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Brief paper

An observer-based control scheme using negative-imaginary theory ☆

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ABSTRACT

This paper presents a full-order state observer-based scheme that transforms a causal, LTI, minimally realized system into a *strongly strict negative-imaginary* system by defining an auxiliary output based on the observed states. The auxiliary output is used for closed-loop control with positive feedback invoking the internal stability condition of negative-imaginary theory. A set of LMI conditions is derived that determines the value of a design parameter μ required for the proposed scheme to transform a given system with a minimal state-space realization into a strongly strict negative-imaginary system. The proposed scheme is applicable for both stable/unstable and square/non-square systems. In this paper, the framework is further explored for robustness analysis of uncertain systems and numerical examples are given to elucidate effectiveness of the proposed results.

1. Introduction

The negative-imaginary systems theory has rapidly attracted the interest of the control community due to its simple internal stability condition for interconnected systems that depends only on the DC loop gain, and its wide applicability in different areas of control systems engineering. For example, such systems arise while considering the transfer functions from input voltage to output voltage in active electrical filters (Patra & Lanzon, 2011), from input voltage to shaft rotational velocity in DC servo motor (Song, Lanzon, Patra, & Petersen, 2012a), from pump input voltage to water level in a linearized coupled tank system. Other applications include nano-positioning systems, flexible spacecrafts, robotic manipulator arms (Mabrok, Kallapur, Petersen, & Lanzon, 2014), etc. The notion of negative-imaginary (NI) and strictly negative-imaginary (SNI) systems has been introduced in Lanzon and Petersen (2008) for robust control of highly resonant flexible structures with colocated position sensors and force actuators (Bhikkaji, Moheimani, & Petersen, 2012; Mabrok et al., 2014). An NI system is a Lyapunov-stable system with equal number of inputs and outputs, having a real-rational, proper transfer function matrix $R(s)$ that satisfies the frequency domain condition $j[R(j\omega) - R^{\star}(j\omega)] \geq 0 \ \forall \omega \in (0, \infty)$. In a SISO setting, a real-rational, proper transfer function is said to have the NI (respectively, SNI) properties if its Nyquist plot lies below (respectively, strictly below) the real axis in the open positive frequency interval (Lanzon & Petersen, 2008; Petersen & Lanzon, 2010). So far in the NI literature, the state-space characterizations of LTI NI and SNI systems have been studied in Lanzon and Petersen (2008), Petersen and Lanzon (2010) and Xiong, Petersen, and Lanzon (2010); NI systems with single and double poles at the origin in Ferrante, Lanzon, and Ntogramatzidis (2016), Ferrante and Ntogramatzidis (2013) and Mabrok et al. (2014); strongly strict negative-imaginary (SSNI) systems in Ferrante et al. (2016) and Lanzon, Song, Patra, and Petersen (2011); lossless NI system properties in Xiong, Petersen, and Lanzon (2012); controller synthesis, robustness and performance analysis in Bhikkaji et al. (2012), Engelken, Patra, Lanzon, and Petersen (2010), Mabrok, Kallapur, Petersen, and Lanzon (2015) and Song, Lanzon, Patra, and Petersen (2010); Song et al. (2012a); Song, Lanzon, Patra, and Petersen (2012b). In recent times, the NI theory has been extended for symmetric and non-rational transfer functions (Ferrante et al., 2016; Ferrante & Ntogramatzidis, 2013) and also, the cooperative control strategy is being applied to control multiple NI agents connected through a communication network (Wang, Lanzon, & Petersen, 2015).

The NI stability result for interconnected systems has drawn profound interest from control theoretic perspective since the stability criteria depends only on the loop gains at zero and infinite frequencies. However, the theory remains inapplicable if the interconnected systems are non-NI which occurs quite often in control theoretic applications. Therefore, if a scheme can be developed which transforms any LTI non-NI system into NI

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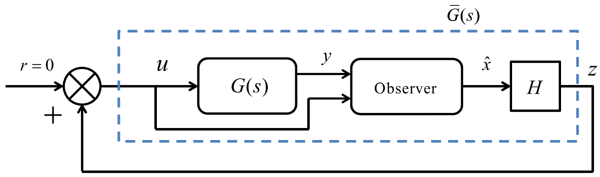

Fig. 1. An observer-based control scheme for stable LTI plants with positive feedback using NI theory.

class then the existing NI theory can be applied to synthesize a stabilizing controller depending on the loop gains at zero and infinite frequencies. So far in the NI literature, to the best of the authors' knowledge, no such procedure is reported by which a non-NI system can be transformed into an NI system. This motivates us to propose a scheme by which an LTI system can be transformed into a negative-imaginary system facilitating the applicability of the NI theory for analysis and controller synthesis, especially widening the applicability of this theory in the presence of unstable and/or non-square plants.

In this paper, we introduce a scheme, as shown in Fig. 1, which transforms a causal, LTI, minimally realized system into a strongly strict negative-imaginary (SSNI) system (Lanzon et al., 2011), a subset of the strictly negative-imaginary (SNI) class (Xiong et al., 2010), by defining an auxiliary output based on the observed states. The proposed observer-based SSNI scheme can also be used to stabilize a system $G(s)$ in closed-loop when the auxiliary output z is positively fed back (see Fig. 1) along with satisfying a particular DC loop gain condition on the transformed system $\bar{G}(s)$. This scheme does not require the original system to be stable or square. In this scheme, a full-order state observer is placed in cascade with the plant and a new output z is defined that depends only on the estimated states $\hat{x}$. For a particular structure of the transformation matrix H, the combined system from input u to the defined output $z := H\hat{x}$ exhibits SSNI properties. A set of LMI-based conditions, involving a design parameter $\mu > 0$, is formulated that yields a feasible finite positive μ value required to design an observer-based control scheme for a given plant. In this work, we have exploited the fact that the SSNI system properties can be defined for uncontrollable systems while maintaining the observability and the full normal rank conditions (Patra & Lanzon, 2011).

