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Brief paper

Supervisor localization of discrete-event systems under partial observation[☆]

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ABSTRACT

Recently we developed *supervisor localization*, a top-down approach to distributed control of discrete-event systems. Its essence is the allocation of monolithic (global) control action among the local control strategies of individual agents. In this paper, we extend supervisor localization by considering partial observation; namely not all events are observable. Specifically, we employ the recently proposed concept of *relative observability* to compute a partial-observation monolithic supervisor, and then design a suitable localization procedure to decompose the supervisor into a set of local controllers. In the resulting local controllers, only observable events can cause state change. We finally illustrate our result by a Transfer Line example.

1. Introduction

In Cai and Wonham (2010a,b, 2015, 2016) and Zhang, Cai, Gan, Wang, and Wonham (2013) we developed a top-down approach, called *supervisor localization*, to the distributed control of multi-agent discrete-event systems (DES). This approach first synthesizes a monolithic supervisor (or a heterarchical array of modular supervisors) assuming that *all* events can be observed, and then decomposes the supervisor into a set of local controllers for the component agents. Localization creates a purely distributed control architecture in which each agent is controlled by its own local controller; this is particularly suitable for applications consisting of many autonomous components, e.g. multi-robot systems. Moreover, localization can significantly improve the comprehensibility of control logic, because the resulting local controllers typically have many fewer states than their parent supervisor. The assumption of full event observation, however, may be too strong in practice, since there often lack enough sensors to observe every event.

In this paper and its conference precursor (Zhang & Cai, 2016a), we extend supervisor localization to address the issue of partial observation. Our approach is as follows. We first synthesize a partial-observation monolithic supervisor using the concept of *relative observability* in Cai, Zhang, and Wonham (2015, 2016). Relative observability is generally stronger than observability (Cieslak, Desclaux, Fawaz, & Varaiya, 1988; Lin & Wonham, 1988), weaker than normality (Cieslak et al., 1988; Lin & Wonham, 1988), and the supremal relatively observable (and controllable) sublanguage of a given language exists. The supremal sublanguage may be effectively computed (Cai et al., 2015), and then implemented by a partial-observation (feasible and nonblocking) supervisor (Wonham, 2016, Chapter 6). We then suitably extend the localization procedure in Cai and Wonham (2010a) to decompose the supervisor into local controllers for individual agents, and moreover prove that the derived local controlled behavior is equivalent to the monolithic one.

The main contributions of this work are as follows. First, we propose the combination of supervisor localization (Cai & Wonham, 2010a) with relative observability (Cai et al., 2015), which leads to a systematic, computationally effective approach to distributed control of DES under partial observation. In particular, local controllers with only observable state transitions are automatically synthesized, and the collective local controlled behavior is guaranteed to be the same as the global nonblocking behavior.

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Second, we identify the linguistic essence of partial-observation localization by developing the following key mappings and concepts (as extensions to their full-observation counterparts). The mappings include E_α, D_α, M and T (see definitions in Section 3.2) which capture the control and marking information of the partial-observation supervisor. Based on these mappings, the new concepts are introduced, including partial-observation control covers and local controllers. In particular, a partial-observation control cover is defined on the state set of the partial-observation supervisor; roughly speaking, the latter corresponds to the powerset of the full-observation supervisor's state set. Moreover, a partial-observation local controller contains only observable state transitions, and uses control functions to determine the existence of selfloops of unobservable controllable events.

Our proposed localization procedure can in principle be used to construct local controllers from a partial-observation supervisor computed by any synthesis method. In particular, the algorithms in Takai and Ushio (2003), and Yin and Lafortune (2016b) compute a nonblocking (maximally) observable sublanguage that is generally incomparable with the supremal relatively observable sublanguage. The reason that we adopt relative observability is first of all that its generator-based computation of the supremal sublanguage is better suited for applying our localization algorithm; by contrast (Yin & Lafortune, 2016b) uses a different transition structure called "bipartite transition system". Another important reason is that the computation of relative observability has been implemented and tested on a set of benchmark examples. This enables us to study distributed control under partial observation of more realistic systems; by contrast, the examples reported in Takai and Ushio (2003) and Yin and Lafortune (2016b) are limited to academic ones.

