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Brief paper

Event-triggered intermittent sampling for nonlinear model predictive control ☆

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ABSTRACT

In this paper, we propose a new aperiodic formulation of model predictive control for nonlinear continuous-time systems. Unlike earlier approaches, we provide event-triggered conditions *without* using the optimal cost as a Lyapunov function candidate. Instead, we evaluate the time interval when the optimal state trajectory enters a local set around the origin. The obtained event-triggered strategy is more suitable for practical applications than the earlier approaches in two directions. First, it does not include parameters (e.g., Lipschitz constant parameters of stage and terminal costs) which may be a potential source of conservativeness for the event-triggered conditions. Second, the event-triggered conditions are necessary to be checked only at certain sampling time instants, instead of continuously. This leads to the alleviation of the sensing cost and becomes more suitable for practical implementations under a digital platform. The proposed event-triggered scheme is also validated through numerical simulations.

1. Introduction

Event-Triggered Control (ETC) and Self-Triggered Control (STC) have been active areas of research in the community of Networked Control Systems (NCSs), due to their potential advantages over the typical time-triggered controllers (Heemels, Johansson, & Tabuada, 2012). In contrast to the time-triggered case where the control signals are executed periodically, ETC and STC trigger the executions based on the violation of certain prescribed control performances, see e.g., Donkers and Heemels (2011) and Wang and Lemmon (2009).

In another line of research, Model Predictive Control (MPC) has been one of the most popular control strategies applied in a wide variety of applications. MPC plays an important role when several constraints, such as actuator or physical limitations, need to be explicitly taken into account. The basic idea of MPC is to obtain the current control action by solving the Optimal Control Problem (OCP) online, based on the knowledge of current state measurement and future behavior prediction through the dynamics.

The application of ETC and STC framework to MPC, generally known as Event-Triggered MPC (ETMPC) and Self-triggered MPC (STMPC), is of particular importance as it potentially alleviates a computational load by reducing the amount of solving OCPs. In ETMPC and STMPC, the OCPs are solved only when some events, generated based on certain control performance criteria, are triggered. These strategies have received an increased attention in recent years; most of the works focus on discrete-time systems, see e.g., Brunner, Gommans, Heemels, and Allgöwer (2015), Brunner, Heemels, and Allgöwer (2014, 2016), Eqtami, Dimarogonas, and Kyriakopoulos (2010), Gommans, Antunes, and Donkers (2014), Gommans and Heemels (2015), Hashimoto, Adachi, and Dimarogonas (2015b) and Henriksson, Quevedo, Peters, Sandberg, and Johansson (2015), and some results include for the continuous-time case, see e.g., Antunes and Heemels (2014), Hashimoto, Adachi, and Dimarogonas (2016) and Kobayashi and Hiraishi (2012) for linear systems and Eqtami, Dimarogonas, and Kyriakopoulos (2011), Eqtami, Heshmati-Alamdari, Dimarogonas, and Kyriakopoulos (2013), Hashimoto, Adachi, and Dimarogonas (2015a, 2017), Li and Shi (2014) and Varutti, Kern, Faulwasser, and Findeisen (2009) for nonlinear systems. In this paper, we are particularly interested in the case of nonlinear continuous-time systems. Among the afore-cited papers for nonlinear continuous-time systems, the results can be further divided into two

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categories, depending on whether disturbances are taken into account; see Varutti et al. (2009) for the disturbance-free case and Eqtami et al. (2011), Eqtami et al. (2013), Hashimoto et al. (2015a, 2017) and Li and Shi (2014) for the presence of disturbance case. In Varutti et al. (2009), an event-triggered MPC strategy has been proposed for nonlinear systems with no disturbances. While a delay compensation strategy has been developed to tackle uncertainties for networked control systems, an explicit form of the event-triggered condition is not provided and beyond the scope of that paper. In Eqtami et al. (2011), a self-triggered strategy is proposed for general nonlinear systems with additive disturbances. The self-triggered condition was derived based on the optimal cost regarded as an ISS Lyapunov function candidate. In Li and Shi (2014), an event-triggered strategy has been proposed for general nonlinear systems with additive bounded disturbances. When deriving the event-triggered strategy, an additional state constraint is imposed such that the optimal cost as a Lyapunov function candidate is decreasing. In Hashimoto et al. (2017), a self-triggered strategy was provided for nonlinear input affine systems based on the optimal cost as a Lyapunov function candidate. In the approach, an additional way to discretize an optimal control trajectory into several control samples was provided so that these can be transmitted to the plant over the network channels.

In this paper, we propose a new event-triggered formulation of MPC for nonlinear continuous-time systems with additive bounded disturbances. The main novelty of the proposed framework with respect to earlier results in this category (Eqtami et al., 2011, 2013; Hashimoto et al., 2015a, 2017; Li & Shi, 2014), is that the event-triggered condition is derived based on a new stability theorem, which does not evaluate the optimal cost as a Lyapunov function candidate. In the stability derivations, we instead evaluate the *time interval*, when the optimal state trajectory enters a local region around the origin. By guaranteeing that this time interval becomes smaller as the OCP is solved, it is ensured that the state enters a prescribed set in finite time.

