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Brief paper

A nonlinear extended state observer based on fractional power functions☆

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ABSTRACT

In this paper, we investigate a nonlinear extended state observer (ESO) constructed from piece-wise smooth functions consisted of linear and fractional power functions. This structure of ESO was first proposed in the 1990's and has been widely used in active disturbance rejection control for engineering controls. Its convergence, however, has remained an open problem up to this day. The main objective of this paper is to provide a convergence theory with explicit error estimation. The performances of this type ESO are studied by numerical simulation and compared with linear ESO. The numerical results show that the ESO proposed in this paper enjoys the advantages of smaller peaking value and better measurement noise tolerance.

1. Introduction

Due to its ability to deal with vast internal and external uncertainty, the active disturbance rejection control (ADRC) (Han, 2009) is becoming an emerging technology in control engineering. The last two decades have witnessed ADRC's success in many industrial applications including DC–DC power converter (Sun & Gao, 2005), flight vehicles control (Xia & Fu, 2013), Gasoline Engines (Xue et al., 2015), hydraulic systems control (Yao, Jiao, & Ma, 2014). The ADRC's characteristics of energy saving has also been demonstrated. For example, a 30% improvement in product performance capability index (Cpk) and 50% reduction in energy consumption were concluded in the test conducted in Parker Hannifin Parflex hose extrusion plant for over a period of eight months (Zheng & Gao, 2012).

The extended state observer (ESO) is central to ADRC. Note that the effect of the so-called "total disturbance" of system, which may contain internal uncertainty, external disturbance, and anything that is hard to model or deal with, can be exhibited in the observable measured output. Through a properly designed ESO, the "total disturbance" can be estimated. Then, it can be naturally canceled in the feedback loop.

A first ESO was proposed by J.Q. Han in late 1980's (Han, 2009) where there are multiple tuning parameters to be tuned to estimate system state and total disturbance. For easy use, Gao (2003) proposed a one-parameter tuning linear ESO in terms of bandwidth, where the high-gain approach is incorporated. The convergence of linear ESO, also known as extended high-gain observer in other context (Praly & Jiang, 2004; Freidovich & Khalil, 2008), is discussed in Zheng, Gao, and Gao (2007). Other types of linear ESO are subsequently proposed for various systems such as control and disturbance unmatched systems (Li, Yang, Chen, & Chen, 2012), and the system without a prior knowledge of nominal control parameter (Jiang, Huang, & Guo, 2015). Very recently, a linear ESO with adaptive gain is investigated in Xue et al. (2015).

In addition to these ESO aforementioned, the nonlinear function commonly used in ESO in practice is of the following form:

$$\mathrm{fal}(\tau,\alpha,\delta)=\begin{cases}\dfrac{\tau}{\delta^{1-\alpha}}, & |\tau|\le\delta,\\ |\tau|^{\alpha}\mathrm{sign}(\tau), & |\tau|>\delta,\end{cases} \quad (1.1)$$

where $0<\alpha<1$ and $\delta>0$ are constants. Based on numerous computer simulations and engineering practices, Han (2009) claimed that the ESO with nonlinear function of type (1.1) is quite effective for state and "total disturbance" estimation, leading to good performance including small peaking value. For nonlinear

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ESO on the other hand, although some progresses have been made in recent papers (Guo & Zhao, 2011; Zhao & Guo, 2015), none of them considers the nonlinear function (1.1). A fundamental theoretical question for this type of ESO is how to choose α and δ so that the convergence is guaranteed. Despite its importance, little is done toward answering this question since (1.1) was proposed. In this paper, we aim at providing an answer to this question by investigating convergence of ESO constructed from nonlinear function (1.1).