The paper is organized as follows. Section 2 reviews the supervisory control problem of DES under partial observation and formulates the partial-observation supervisor localization problem. Section 3 develops the partial-observation localization procedure, and Section 4 illustrates the procedure by a Transfer Line example. Finally Section 5 states our conclusions.

2. Preliminaries and problem formulation

2.1. Supervisory control of DES under partial observation

A DES plant is given by a generator

$$\mathbf{G} = (Q, \Sigma, \delta, q_0, Q_m) \tag{1}$$

where Q is the finite state set; $q_0 \in Q$ is the initial state; $Q_m \subseteq Q$ is the subset of marker states; Σ is the finite event set; $\delta : Q \times \Sigma \to Q$ is the (partial) state transition function. In the usual way, δ is extended to $\delta : Q \times \Sigma^* \to Q$, and we write $\delta(q, s)!$ to mean that $\delta(q, s)$ is defined. Let Σ^* be the set of all finite strings, including the empty string ϵ. The *closed behavior* of $\mathbf{G}$ is the language $L(\mathbf{G}) = \{s \in \Sigma^* | \delta(q_0, s)!\}$ and the *marked behavior* is $L_m(\mathbf{G}) = \{s \in L(\mathbf{G}) | \delta(q_0, s) \in Q_m\} \subseteq L(\mathbf{G})$. A string s_1 is a *prefix* of a string s, written $s_1 \le s$, if there exists s_2 such that $s_1 s_2 = s$. The *(prefix) closure* of $L_m(\mathbf{G})$ is $\overline{L_m(\mathbf{G})} := \{s_1 \in \Sigma^* | (\exists s \in L_m(\mathbf{G}))\, s_1 \le s\}$. In this paper, we assume that $\overline{L_m(\mathbf{G})} = L(\mathbf{G})$; namely, $\mathbf{G}$ is *nonblocking*.

For supervisory control, the event set Σ is partitioned into Σ_c, the subset of controllable events and Σ_{uc}, the subset of uncontrollable events (i.e. $\Sigma = \Sigma_c \dot\cup \Sigma_{uc}$). For partial observation, Σ is partitioned into Σ_o, the subset of observable events, and Σ_{uo}, the subset of unobservable events (i.e. $\Sigma = \Sigma_o \dot\cup \Sigma_{uo}$). Bring in the *natural projection* $P : \Sigma^* \to \Sigma_o^*$ defined by: (i) $P(\epsilon) = \epsilon$; (ii) $P(\sigma) = \sigma$ if $\sigma \in \Sigma_o$; (iii) $P(\sigma) = \epsilon$ if $\sigma \notin \Sigma_o$; (iv) $P(s\sigma) = P(s)P(\sigma)$, for all $s \in \Sigma^*$ and $\sigma \in \Sigma$. As usual, P is extended to $P : Pwr(\Sigma^*) \to Pwr(\Sigma_o^*)$, where $Pwr(\cdot)$ denotes powerset. Write $P^{-1} : Pwr(\Sigma_o^*) \to Pwr(\Sigma^*)$ for the *inverse-image function* of P.

A *supervisory control* for $\mathbf{G}$ is any map $V : L(\mathbf{G}) \to \Gamma$, where $\Gamma := \{\gamma \subseteq \Sigma | \gamma \supseteq \Sigma_{uc}\}$. Then the closed-loop system is $V/\mathbf{G}$, with closed behavior $L(V/\mathbf{G})$ and marked behavior $L_m(V/\mathbf{G})$ (Wonham, 2016). Under partial observation $P : \Sigma^* \to \Sigma_o^*$, we say that V is *feasible* if

$$(\forall s, s' \in L(\mathbf{G}))\quad P(s) = P(s') \Rightarrow V(s) = V(s'), \tag{2}$$

and V is *nonblocking* if $\overline{L_m(V/\mathbf{G})} = L(V/\mathbf{G})$.