The derivation of the new stability is motivated by the fact that the earlier event-triggered strategies may include Lipschitz constant parameters for the stage and terminal cost (see e.g., Eqtami et al., 2013; Hashimoto et al., 2017). When standard quadratic costs are utilized, the corresponding Lipschitz parameters are characterized by the maximum distance of the state from the origin (Eqtami et al., 2013), and the triggering condition becomes largely affected by the state domain considered in the problem formulation. That is, as a larger state domain is considered, the event-triggered condition may become more conservative. Depending on the problem formulation, therefore, it may not be desirable to include these parameters in the event-triggered condition. Since our approach does not evaluate the optimal cost as a Lyapunov function candidate, the corresponding event-triggered conditions do not include such unsuitable parameters even though quadratic cost functions are used. We will also illustrate through a simulation example that the proposed approach attains much less conservative result than our previous result presented in Hashimoto et al. (2017).

As another contribution of this paper with respect to the afore-cited papers of ETMPC for continuous-time systems (including the linear case), we will additionally incorporate Periodic Event-Triggered Control (PETC) framework (Heemels & Donkers, 2013). In PETC, triggering conditions are evaluated only at certain sampling time instants, instead of continuously. This approach has certain advantages over the existing ETMPC strategies, since it alleviates a sensing load to evaluate the event-triggered conditions and becomes more suitable to be implemented under digital platforms. In the general PETC framework, the sampling time to evaluate the event-triggered condition is constant for all update times (Heemels & Donkers, 2013). In our proposed approach, on the other hand, the sampling time is selected in an adaptive way; for each time of solving OCP, the controller adaptively determines the sampling time to check the event-triggered condition, such that the desired control performance can be guaranteed.

This paper is organized as follows. In Section 2, the optimal control problem is formulated. In Section 3, feasibility of the OCP is analyzed. In Section 4, our main proposed algorithm is presented, and the stability is shown in Section 5. A simulation example validates our proposed method in Section 6. We finally conclude in Section 7.

Notations. Let $\mathbb{R}, \mathbb{R}_{>0}, \mathbb{R}_{\geq 0}, \mathbb{N}_{\geq 0}, \mathbb{N}_{\geq 1}$ be the real, positive real, non-negative real, non-negative integers and positive integers, respectively. For a given matrix Q, we use $Q \succ 0$ to denote that the matrix Q is positive definite. The notation $\lambda_{\min}(Q)$ is used to denote the minimal eigenvalue of the matrix Q. We denote $\|x\|$ as the Euclidean norm of vector x, and $\|x\|_P$ as a weighted norm of vector x, i.e., $\|x\|_P = \sqrt{x^\mathsf{T} P x}$. Given a compact set $\Phi \subseteq \mathbb{R}^n$, we denote by $\partial\Phi$ the boundary of Φ. The function $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is called Lipschitz continuous in $\mathbb{R}^n$ with a weighted matrix P, if there exists $0 \leq L_f < \infty$ such that $\|f(x_1, u) - f(x_2, u)\|_P \leq L_f \|x_1 - x_2\|_P$, $\forall x_1, x_2 \in \mathbb{R}^n$, $\forall u \in \mathbb{R}^m$.

2. Problem formulation

2.1. Dynamics and optimal control problem

In this section the problem formulation is defined. We consider to apply MPC to the following nonlinear systems with additive disturbances:

$$\dot{x}(t) = f(x(t), u(t)) + w(t), \quad t \geq t_0, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control input, $w(t) \in \mathbb{R}^n$ is an additive bounded disturbance, and $t_0 \in \mathbb{R}$ denotes the initial time. The control input u and the disturbance w are assumed to satisfy the following constraints:

$$u(t) \in \mathcal{U} \subseteq \mathbb{R}^m, \qquad w(t) \in \mathcal{W} \subseteq \mathbb{R}^n, \quad \forall t \geq t_0. \tag{2}$$

Regarding the constraint (2) and the plant model (1), we make the following standard assumptions (Chen & Allgöwer, 1998):

Assumption 1. (i) The constraint sets $\mathcal{U}$ and $\mathcal{W}$ are compact, convex and $0 \in \mathcal{U}$; (ii) the function $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is twice continuously differentiable, and $f(0, 0) = 0$; (iii) the system (1) has a unique, absolutely continuous solution for any initial state $x(t_0)$ and any piecewise continuous control and disturbance $u : [t_0, \infty) \to \mathcal{U}$, $w : [t_0, \infty) \to \mathcal{W}$; (iv) for the linearized system around the origin with no disturbances, i.e., $\dot{x}(t) = A_f x(t) + B_f u(t)$, where $A_f = \partial f/\partial x(0, 0)$ and $B_f = \partial f/\partial u(0, 0)$, the pair (A_f, B_f) is stabilizable.

Let t_k, $k \in \mathbb{N}_{\geq 0}$ be the update time instants when OCPs are solved, and let $\Delta_k = t_{k+1} - t_k$ be the inter-event times. At t_k, the controller solves an OCP based on the state measurement $x(t_k)$ and the predictive behavior of the systems described by (1). In this paper, we consider the following cost to be minimized:

$$J(x(t_k), u(\cdot)) = \int_{t_k}^{t_k+T_k} \|\hat{x}(\xi)\|_Q^2 + \|u(\xi)\|_R^2 \mathrm{d}\xi, \tag{3}$$

where $Q = Q^\mathsf{T} \succ 0$, $R = R^\mathsf{T} \succ 0$ and $T_k > 0$ is the prediction horizon. $\hat{x}(\xi)$ denotes the nominal trajectory of (1) given by $\dot{\hat{x}}(\xi) = f(\hat{x}(\xi), u(\xi))$ for all $\xi \in [t_k, t_k + T_k]$ with $\hat{x}(t_k) = x(t_k)$. Here, the prediction horizon T_k is not constant but is adaptively selected such that it is strictly decreasing. More characterization of T_k is provided in this section when formulating the OCP.

The following lemma states that there exists a stabilizing, state feedback controller in a prescribed local set around the origin: