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Brief paper

Stabilization of systems with asynchronous sensors and controllers☆

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ABSTRACT

We study the stabilization of networked control systems with asynchronous sensors and controllers. Offsets between the sensor and controller clocks are unknown and modeled as parametric uncertainty. First we consider multi-input linear systems and provide a sufficient condition for the existence of linear time-invariant controllers that are capable of stabilizing the closed-loop system for every clock offset in a given range of admissible values. For first-order systems, we next obtain the maximum length of the offset range for which the system can be stabilized by a single controller. Finally we illustrate the results with a numerical simulation.

1. Introduction

In networked and embedded control systems, the outputs of plants are often sampled in a nonperiodic fashion and sent to controllers with time-varying delays. To address robust control with such imperfections, various techniques have been developed, for example, the input-delay approach (Fridman, Seuret, & Richard, 2004; Mirkin, 2007), the gridding approach (Donkers, Heemels, van de Wouw, & Hetel, 2011; Fujioka, 2009; Oishi & Fujioka, 2010), and the impulsive systems approach based on Lyapunov functionals (Naghshtabrizi, Hespanha, & Teel, 2010), on looped functionals (Briat & Seuret, 2012), and on clock-dependent Lyapunov functions (Briat, 2013); see also the surveys (Hespanha, Naghshtabrizi, & Xu, 2007; Hetel et al., 2017). In contrast to the references mentioned above, here we assume that time-stamps are used to provide the controller with information about the sampling times and the communication delays incurred by each measurement. In this approach, sensors send measurements to controllers together with time-stamps, and the controllers exploit this information to mitigate the effect of variable delays and sampling periods (Garcia, Antsaklis, & Montestruque, 2014; Graham & Kumar, 2004; Nakamura, Hirata, & Sugimoto, 2008). However, when the local clocks at the sensors and at the controllers are not synchronized, the time-stamps and the true sampling instants do not match. Protocols to establish synchronization have been actively studied as surveyed in Rhee, Lee, Kim, Serpedin, and Wu (2009), and synchronization by the global positioning system (GPS) or radio clocks has been utilized in some systems. Nevertheless, synchronizing clocks over networks has fundamental limits (Freris, Graham, & Kumar, 2011), and a recent study (Jiang, Zhang, Harding, Makela, & Domíngues-García, 2013) has shown that synchronization based on GPS signals is vulnerable against attacks.

In this paper, we study the stabilization problem of systems with asynchronous sensing and control. We assume that the controller can use the time-stamps but does not know the offset between the sensor and controller clocks, but we do assume that this offset is essentially constant over the time scales of interest. Our objective is to find linear time-invariant (LTI) controllers that achieve closed-loop stability for every clock offset in a given range.

We formulate the stabilization of systems with clock offsets as the problem of stabilizing systems with parametric uncertainty, which can be regarded as the simultaneous stabilization of a family of plants, as studied in Vidyasagar (1985, Sec. 5.4) and Vidyasagar and Viswanadham (1982). However, we had to overcome a few technical difficulties that distinguish the problem considered here from previously published results:

Infinitely many plants: We consider a family of plant models that is indexed by a continuous-valued parameter. Such a family includes

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infinitely many plants, but the approaches for simultaneous stabilization, e.g., in Shi and Qi (2009) exploit the property that the number of plant models is finite.

Nonlinearity of the uncertain parameter: In this work, the uncertain parameter appears in a non-linear form. Therefore, it is not suitable to use the techniques based on linear matrix inequalities (LMIs) in de Oliveira, Bernussou, and Geromel (1999) for the robust stabilization of systems with polytopic uncertainties. Although the robust stability analysis based on continuous paths of systems with respect to the ν-gap metric was developed in Cantoni, Jönsson, and Kao (2012), controller designs based on this approach have not been fully investigated.

Common unstable poles and zeros: Earlier studies on simultaneous stabilization consider a restricted class of plants. For example, the sufficient condition in Blondel, Campion, and Gevers (1993) is obtained for a family of plants with no common unstable zeros or poles. The set of plants in Maeda and Vidyasagar (1984) has common unstable zeros (or poles) but all the plants are stable (or minimum-phase). These assumptions are not satisfied for the systems in the present paper.

We make the following technical contributions for multi-input systems and first-order systems: First we consider multi-input systems and obtain a sufficient condition for stabilization with asynchronous sensing and control. We construct a stabilizing controller from the solution of an appropriately defined $\mathcal{H}^\infty$ control problem. The above mentioned difficulties found in the simultaneous stabilization problem we consider are circumvented by exploiting geometric properties on $\mathcal{H}^\infty$. For first-order systems, we obtain an explicit formula for the exact bound on the clock offset that can be allowed for stability. This result is based on the stabilization of interval systems (Ghosh, 1988; Olbrot & Nikodem, 1994), to which our problem can be reduced for first-order plants. We start by formulating the problem in the context of state feedback without disturbances and noise, but we show in Section 3.2 that the above results also apply for output feedback with disturbances and noise.

The authors in the previous study (Okano, Wakaiki, & Hespanha, 2015) have considered systems with time-varying clock offsets and have proposed a stabilization method with causal controllers, based on the analysis of data rate limitations in quantized control. The stability analysis and the $\mathcal{L}^2$-gain analysis of systems with variable clock offsets have been investigated in Wakaiki, Okano, and Hespanha (2015, 2016), respectively. The major difference with respect to those studies is that here we consider only constant offsets but design stabilizing LTI controllers. This paper is based on the conference paper (Wakaiki, Okano, & Hespanha, 2015), but here we extend the preliminary results for single-input systems to the multi-input case.