It is well-known (Lin & Wonham, 1988) that under partial observation, a feasible and nonblocking supervisory control V exists which synthesizes a (nonempty) sublanguage $K \subseteq L_m(\mathbf{G})$ if and only if K is both controllable and observable (Wonham, 2016). When K is not observable, however, there generally does not exist the supremal observable (and controllable) sublanguage of K. Recently in Cai et al. (2015), a new concept of *relative observability* is proposed, which is stronger than observability but permits the existence of the supremal relatively observable sublanguage.

Formally, a sublanguage $K \subseteq L_m(\mathbf{G})$ is *controllable* (Wonham, 2016) if $\overline{K}\Sigma_{uc} \cap L(\mathbf{G}) \subseteq \overline{K}$. Let $C \subseteq L_m(\mathbf{G})$. A sublanguage $K \subseteq C$ is *relatively observable* with respect to C (or C-observable) if for every pair of strings $s, s' \in \Sigma^*$ that are lookalike under P, i.e. $P(s) = P(s')$, the following two conditions hold (Cai et al., 2015):

$$\text{(i)}\ (\forall \sigma \in \Sigma)\quad s\sigma \in \overline{K}, s' \in \overline{C}, s'\sigma \in L(\mathbf{G}) \Rightarrow s'\sigma \in \overline{K} \tag{3}$$

$$\text{(ii)}\ s \in K, s' \in \overline{C} \cap L_m(\mathbf{G}) \Rightarrow s' \in K. \tag{4}$$

For $E \subseteq L_m(\mathbf{G})$ write $\mathcal{CO}(E)$ for the family of controllable and C-observable sublanguages of E. Then $\mathcal{CO}(E)$ has a unique supremal element $\sup \mathcal{CO}(E)$ which may be effectively computed (Cai et al., 2015).

2.2. Formulation of partial-observation localization problem

Let the plant $\mathbf{G}$ be comprised of N (>1) component agents

$$\mathbf{G}_k = (Q_k, \Sigma_k, \delta_k, q_{0,k}, Q_{m,k}), \quad k = 1, \ldots, N.$$

Then $\mathbf{G}$ is the synchronous product (Wonham, 2016) of $\mathbf{G}_k$ (k in the integer range $\{1, \ldots, N\}$, denoted as $[1, N]$), i.e. $\mathbf{G} = \|_{k\in[1,N]} \mathbf{G}_k$. Here Σ_k need not be pairwise disjoint. These agents are implicitly coupled through a specification language $E \subseteq \Sigma^*$ that imposes a constraint on the global behavior of $\mathbf{G}$ (E may itself be the synchronous product of multiple component specifications). For the plant $\mathbf{G}$ and the imposed specification E, let the generator $\mathbf{SUP} = (X, \Sigma, \xi, x_0, X_m)$ be such that

$$L_m(\mathbf{SUP}) := \sup \mathcal{CO}(E \cap L_m(\mathbf{G})) \tag{5}$$

and $L(\mathbf{SUP}) = \overline{L_m(\mathbf{SUP})}$ (i.e. $\mathbf{SUP}$ is nonblocking). We call $\mathbf{SUP}$ the *controllable and observable behavior.*[1] To rule out the trivial case, we assume that $L_m(\mathbf{SUP}) \neq \emptyset$.

Now let $\alpha \in \Sigma_c$ be an arbitrary controllable event, which may or may not be observable. We say that a generator

$$\mathbf{LOC}_\alpha = (Y_\alpha, \Sigma_\alpha, \eta_\alpha, y_{0,\alpha}, Y_{m,\alpha}), \quad \Sigma_\alpha \subseteq \Sigma_o \cup \{\alpha\}$$

is a *partial-observation local controller* for α if (i) $\mathbf{LOC}_\alpha$ enables/disables the event α (and only α) consistently with $\mathbf{SUP}$, and (ii) if α is unobservable, then α-transitions are selfloops in $\mathbf{LOC}_\alpha$, i.e.

$$(\forall y \in Y_\alpha)\quad \eta_\alpha(y, \alpha)! \Rightarrow \eta_\alpha(y, \alpha) = y.$$

[1] Note that **SUP**, defined over the entire event set Σ, is *not* a representation of a partial-observation supervisor. The latter can only have observable events as state transitions, according to the definition in Section 3.1.