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Brief paper

Optimal role and position assignment in multi-robot freely reachable formations☆

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ABSTRACT

Many multi-robot problems require the achievement of formations as part of the overall mission. This work considers a scenario in which unlabeled homogeneous robots must adopt a given formation pattern buildable anywhere in the environment. This involves finding the relative pose of the formation in regard to the initial robot positions, understood as a translation and a rotation; and the optimal assignment of the role of each robot within the formation. This paper provides an optimal solution for the combined parameters of translation, rotation and assignment that minimizes total displacement. To achieve this objective we first formally prove that the three decision variables are separable. Since computing the optimal assignment without accounting for the rotation is a computationally expensive problem, we propose an algorithm that efficiently computes the optimal roles together with the rotation. The algorithm is provably correct and finds the optimal solution in finite time. A distributed implementation is also discussed. Simulation results characterize the complexity of our solution and demonstrate its effectiveness.

1. Introduction

The problem of a team of robots reaching and maintaining a specific formation finds application in many multi-robot tasks such as cooperative manipulation (Alonso-Mora, Knepper, Siegwart, & Rus, 2015; Michael, Fink, & Kumar, 2011), or environmental surveillance (Schwager, Julian, Angermann , & Rus, 2011). Typically, this problem is addressed by defining pairwise desired values between the robots, and then designing control laws that drive them towards these values (Oh, Park, & Ahn, 2015). In some cases the robots use relative positions (Cortés, 2009), bearing angles (Franchi & Giordano, 2012) or inter-robot distances (Oh & Ahn, 2011). Other approaches include obstacle avoidance (Ayanian & Kumar, 2010) and vision sensors (Das, Fierro, Kumar, Ostrowski, Spletzer, & Taylor, 2002; Montijano, Cristofalo, Zhou, Schwager, & Sagues, 2016). A common assumption in all these approaches is a predefined assignment of the role of each robot in the formation, i.e., each robot has a known "label" within the formation, which defines the pairwise desired values with its neighbors based on it. Depending on the initial conditions, this can result in large displacements of all the robots or, in some cases, even convergence to undesired equilibria, as observed in some distance-based solutions (Krick, Broucke, & Francis, 2008).

A possible way to overcome this limitation is to include as a part of the problem the role assignment of each robot within the formation, i.e., given a specific pattern decide which robot should occupy which position on it. When taking this route, many solutions in the literature make the strong assumption that final coordinates for the formation positions are known at the time of role allocation: this particular optimization problem is known as the optimal or linear assignment problem (Munkres, 1957), where the objective is, given a set of robots and a set of targets, to find a bipartite matching that minimizes some cost function, e.g., total displacement. This well-known classical problem can be centrally solved using, for example, the Hungarian (Kuhn, 1955) or simplex (Dantzig, Orden, Wolfe, et al., 1955) methods. Since this problem naturally appears in many robotic contexts, it has been exhaustively analyzed from a general multi-robot task allocation perspective (Gerkey & Matarić, 2004; Korsah, Stentz , & Dias, 2013; Smith & Bullo, 2007) and, more specifically, in formation-oriented works aiming at relevant bottlenecks in the role allocation like

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communications and scalability (Bertsekas, 1990; Morgan, Subramanian, Chung, & Hadaegh, 2016; Yu, Chung, & Voulgaris, 2015; Zavlanos, Spesivtsev, & Pappas, 2008).

Lifting the assumptions in the previous examples, known roles and known final positions, gives rise to a more general formulation, which our work studies. There are practical advantages to this approach, that involves considering the computation of the target positions and the role assignments together in the formation problem. The joint computation can lead to overall smaller displacements of the robots, reducing, e.g., total fuel consumption, which is useful in any domain where the formation is not fixed within the global frame (Yim, Shen, Salemi, Rus, Moll, Lipson, Klavins, & Chirikjian, 2007). In addition, the solution of the joint problem implicitly satisfies proximity to the final formation, which in a distance-based context can increase the chances of convergence to the desired equilibrium. On the other hand, the joint problem poses a non-linear optimization problem with a non-trivial solution, because all the variables are intrinsically related. To overcome these issues, partial optimal solutions are computed in Ji, Azuma, and Egerstedt (2006), provided one of the variables is fixed. These results are used afterwards to propose different suboptimal iterative algorithms for the remaining variables. The solution in Zavlanos and Pappas (2007) guarantees a consistent control law while using a market-based algorithm to assign formation roles to agents. This work makes no claims on the optimality of the assignment, only ensuring its soundness in regard to the absence of inconsistent allocations. Another solution is presented in Kanjanawanishkul and Zell (2010), where optimality is sacrificed by using a two-step approach in which the formation translation and rotation are found by consensus and role allocations are corrected during an on-line phase. Inversely, Derenick and Spletzer (2007) starts from a given assignment for a previous formation to find with convex optimization the optimal translation, rotation and scale for a new formation. The previous assignment is kept for the formation transition and hence the solution will be typically suboptimal unless precisely the same assignment were optimal for both formations.

