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Brief paper

# A complete greedy algorithm for infinite-horizon sensor scheduling[☆]

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ABSTRACT

In this paper we study the problem of scheduling sensors to estimate the state of a linear dynamical system. The estimator is a Kalman filter and our objective is to optimize the *a posteriori* error covariance over an infinite time horizon. We focus on the case where a fixed number of sensors are selected at each time step, and we characterize the exact conditions for the existence of a schedule with uniformly bounded estimation error covariance. Using this result, we construct a scheduling algorithm that guarantees that the error covariance will be bounded if the existence conditions are satisfied. We call such an algorithm complete. Finally, we provide simulations to compare the performance of the algorithm against other known techniques.



## 1. Introduction

One technique for monitoring an environmental process is to deploy a sensor network. Each sensor can be equipped with the ability to make a range of measurements. Sensor networks have been used in various applications including determining a robot's state (Hovland & McCarragher, 1997), tracking the position of a target (Isler & Bajcsy, 2005), selecting the frequency in radar and sonar applications, or monitoring tasks such as chemical processes (Kookos & Perkins, 1999), seismic activity or toxin levels at a factory. Sensor scheduling techniques can also be applied to problems such as adaptive compressed sensing (Liu, Chong, & Scharf, 2012).

The collection of data can be done by operating every sensor continuously; however, the network may be required to have a long life span and so this strategy may not be viable due to energy and communication constraints. To overcome these restrictions, sensors can alternate between awake and asleep modes. Unless the network provides enough redundancy, this method could result in an incomplete picture of the phenomenon of interest. Therefore, a sensing schedule has to be constructed in an intelligent way in order to obtain as much information as possible. This is, in essence, the sensor scheduling problem.

The sensor scheduling problem has received considerable attention in recent years. In the context of linear Gaussian systems, a Kalman filter is the optimal estimator in that it produces an estimate with the least mean square error. Thus, the Kalman filter is commonly used as the basis for the sensor scheduling problem. An exception is Ilkturk (2015), where the condition number of the sequence observability matrix is used as a metric to find a sensor schedule. In this paper we will use a metric on the error covariance of the Kalman filter as our objective function. With this setting, the infinite horizon sensor scheduling problem is studied in Zhang, Vitus, Hu, Abate, and Tomlin (2010). Under some mild conditions, it is shown that the optimal infinite horizon schedule is independent of the initial covariance. Also, it is shown that given an optimal schedule, its cost can be estimated arbitrarily closely by a periodic schedule, with a finite period. However, if the optimal schedule is not known, the analysis does not provide a constructive method for efficiently computing an approximate periodic schedule.

Numerous approaches have been proposed to tackle the sensor scheduling problem. The results in Zhang et al. (2010) serve as a reason to find optimal periodic schedules for infinite horizon scheduling problem. The authors in Shi and Chen (2013b) find a periodic schedule using a branch-and-bound approach. In Shi and Chen (2013a) the authors find an optimal periodic schedule by approximating the objective function of the sensor scheduling problem. A locally optimal solution to periodic scheduling was proposed in Liu, Fardad, Varshney, and Masazade (2014) with constraints on the number of times each sensor can be used in a period. Their objective function incorporates both the estimation error and the number of sensors used per time step. A drawback to these approaches is that the optimal period is unknown, and thus the desired period must be given as an input.

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Optimal and semi-optimal algorithms for the finite horizon problem that use tree pruning techniques are provided in Vitus, Zhang, Abate, Hu, and Tomlin (2012). In Gupta, Chung, Hassibi, and Murray (2004), three different approaches (sliding window, greedy thresholding and random selection) are empirically compared. The authors further develop the random selection method in Gupta, Chung, Hassibi, and Murray (2006), where a strategy for stochastically selecting measurements based on an intelligently constructed probability distribution is described and bounded. In Maheswararajah, Halgamuge, and Premaratne (2009), a few different approaches are studied, including a best step look ahead algorithm, an approach based on the Viterbi algorithm and another by casting the problem as a duality problem. The algorithms are described and empirically compared in terms of performance and computation time.

A convex relaxation based approach is discussed in Weimer, Sinopoli, and Krogh (2008) and applied to the monitoring of $CO_2$ using a wireless sensor network. Another convex relaxation approach is given in Joshi and Boyd (2009) along with solution dependent bounds. In Shamaiah, Banerjee, and Vikalo (2010), this approach is, however, empirically shown to be worse than a greedy algorithm. In Jawaid and Smith (2015), authors studied some properties of greedy sensor scheduling algorithms and their relation to submodular set functions.

A general framework for the sensor scheduling problem is presented in Mo, Ambrosino, and Sinopoli (2011). A number of problems can be addressed in this framework such as minimizing the final covariance over a time horizon, the average covariance, the variance of a single state, or even the cost of a finite horizon LQG regulator. A number of network constraints can also be included. The problem is framed as a relaxed quadratic program, and a greedy approach is described although the error bound is not necessarily uniformly bounded for unstable systems. In Maity and Baras (2015), a continuous time sensor scheduling problem is considered for an objective capturing both estimation error and sensor switching costs.

In this paper we consider infinite-horizon sensor scheduling. Based on the discussion above, existing approaches for this problem are (1) to fix a period and compute a periodic schedule; (2) to repeatedly apply a finite-horizon algorithm; or (3) to greedily select sensors at each time step. For each of these methods, there are no guarantees that the resulting schedule will produce a uniformly bounded sequence of covariance matrices. In fact, we do not know of any results that characterize the exact conditions under which an infinite horizon sensor schedule exists that results in a uniformly bounded sequence of covariance matrices.