The remainder of the paper is organized as follows. Section 2 introduces the closed-loop system we consider and presents the problem formulation. Section 3 is devoted to the discretization of the closed-loop system. In Section 4, we obtain a sufficient condition for the stabilizability of general-order systems. In Section 5, we derive the exact bound on the permissible clock offset for first-order systems. A numerical example is presented in Section 6.

Notation and definitions: We denote by $\mathbb{Z}_+$ the set of non-negative integers. The symbols $\mathbb{D}$ and $\mathbb{T}$ denote the open unit disc $\{z \in \mathbb{C} : |z| < 1\}$ and the unit circle $\{z \in \mathbb{C} : |z| = 1\}$, respectively. We denote by $\mathbb{D}^c$ the complement of the open unit disc $\{z \in \mathbb{C} : |z| \geq 1\}$.

A square matrix F is said to be *Schur stable* if all its eigenvalues lie in the unit disc $\mathbb{D}$. We say that a discrete-time LTI system $\xi_{k+1} = F\xi_k + Gu_k$, $y_k = H\xi_k$ is *stabilizable* (*detectable*) if there exists a matrix K (L) such that $F - GK$ ($F - LH$) is Schur stable. We also use the terminology (F, G) is stabilizable (respectively, (F, H) is detectable) to denote this same concept.

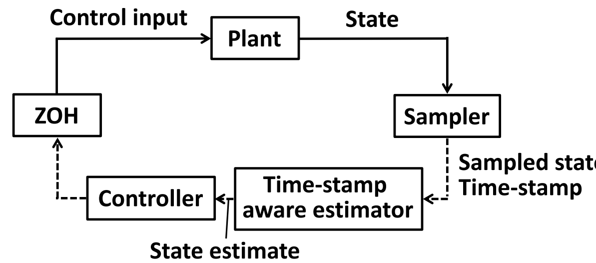

Fig. 1. Closed-loop system with a time-stamp aware estimator.

We denote by $\mathcal{RH}^\infty$ the space of all bounded holomorphic real-rational functions in $\mathbb{D}$. The field of fractions of $\mathcal{RH}^\infty$ is denoted by $\mathcal{RF}^\infty$. For a commutative ring R, $\mathbf{M}(R)$ denotes the set of matrices with entries in R, of any order. For $M \in \mathbf{M}(\mathbb{C})$, $\|M\|$ denotes the induced 2-norm. For $G \in \mathbf{M}(\mathcal{RH}^\infty)$, the $\mathcal{H}^\infty$-norm is defined as $\|G\|_\infty = \sup_{z\in\mathbb{D}} \|G(z)\|$. For $G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \in \mathbf{M}(\mathcal{RF}^\infty)$ and $Q \in \mathbf{M}(\mathcal{RF}^\infty)$, we define a lower linear fractional transformation of G and Q as $\mathcal{F}_\ell(G, Q) := G_{11} + G_{12}Q(I - G_{22}Q)^{-1}G_{21}$.

A pair (N, D) in $\mathbf{M}(\mathcal{RH}^\infty)$ is said to be *right coprime* if the Bezout identity $XN + YD = I$ holds for some $X, Y \in \mathbf{M}(\mathcal{RH}^\infty)$. $P \in \mathbf{M}(\mathcal{RF}^\infty)$ admits a *right coprime factorization* if there exist D, $N \in \mathbf{M}(\mathcal{RH}^\infty)$ such that $P = ND^{-1}$ and the pair (N, D) is right coprime. Similarly, a pair $(\tilde{D}, \tilde{N})$ in $\mathbf{M}(\mathcal{RH}^\infty)$ is *left coprime* if the Bezout identity $\tilde{N}\tilde{X} + \tilde{D}\tilde{Y} = I$ holds for some $\tilde{X}, \tilde{Y} \in \mathbf{M}(\mathcal{RH}^\infty)$. $P \in \mathbf{M}(\mathcal{RF}^\infty)$ admits a *left coprime factorization* if there exist $\tilde{D}, \tilde{N} \in \mathbf{M}(\mathcal{RH}^\infty)$ such that $P = \tilde{D}^{-1}\tilde{N}$ and the pair $(\tilde{D}, \tilde{N})$ is left coprime. If P is a scalar-valued function, then we use the expressions *coprime* and *coprime factorization*.

2. Problem statement

Consider the following LTI plant:

$$\Sigma_P : \dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are the state and the input of the plant, respectively. As shown in Fig. 1, this plant is connected through a sampler and a zero-order hold (ZOH) to a time-stamp aware estimator and a controller, which will be described soon.

Let $s_1, s_2, \dots$ be sampling instants from the perspective of the controller clock. A sensor measures the state $x(s_k)$ and sends it to a controller together with a time-stamp. However, since the sensor and the controller may not be synchronized, the time-stamp determined by the sensor typically includes an unknown offset with respect to the controller clock. In this paper, we assume that the clock offset is constant. Although clock properties are affected by environments such as temperature and humidity, the change of such properties is slow for the time scales of interest. Furthermore, the difference of clock frequencies can be ignored. This is justified by noting that time synchronization techniques, like the one proposed in He, Cheng, Shi, Chen, and Sun (2014), can achieve asymptotic convergence of the clock frequencies (in the mean-square sense), even in the presence of random network delays. We thus assume that the time-stamp $\hat{s}_k$ reported by the sensor is given by

$$\hat{s}_k = s_k + \Delta \quad (k \in \mathbb{N}) \tag{2}$$

for some unknown constant $\Delta \in \mathbb{R}$.

Let $h > 0$ be the update period of the ZOH. The control signal $u(t)$ is assumed to be piecewise constant and updated periodically at times $t_k = kh$ $(k \in \mathbb{N})$ with values u_k computed by the controller: $u(t) = u_k$ for $t \in [t_k, t_{k+1})$. We place a basic assumption for stabilization of sampled-data systems.