For the sake of exposition, we suppose in this paper that $\delta = 1$ since other cases can be similarly dealt with. Consider the following lower triangle nonlinear system with vast uncertainty:

$$\begin{cases} \dot{x}_1(t) = x_2(t) + \phi_1(t, u(t), x_1(t)), \\ \qquad\vdots \\ \dot{x}_{n-1}(t) = x_n(t) + \phi_{n-1}(t, u(t), x_1(t), \dots, x_{n-1}(t)), \\ \dot{x}_n(t) = f(t, x(t), w(t)) + \phi_n(t, u(t), x(t)), \\ y(t) = x_1(t), \end{cases} \tag{1.2}$$

where $x(t) = (x_1(t), \dots, x_n(t)) \in \mathbb{R}^n$ is the state, $\phi_i \in C(\mathbb{R}^{i+2}, \mathbb{R})$ are known system functions, $f \in C(\mathbb{R}^{n+2}, \mathbb{R})$ is an unknown system function, $y(t)$ is the measured output, $u(t)$ is the control input, $w(t)$ is the external disturbance. The "total disturbance" or "extended state" is denoted by

$$x_{n+1}(t) \triangleq f(t, x(t), w(t)). \tag{1.3}$$

We propose the following ESO for system (1.2):

$$\begin{cases} \dot{\hat{x}}_1(t;r) = \hat{x}_2(t;r) + \dfrac{k_1}{r^{n-1}} \mathscr{G}_1(r^n(x_1(t) - \hat{x}_1(t;r))) \\ \qquad + \phi_1(t, u(t), x_1(t)), \\ \qquad\vdots \\ \dot{\hat{x}}_n(t;r) = \hat{x}_{n+1}(t;r) + k_n \mathscr{G}_n(r^n(x_1(t) - \hat{x}_1(t;r))) \\ \qquad + \phi_n(t, u(t), x_1(t), \hat{x}_2(t;r), \dots, \hat{x}_n(t;r)), \\ \dot{\hat{x}}_{n+1}(t;r) = r k_{n+1} \mathscr{G}_{n+1}(r^n(x_1(t) - \hat{x}_1(t;r))), \end{cases} \tag{1.4}$$

where r is a constant gain, k_i's are constants to be chosen so that the following matrix is Hurwitz:

$$K = \begin{pmatrix} -k_1 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -k_n & 0 & 0 & \cdots & 1 \\ -k_{n+1} & 0 & 0 & \cdots & 0 \end{pmatrix}_{(n+1)\times(n+1)}, \tag{1.5}$$

and $\{\mathscr{G}_i\}_{i=1}^n$ is of the form:

$$\mathscr{G}_i(\tau) = \mathrm{fal}(\tau, \theta_i, 1) \tag{1.6}$$

with $\theta_i \in (0, 1)$, $i = 1, 2, \dots, n+1$, being positive constants to be specified later.

The remaining part of the paper is organized as follows. In Section 2, we present convergence result of the ESO with fractional power function $\mathscr{G}_i(\cdot)$'s defined in (1.6). State observer reduced from ESO is also introduced. Since the proof of the main result is lengthy and needs some mathematical techniques, it is carried out separately in Section 3. The numerical simulations are presented in Section 4 to demonstrate the convergence as well as other properties including peaking value reduction and measurement noise tolerance.

2. Main results

In this section, we present the convergence of ESO (1.4) based on fractional power function (1.6). To this purpose, we make some basic assumptions on the plant.

Assumption A1. All the functions including the disturbance $w(t)$ and its derivative $\dot{w}(t)$, and the solution of (1.2) are supposed to be uniformly bounded. The unknown function is supposed to be $f \in C^1(\mathbb{R}^{n+2}, \mathbb{R})$ and there exists continuous function $\tilde{f} : \mathbb{R}^{n+1} \to \mathbb{R}$ such that $|f(t, \xi)| + \left|\frac{\partial f(t,\xi)}{\partial t}\right| \le \tilde{f}(\xi)$, $\forall\, t \in [0, \infty)$, $\xi \in \mathbb{R}^{n+1}$. For the known functions $\phi_i \in C(\mathbb{R}^{i+2}, \mathbb{R})$, there exist continuous bounded function $\mathscr{L} \in C(\mathbb{R}^2, \mathbb{R})$ and continuous functions $\tilde{\phi}_i \in \mathbb{C}(\mathbb{R}^i, \mathbb{R})$ such that