The main contribution of this paper is an exact and provably correct solution able to find all the optimal parameters in finite time for the problem of simultaneous role assignment and formation placement. In order to do so, we first demonstrate that the variables to optimize (translation, rotation, and role assignment) can be computed separately. Then, since the computation of the roles results in a quadratic assignment problem, computationally intractable, our second contribution is an algorithm that exploits complex-number properties of our model to efficiently compute the optimal roles and rotation simultaneously. Our third contribution is a distributed implementation of such algorithm. Taken together, these results provide a complete and efficient solution to the problem being addressed. A preliminary version of this paper was presented in Montijano and Mosteo (2014). This work extends our previous results to account for rotations in the formation, which considerably increase the complexity of the problem.

Our work provides a solution prior to the motion phase and relies on properties of the squared L^2 norm. Hence, it is optimal for holonomous robots and can be useful to other motion models for which this norm is a good approximation. Finally, any problem that requires an initial matching of roles can also benefit by using it as a generic allocation algorithm, such as control laws or motion plans that optimize the action (Karaman & Frazzoli, 2011; Morgan et al., 2016; Şucan & Kavraki, 2012) during the motion phase.

The rest of the paper is organized as follows: Section 2 formally defines the problem. A mathematical decomposition to find the optimal solution is described in Section 3. An efficient algorithm using a recursive search is described in Section 4. Section 5 provides a distributed implementation of the algorithm. Empirical evaluation with simulations is provided in Section 6. Finally, the conclusions of the work are given in Section 7.

2. Problem definition

Let us consider a team of N homogeneous robots, $\mathcal{V} = \{1, \ldots, N\}$, moving on the plane. The position of robot i is denoted by $\mathbf{p}_i \in \mathbb{R}^2$, and $\mathbf{p} = [\mathbf{p}_1^T, \ldots, \mathbf{p}_N^T]^T \in \mathbb{R}^{2N\times 1}$ denotes the concatenation of all the positions. The objective of the team is to self-organize, distributing themselves in a known specific formation pattern, which can be described by a set of points, $\mathbf{b}_j$, and $\mathbf{b} = [\mathbf{b}_1^T, \ldots, \mathbf{b}_N^T]^T$. Without loss of generality, the pattern is defined in such a way that

$$\bar{\mathbf{b}} = \sum_{k=1}^{N} \mathbf{b}_k = \mathbf{0}_2, \tag{1}$$

which can be accomplished simply by shifting the coordinates of the different $\mathbf{b}_j$ by their original centroid. We assume that there are no restrictions about the role of the different robots in the formation, or about where it is achieved in the environment. However, we assume that a limited amount of energy is available, and thus the formation should be achieved minimizing the total distance covered by the whole team of robots in order to reduce the total fuel consumption.

In this regard, denote by $\mathbf{q} = [\mathbf{q}_1^T, \ldots, \mathbf{q}_N^T]^T$ a particular realization of the pattern, expressed as a function of $\mathbf{b}$ with the application of a translation, $\bar{\mathbf{q}}$, and a rotation, ψ, so that

$$\mathbf{q} = \mathbf{1}_N \otimes \bar{\mathbf{q}} + \left(\mathbf{I}_N \otimes \mathbf{R}_\psi\right)\mathbf{b} \tag{2}$$

with

$$\mathbf{R}_\psi = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix}$$

a rotation matrix, $\mathbf{I}_N$ the identity matrix of dimension N, $\mathbf{1}_N$ a vector with all its components equal to one and the operator $\otimes$ used to describe the Kronecker product.

In addition, in order to identify which robot takes which role within the formation, we define the set of assignment variables $x_{ij}, i, j \in \{1, \ldots, N\}$. As it is standard in assignment problems, $x_{ij} = 1$ denotes that robot i assumes the role j within the formation, whereas otherwise $x_{ij} = 0$. Since each robot can only be assigned one role and there cannot be two robots with the same role,

$$\sum_{j=1}^{N} x_{ij} = \sum_{i=1}^{N} x_{ij} = 1 \text{ and } x_{ij} \in \{0, 1\} \tag{3}$$

for all $i, j \in \{1, \ldots, N\}$. The assignment variables can also be arranged in a matrix, $\mathbf{X} = [x_{ij}]$. When all the constraints in (3) are satisfied, the matrix $\mathbf{X}$ is a permutation and, as such, $\mathbf{X}^{-1} = \mathbf{X}^T$.

Finally, given a formation, we consider that the energy (cost) required to move robot ith to the jth position in the formation is given by the squared distance the robots need to move to reach that point, i.e., $\|\mathbf{p}_i - \mathbf{q}_j\|_2^2$. Therefore, the goal of the paper is to find the translation, $\bar{\mathbf{q}}$, rotation, ψ, and role assignment, $\mathbf{X}$, that result in the minimum total displacement of the whole team of robots. This can be presented as an optimization problem of the form

$$\begin{aligned} \underset{\mathbf{X},\bar{\mathbf{q}},\psi}{\text{minimize}} \quad & \sum_{i=1}^{N}\sum_{j=1}^{N} x_{ij}\|\mathbf{p}_i - \mathbf{q}_j\|_2^2, \\ \text{subject to} \quad & (3). \end{aligned} \tag{4}$$

This formulation differs from traditional task assignment problems in that, due to the new decision variables $\mathbf{q}_j$ and ψ, individual robot costs cannot be precomputed. This sets the problem apart from the regular instantaneous optimal assignment problem (Gerkey & Matarić, 2004), requiring a new analysis for an optimal and efficient algorithm. It also differs from typical formation control strategies in that robots do not know a priori specific desired pairwise values, since they are dependent on the assignment.