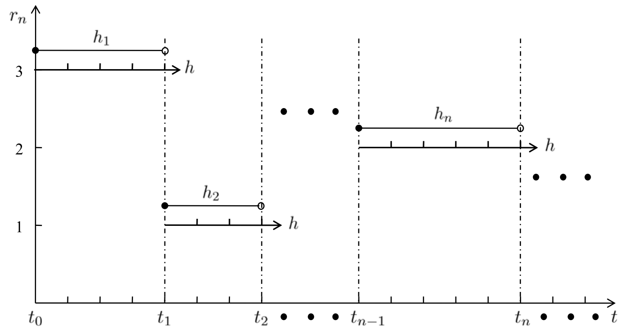

Fig. 1. One possible evolution of SPs r_n, t_n and h_n ($N = 3$).

bring much difficulty for the analysis and synthesis of semi-MJSs, which also results in little presence of numerically solvable synthesis criteria for semi-MJSs. It is thus of great significance, by wide application points of view, to further study the analysis and synthesis problems of semi-MJSs, which partially motivates this research.

On the other hand, sliding mode control (SMC), which is with intrinsically strong robustness and fast response, has been well investigated for the stabilization of uncertain systems and nonlinear systems (Zhang, Shi, & Lin, 2016; Zhang & Xia, 2010). The basic procedure of SMC is to firstly determine a switching function such that the sliding motion satisfies some design specifications. Then on the basis of Lyapunov stability theory, one synthesizes a controller to ensure the switching function attractive to the system states (Pisano, Tanelli, & Ferrara, 2016; Shi, Xia, Liu, & Rees, 2006). Notice that the designed control law is not necessarily continuous. Intuitively, in the SMC scheme, a variable structure control system is constructed to drive and then constrains the system states to locate within a neighborhood of the switching function. Obviously, with the particular selection of switching function, the dynamical behaviors of the underlying systems can be tailored. Meantime, the closed-loop system response becomes totally insensitive to a particular class of uncertainties. All these advantages inspire a substantial body of results on the SMC of stochastic systems and nonlinear systems (Chen, Niu, & Huang, 2011; Ma & Boukas, 2009; Shi et al., 2006; Wu, Shi, & Gao, 2010). Especially, the authors in Ma and Boukas (2009) studied the SMC of MJSs by a state-feedback formulation; the robust output-based sliding mode controller design procedure was presented in Chen et al. (2011) for the stabilization of Itô stochastic systems with Markovian jumping parameters; the observer-based SMC problem was addressed in Wu et al. (2010) for singular Markovian jump systems. Notice that for many practical applications, the state variables are not generally completely accessible. In this case, attempts to develop the output-based SMC strategy for semi-MJSs are more reasonable and promising. In particular, the numerically tractable synthesis conditions on output-feedback-based SMC problem for continuous-time semi-MJSs have not been well solved, which inspires us for this study.

With those motivations, in this article, we will tackle the output-feedback-based SMC problem for continuous-time semi-MJSs with parametric uncertainties. Specifically, by fully exploring the dynamical properties of the original system, combining with the switching functions, a descriptor system will be firstly constructed to describe the holonomic dynamics of the sliding mode. Then, based upon a semi-Markovian Lyapunov function, an SS analysis criterion will be derived for the sliding mode dynamical systems, where the switching surface will simultaneously be synthesized. Furthermore, an SMC law will be constructed to guarantee that the resulting closed-loop system converges towards the sliding surface in finite time, locally for a given sliding region. Simulation studies will be finally carried out to validate the effectiveness of the developed strategy.

Notations. The notations utilized are standard. $\mathbb{R}_+$ and $\mathbb{Z}_+$ refer to, respectively, the set of non-negative real numbers and the set of non-negative integers; $\mathtt{Sym}\{A\}$ refers to $A + A^\top$; $\mathcal{E}[\cdot]$ represents the mathematical expectation; $\|\cdot\|$ refers to the Euclidean norm of a vector or its induced norm of a matrix.

2. Model description

To introduce the semi-Markov process formally, we first give the following three stochastic processes (SPs).