$$\begin{cases} |\phi_i(t, u, \nu_1, \nu_2, \dots, \nu_i) - \phi_i(t, u, \nu_1, \tilde{\nu}_2, \dots, \tilde{\nu}_i)| \\ \quad \le \mathscr{L}(t, u) \|(\nu_2 - \tilde{\nu}_2, \dots, \nu_i - \tilde{\nu}_i)\|^{\alpha_i}, \\ |\phi_i(t, u, \nu_1, \dots, \nu_i)| \le \tilde{\phi}_i(\nu_1, \dots, \nu_i), \\ \alpha_i \in (0, 1],\ \nu_i, \tilde{\nu}_i \in \mathbb{R},\ i = 1, 2, \dots, m. \end{cases} \tag{2.1}$$

Remark 2.1. It is important to stress that we focus only on convergence of ESO for open loop system. The boundedness of state is used for estimation of state-dependent total disturbance. If the "total disturbance" is state-independent or only the state is estimated, the boundedness of the state can be removed, see Theorem 2.1 and Corollary 2.1 later. In addition, the state is bounded in most practical control systems such as those for faults diagnosis (Yan, Tian, Shi, & Wang, 2008). Finally, since the ESO is designed for control purpose, in case that the system is not bounded, we can also use feedback to make the system bounded, which will be investigated in the forthcoming paper.

Let

$$\alpha = \max_{1\le i\le n} (n+1-i)(1-\alpha_i), \qquad \alpha^* = \min_{1\le i\le n} \alpha_i. \tag{2.2}$$

The main result is stated as Theorem 2.1.

Theorem 2.1. *Suppose that in system (1.2), $\alpha_i \in (0, 1]$, $\alpha < 1$, and Assumption A1 holds. Let $\theta_i = i\theta - (i-1)$, $i = 1, 2, \dots, n+1$ in ESO (1.4). Then there exist $\theta^* \in (n/(n+1), 1)$ and $r^* > 0$ such that for any $\theta \in [\theta^*, 1)$, $r > r^*$, and any initial state $(x_{10}, x_{20}, \dots, x_{n0})$ of system (1.2) and initial state $(\hat{x}_{10}, \hat{x}_{20}, \dots, \hat{x}_{n0}, \hat{x}_{(n+1)0})$ of ESO (1.4), the observer errors satisfy, for any $t > t_r$, $i = 1, 2, \dots, n+1$, that*

$$|x_i(t) - \hat{x}_i(t;r)| \le \Gamma (1/r)^{n+1-i+\frac{1}{(1-\alpha)(2-\alpha^*)}}, \tag{2.3}$$

where $t_r > 0$ is r-dependent and satisfies $\lim_{r\to\infty} t_r = 0$, $x_{n+1}(t)$ is the total disturbance defined in (1.3), and Γ is an r-independent constant defined in (3.56).

Moreover, if the "total disturbance" (1.3) is independent of the state, that is, $f(\cdot) = w(t)$, then (2.3) holds without assuming the boundedness of the system state.

We first point out two features of ESO (1.4) where $\mathscr{G}_i(\tau)'s$ play the role of somehow saturation-like behaviors. The other two merits of peaking value reduction and noise tolerance will be discussed at the end of Section 4.

It is seen from (2.3) that the error between the state of ESO (1.4) and state of system (1.2) including total disturbance can be made as small as desired by tuning gain parameter r to be large enough. In fact, (2.3) together with $\lim_{r\to\infty} t_r = 0$ implies that for any $T > 0$, $i = 1, 2, \dots, n+1$,

$$\lim_{r\to\infty} \sup_{t\in[T,\infty)} |x_i(t) - \hat{x}_i(t;r)| = 0. \tag{2.4}$$

Generally speaking, to make state of ESO approximate state and total disturbance to an acceptable small error, the gain parameter r should be tuned according to variation speed of the total disturbance: The smaller the variation speed of the total disturbance, the smaller tuning parameter r. If the total disturbance is not varying with time (a constant: $f(\cdot) = \bar{d} \in \mathbb{R}$), then a small tuning gain can guarantee asymptotic convergence.