*Contributions*: We give necessary and sufficient conditions for the existence of an infinite horizon sensor schedule with a bounded error covariance (Section 4). We then provide a complete algorithm for sensor scheduling (Section 5): That is, our algorithm outputs a uniformly bounded sensor schedule if one exists. The algorithm has the same runtime as the simple greedy algorithm and we show in simulations (Section 6) that our proposed algorithm outperforms the greedy algorithm, and can be used to efficiently compute schedules for high-dimensional linear systems with a large number of sensors.

A preliminary version of this paper was presented in Jawaid and Smith (2014). Relative to this early version, we now provide a more efficient algorithm along with details on its implementation. We also extend both the algorithm and analysis to the general problem of $k$ sensors per time step, and provide complete proofs of the correctness of the proposed greedy algorithm. Finally, we present more extensive simulation results on high-dimensional linear systems, including a system obtained by discretizing the heat equation.

## 2. Preliminaries

Consider the discrete-time linear stochastic system

$$\begin{aligned} x_{t+1} &= Ax_t + w_t, \quad x_t \in \mathbb{R}^n, \\ y_t &= C_t x_t + v_t, \quad y_t \in \mathbb{R}^k \end{aligned} \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$ and $C \in \mathbb{R}^{m\times n}$. The matrix $C_t$ is a subset of $k$ rows of $C$. This is the standard sensor selection model, as in Mo et al. (2011) and Vitus et al. (2012). The process noise $w_t$ and measurement noise $v_t$ are zero mean Gaussian noise vectors with covariance matrices $W, V \in \mathbb{R}^{n\times n}$, respectively, with $W \succeq 0$ and $V \succ 0$. We assume that the noises are independent over time.

For the case $C_t = C$ (LTI system), the system is said to be observable if its observability matrix $\Theta = \mathrm{col}(C, CA, \dots, CA^{n-1})$ has rank $n$.

If the observability matrix is not full rank then a similarity transform $T$ can be used to convert the system into standard form for unobservable systems.

$$\bar{A} = T^{-1}AT = \begin{bmatrix} A_{\bar{o}} & A_{12} \\ 0 & A_o \end{bmatrix}, \qquad \bar{C} = CT = \begin{bmatrix} 0 & C^o \end{bmatrix}. \tag{2}$$

Here $(A_o, C^o)$ is observable. If $A_{\bar{o}}$ is stable, the system is said to be detectable.

Consider a sequence of measurements $\sigma = (\sigma_0, \sigma_1, \dots)$, and the corresponding sequence of matrices $(C_0, C_1, \dots)$. For a given time $t$ and time window $k$, the sequence observability matrix for the given system can be written as

$$B_\sigma(t, t+k) = \mathrm{col}(C_t, C_{t+1}A, \dots C_{t+k}A^k).$$

The following definition follows from the definition of Uniform Detectability in Anderson and Moore (1981).

**Definition 1** (*Uniform Detectability*). For the system in (1), the sequence of measurements $\sigma$ is uniformly detectable if there exist non-negative integers $s$, $r$ and constants $\alpha \in [0, 1)$ and $\beta > 0$, such that for all $\{x \in \mathbb{R}^n \mid \|x\| = 1\}$ and all times $t$,

$$\left\|A^r x\right\| \geq \alpha \Longrightarrow \|B(t, t+s)x\| \geq \beta > 0. \tag{3}$$

Additionally, for the given system, the sequence is uniformly observable if there exist integer $s$ and positive constants $\beta_1, \beta_2$ such that

$$\begin{aligned} & 0 < \beta_1 \leq \|B(t, t+s)x\| \leq \beta_2 \\ \iff \quad & \mathrm{rank}(B(t, t+s)) = n. \end{aligned} \tag{4}$$

Note that for a general time varying system, the equivalence in (4) holds only in the forward direction. For example, the reverse direction does not necessarily hold when $C_t$ or $A_t$ can take on infinitely many values. This, however, is not the case for the system in (1), where $A$ is fixed and $C_t$ is a subset of rows of a time invariant $C$.

Finally, we will use the following result for some of our proofs.

**Lemma 2.** *Suppose $(A, C)$ is observable and $A$ is full rank. Then, letting $c_i$ be the ith row of $C$,*

$$B \triangleq col(c_1, c_1A, \dots, c_1A^{n-1}, c_2A^n, \dots, c_mA^{mn-1}) \tag{5}$$

*is full rank.*

**Proof.** Since $\mathrm{rank}(\Theta) = n$, it suffices to show that each of the rows of $\Theta$ can be written as a linear combination of the rows in $B$. Let $X_i = \{c_i, c_iA, \dots, c_iA^{n-1}\}$ for $i = 1, \dots, m$. Note that the rows of $\Theta$ comprise of the vectors in the multiset $\bigcup_{i=1}^m X_i$. Also, note that $x \in X_i \Longrightarrow xA^{(i-1)n}$ is a row of $B$. Let $X_i^b = X_iA^{(i-1)n} = \{c_iA^{(i-1)n}, c_iA^{(i-1)n+1}, \dots, c_iA^{in-1}\}$. So the rows of $B$ comprise of elements of the multiset $\bigcup_{i=1}^m X_i^b$.