(i) The SP $\{r_n\}_{n\in\mathbb{Z}_+}$ takes values in $\mathcal{I} := \{1, 2, \ldots, N\}$, where r_n refers to the index of system mode at the nth transition;
(ii) The SP $\{t_n\}_{n\in\mathbb{Z}_+}$ takes values in $\mathbb{R}_+$, where t_n represents the time at the nth transition. Notice that $t_0 = 0$ and t_n increases monotonically with n;
(iii) The SP $\{h_n\}_{n\in\mathbb{Z}_+}$ takes values in $\mathbb{R}_+$, where $h_n = t_n - t_{n-1}$, $\forall n \in \mathbb{Z}_{\geq 1}$ represents the sojourn-time of mode r_{n-1} between the $(n-1)$th transition and nth transition, and $h_0 = 0$.

To intuitively characterize those three SPs, one possible evolution is shown in Fig. 1.

Consider a stochastic switched system defined by

$$\Sigma : \dot{x}(t) = A(r(t))x(t), \quad t_n \leq t < t_{n+1}, \tag{1}$$

where $x(0) = x_0 \in \mathbf{R}^{n_x}$ is a constant vector; $A(r(t)) \in \mathbf{R}^{n_x\times n_x}$, $r(t) \in \mathcal{I}$, are real matrices; and suppose that the initial condition $t_0 = 0$ and $r(0)$ is a constant.

Definition 2.1 (*Ouhbi & Limnios, 2002; Stone, 1973*)**.** We say that the SP $r(t) := r_n$, $t \in [t_n, t_{n+1})$, is a homogeneous semi-Markov process, and Σ is a continuous-time homogeneous semi-MJS if the subsequent two conditions are true for every $i, j \in \{1, \ldots, N\}$, $t_0, t_1, \ldots, t_n \geq 0$.

(i) It holds that $\Pr(r_{n+1} = j, h_{n+1} \leq h|r_n, \ldots, r_0, t_n, \ldots, t_0) = \Pr(r_{n+1} = j, h_{n+1} \leq h|r_n)$.
(ii) The probability $\Pr(r_{n+1} = j, h_{n+1} \leq h|r_n = i)$ is independent of n.

Notice that conditions (i) and (ii) in particular show that the process $\{(r_n, t_n)\}_{n=0}^{\infty}$ is a time-homogeneous Markov renewal process, and therefore $\{r_n\}_{n=0}^{\infty}$ is a time-homogeneous Markov process. It is also straightforward from Definition 2.1 that the transition probabilities of homogeneous semi-Markov process $r(t) := r_n$, $t \in [t_n, t_{n+1})$, $n \in \mathbb{N}_{\geq 1}$, are merely dependent on the sojourn-time h_n instead of system-operation time t, and thus the TRs of homogeneous semi-Markov process are only characterized by sojourn-time h. In addition, the cumulative distribution function (CDF) of sojourn-time for the ith system mode, $i \in \mathcal{I}$, is specified as $G_i(h) = \Pr\{h_{n+1} \leq h|r_n = i\}$.

In this paper, we focus on the subsequent continuous-time semi-MJSs with parametric uncertainties,

$$\begin{cases} \dot{x}(t) = (A(r(t)) + \Delta A(t, r(t)))x(t) \\ \quad + B(r(t))(u(t) + f(t, x(t), r(t))) \\ y(t) = C(r(t))x(t) \end{cases} \tag{2}$$

where $x(t) \in \Re^{n_x}$ refers to the state vector; $u(t) \in \Re^{n_u}$ denotes the control input; $y(t) \in \Re^{n_y}$ refers to the measurement output, and $n_y \geq n_u$. $\{r(t), h\}_{t\geq 0} := \{r_n, h_n\}_{n\in\mathbb{N}_{\geq 1}}$ denotes a continuous-time and discrete-state homogeneous semi-Markov process with right continuous trajectories, which takes values in a finite set $\mathcal{I} := \{1, \ldots, N\}$ with TR matrix $\Lambda(h) := [\lambda_{ij}(h)]_{N\times N}$ characterized with Foucher et al. (2005) and Janssen and Manca (2006)

$$\begin{cases} \Pr\{r_{n+1} = j, h_{n+1} \leq h + \delta|r_n = i, h_{n+1} > h\} \\ \quad = \lambda_{ij}(h)\delta + o(\delta), \quad i \neq j \\ \Pr\{r_{n+1} = j, h_{n+1} > h + \delta|r_n = i, h_{n+1} > h\} \\ \quad = 1 + \lambda_{ii}(h)\delta + o(\delta), \quad i = j \end{cases} \tag{3}$$