So far, various observer-based control schemes have been introduced in the literature. But none of these techniques is dedicated for the NI framework. To mention a few: An observer-based state feedback control scheme is designed in Arcak and Kokotovic (2001) to ensure global asymptotic stability of linear systems with slope-restricted nonlinearities in feedback; Lyapunov stability theory and LMI-based approach have been adopted in Lien (2004) to design observer-based control for a class of linear systems with state perturbations where the controller and observer gains can be obtained from the LMI formulations; A state observer-based robust trajectory-tracking control scheme is proposed in Oya, Su, and Kobayashi (2004) for electrically driven robot manipulators where the observer is used to estimate the joint angular velocities; An observer-based robust control technique is designed in Li and Yang (2012) that also facilitates the acquisition of information used for fault detection in feedback control systems; Further in Collado, Lozano, and Johansson (2007), Johansson and Robertsson (2002), the authors have proposed an observer-based transformation scheme in strictly positive-real framework. In this paper, the proposed observer-based transformation and the associated feedback control scheme broaden the scope of applicability of the NI theory for robust control analysis and synthesis.

The remainder of the paper proceeds as follows: in Section 2, useful notations are given. Section 3 illustrates a few necessary definitions, lemmas and theorems which streamline the main results of this paper. Section 4 deals with the observer-based scheme that transforms a causal, LTI system into an SSNI system. Section 5 formulates a set of LMI-based conditions by which the proposed observer-based feedback control scheme guarantees internal stability of LTI systems with positive feedback. In Section 6, robust stability analysis of LTI uncertain systems using the proposed observer-based control strategy is explained through an example. Section 7 concludes the paper.

2. Notations

The notations and acronyms are standard throughout. The fields of real and complex numbers are denoted by $\mathbb{R}$ and $\mathbb{C}$, respectively. The sets of real and complex matrices of dimension $(n \times n)$ are denoted by $\mathbb{R}^{n\times n}$ and $\mathbb{C}^{n\times n}$, respectively. The maximum and the minimum eigenvalues of a matrix $A \in \mathbb{C}^{n\times n}$ that has only real eigenvalues are indicated by $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$, respectively. The determinant and the rank of a matrix A are denoted by $\det[A]$ and $\mathrm{rank}[A]$, respectively. The real and imaginary parts are represented by $\Re(\cdot)$ and $\Im(\cdot)$, respectively. A^T, $A^\star$ and $\bar{A}$ denote the transpose, the complex-conjugate transpose and the complex-conjugate of a matrix A. The shorthands for $\left(A^{-1}\right)^\star$ and $\left(A^{-1}\right)^T$ are represented by $A^{-\star}$ and A^{-T}, respectively. The terminology that $R(s)$ is a 'proper' transfer function matrix includes the cases $R(\infty) \equiv 0$, $\det[R(\infty)] = 0$ and $\det[R(\infty)] \neq 0$. The set of all proper, real-rational transfer function matrices is denoted by $\mathscr{R}^{n\times n}$ whereas the set of all proper, real-rational and asymptotically stable transfer function matrices is denoted by $\mathscr{RH}_\infty^{n\times n}$, both of dimensions $(n \times n)$. For a transfer function matrix $R(s)$, $R^\star(j\omega) = R^T(-j\omega)$ and $R^\sim(s) = R^T(\bar{s})$ where $\bar{s}$ denotes the complex-conjugate of s. The minimal state-space realization of a system $R(s)$ is represented by $R(s) \underset{\min}{=} \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$.

3. Preliminaries

In this section, some preliminary results are given which provide a background to establish the main results of this paper. We now describe the NI, SNI and SSNI systems through the following definitions and the lemma. For complete details of the NI theory and its applications literatures (Bhikkaji et al., 2012; Bhowmick & Patra, 2016, 2017; Dey, Patra, & Sen, 2016; Lanzon & Petersen, 2008; Lanzon et al., 2011; Mabrok et al., 2014, 2015; Petersen & Lanzon, 2010; Xiong et al., 2010) may be referred.

Definition 1 (***NI System*** *(Mabrok et al., 2014, 2015)*)**.** A square, causal, real-rational, proper transfer function matrix $R(s)$ is negative-imaginary (NI) if the following conditions are satisfied:

(1) $R(s)$ has no poles in $\Re[s] > 0$;
(2) $j[R(j\omega) - R^\star(j\omega)] \geq 0$ $\forall \omega \in (0, \infty)$ except the values of ω where $j\omega$ is a pole of $R(s)$;
(3) If $j\omega_0$, $\omega_0 \in (0, \infty)$, is a pole of $R(s)$, it is at most a simple pole and the residue matrix $K_0 := \lim_{s\to j\omega_0}(s - j\omega_0)jR(s)$ is positive semidefinite Hermitian;
(4) If $s = 0$ is a pole of $R(s)$ then, $\lim_{s\to 0} s^k R(s) = 0$ for all $k \geq 3$ and $\lim_{s\to 0} s^2 R(s)$ is positive semidefinite Hermitian.

According to Definition 1, the NI class includes systems having complex pole-pair on the $j\omega$ axis or pole(s) at the origin, however, in this paper, we have considered only asymptotically stable NI systems.

Definition 2 (***SNI System*** *(Petersen & Lanzon, 2010; Xiong et al., 2010)*)**.** A square, causal, real-rational, proper transfer function matrix $R(s)$ is strictly negative-imaginary (SNI) if the following conditions are satisfied: