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Brief paper

Receding horizon control for multiplicative noise stochastic systems with input delay[☆]

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ABSTRACT

This paper is concerned with finite horizon stabilization control for a class of discrete time stochastic systems subject to multiplicative noise and input delay. By constructing a new cost function, a complete solution to the problem of finite horizon stabilization is given for the first time based on previous work Zhang et al. (2015). It is shown that the system can be stabilized in the mean square sense with the receding horizon control (RHC) if and only if two new inequalities on the terminal weighting matrices are satisfied. Moreover, the two inequalities can be solved by using iterative algorithm. The explicit stabilizing controller is derived by solving a finite horizon optimal control problem. Simulations demonstrate the effectiveness of the proposed method.

1. Introduction

In the past few decades, receding horizon control (RHC, also known as model predictive control) has attracted interest from the control community because of its applicability in chemical, automotive, and aerospace processes. A considerable amount of research effort has been devoted to RHC, e.g., see Garcia, Prett, and Morari (1989), Mayne (2014), Mayne, Rawlings, Rao, and Scokaert (2000), Richalet, Rault, Testud, and Papon (1978) and the references therein. The basic concept of RHC is to solve an optimization problem on the finite horizon at the current time and implement only the first solution as the current control. This procedure is then repeated at the next time step.

The stabilization problem as one of fundamental problems has been studied extensively based on RHC. Kwon et al. (Kwon & Pearson, 1977) originally studied the stabilizing property of the RHC law for linear systems. This idea was then generalized to stochastic systems and time delay systems (Bernardini & Bemporad, 2012; Cannon, Kouvaritakis, & Wu, 2009a,b; Chatterjee, Hokayem, & Lygeros, 2011; Chatterjee & Lygeros, 2015; Hessem & Bosgra, 2003; Hokayem, Cinquemani, Chatterjee, Ramponi, & Lygeros, 2012; Kwon, Lee, & Han, 2004; Lee & Han, 2015; Park, Yoo, Han, & Kwon, 2008; Perez & Goodwin, 2001; Primbs & Sung, 2009; Wei & Visintini, 2014). Chatterjee et al. (2011) and Hokayem et al. (2012) investigated the RHC for additive noise systems. In Chatterjee et al. (2011), the optimization problem was solved by using a vector space method that ensured the variance of state was bounded. In Hokayem et al. (2012), incomplete state information was considered and a Kalman filter was used to estimate the optimal state. RHC bounded the state of the overall systems in the mean square sense. Refs. Cannon et al. (2009a) and Primbs and Sung (2009) studied RHC for systems with multiplicative noise. In Cannon et al. (2009a), the concept of probability invariance was introduced to ensure the stability of a closed-loop system, whereas Primbs and Sung (2009) used semi-definite programming to solve the optimization problem and the stability of the closed-loop system was ensured under a specific terminal weight and terminal constraint. In Cannon et al. (2009b), a system with both additive and multiplicative noise was considered based on RHC. Other related RHC stochastic problems can be found in Bernardini and Bemporad (2012), Chatterjee and Lygeros (2015), Hessem and Bosgra (2003), Perez and Goodwin (2001) and Wei and Visintini (2014). Systems with time delay have also been subjected to RHC (Kwon et al., 2004; Lee & Han, 2015; Park et al., 2008). For

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instance, Kwon et al. (2004) studied RHC for a system with state delay and used a linear matrix inequality (LMI) condition on the terminal weighting matrices to guarantee the stability of a closed-loop system. In Park et al. (2008), a system with input delay was investigated based on RHC, and a stabilization condition was derived based on LMI. Ref. Lee and Han (2015) also considered RHC stabilization for a system with state delay. By proposing a more generalized cost function, a delay dependent stability condition was obtained. However, it is notable that time delay and multiplicative noise have been considered separately in all of the aforementioned studies and the references therein. When a system has both time delay and multiplicative noise, the control problem is particularly difficult. One of the obstacles is that the separation principle does not hold for stochastic systems with multiplicative noise.

In this article, we discuss the RHC stabilization of discrete time linear systems with both multiplicative noise and input delay. Our aim is to determine the RHC stabilization condition, and derive the RHC stabilization controller when this condition is met. The main contributions of this paper are three-fold: First, we construct a novel cost function that includes two terminal weighting matrices. An explicit stabilizing controller is obtained by solving this finite horizon optimal control problem. Second, a necessary and sufficient condition for the stabilization of delayed stochastic systems is developed. Under some mild assumptions, it is shown that the system can be stabilized in the mean square sense if and only if two inequalities regarding terminal weighting matrices are satisfied. Third, by introducing a slack variable, an iterative algorithm has been proposed to solve the two inequalities.

The remainder of this paper is organized as follows. Section 2 presents the formulation of the problem for stochastic systems with multiplicative noise and input delay. In Section 3 the corresponding RHC law and the necessary and sufficient condition for the asymptotic mean square stability of the closed-loop system are derived. The iterative algorithm to solve the two inequalities is also discussed in Section 3. A numerical example to validate the performance of the proposed RHC is provided in Section 4. Finally, our conclusions are given in Section 5.

The following notations are used throughout the paper. $\mathcal{R}^n$ denotes the n dimensional Euclidean space. The subscript $'$ represents the matrix transpose; a symmetric matrix $M > 0 (\geq 0)$ means that it is strictly positive definite (positive semi-definite). $\{\Omega, \mathcal{F}, \mathcal{P}, \{\mathcal{F}_k\}_{k\geq 0}\}$ denotes a complete probability space on which some scalar white noise ω_k is defined such that $\{\mathcal{F}_k\}_{k\geq 0}$ is the natural filtration generated by ω_k, i.e., $\mathcal{F}_k = \sigma\{\omega_0, \ldots, \omega_k\}$. Let $\hat{x}_{k|m} = E_{m-1}(x_k)$, where $E_{m-1}(x_k)$ is the conditional expectation of x_k with respect to $\mathcal{F}_{m-1}$. $E(\cdot)$ denotes the mathematical expectation over the noise $\{\omega_k, k \geq 0\}$.

2. Problem statement

Consider the following linear discrete time stochastic system with input delay:

$$x_{k+1} = (A + \omega_k\bar{A})x_k + (B + \omega_k\bar{B})u_{k-d}, \quad (1)$$

with the initial condition $x_0, u_{-d}, u_{-d+1}, \ldots, u_{-1}$. For the convenience of later discussion, let $A_k = A + \omega_k\bar{A}$, $B_k = B + \omega_k\bar{B}$. Then, system (1) becomes

$$x_{k+1} = A_k x_k + B_k u_{k-d}, \quad (2)$$

where $x_k \in \mathcal{R}^n$ is the state; $u_k \in \mathcal{R}^m$ is the input with delay $d > 0$; $\bar{A}$, $\bar{B}$, A, and B are matrices of appropriate dimensions; and ω_k is a scalar random white noise with zero mean and variance σ.

The problem to be solved in this paper is formulated as follows: Find the $\mathcal{F}_{k-d-1}$-measurable controller $u_{k-d} = H\hat{x}_{k|k-d}, k \geq d$, such that the closed-loop system $x_{k+1} = A_k x_k + B_k H\hat{x}_{k|k-d}$ is asymptotically mean square stable, i.e., $\lim_{k\to\infty} E(x_k' x_k) = 0$.

Remark 1. Note that the results presented in this paper are applicable to more general systems of multiple multiplicative noises with no substantial difference:

$$x_{k+1} = (A + \omega_k^{(1)}\bar{A})x_k + (B + \omega_k^{(2)}\bar{B})u_{k-d},$$

where $\omega_k^{(1)} \neq \omega_k^{(2)}$.

3. Receding horizon control for discrete time stochastic systems with input delay

In this section, we present results for the asymptotic mean square stability for discrete time stochastic systems with input delay (1). The RHC solution is given first.

3.1. Receding horizon control

To solve the problem formulated in Section 2, we first introduce the following function:

$$\begin{aligned} &J(x_k, \mathcal{U}_k^{(d)}, k, k+N, \mathcal{U}_k) \\ &= \sum_{i=0}^{N} x_{k+i}' Q x_{k+i} + \sum_{i=0}^{N-d} u_{k+i}' R u_{k+i} + (x_{k+N+1})' \\ &\quad \times P^{(1)} x_{k+N+1} + (x_{k+N+1})' \sum_{i=2}^{d+1} P^{(i)} \hat{x}_{k+N+1|k+N+i-d-1}, \end{aligned}$$

where x_k and $\mathcal{U}_k^{(d)} = (u_{k-1}, \ldots, u_{k-d})$ are known values at time k; $\mathcal{U}_k = (u_k, \ldots, u_{k+N-d})$ is the control to be determined; $Q \geq 0$, $R > 0$, and N is a finite positive integer. For the convenience of discussions in the below, we denote the cost function as:

$$\begin{aligned} &J_{k-1}(x_k, \mathcal{U}_k^{(d)}, k, k+N, \mathcal{U}_k) \\ &= E_{k-1}\Big[J(x_k, \mathcal{U}_k^{(d)}, k, k+N, \mathcal{U}_k)\Big], \end{aligned} \quad (3)$$

where $E_{k-1}[J(x_k, \mathcal{U}_k^{(d)}, k, k+N, \mathcal{U}_k)]$ is the conditional mathematical expectation given $\mathcal{F}_{k-1} = \sigma\{\omega_0, \ldots, \omega_{k-1}\}$.

It is assumed that the weighting matrices $P^{(i)}$, $i = 1, 2, \ldots, d+1$ satisfy $P^{(1)} > 0$, $P^{(2)} \leq 0$, and

$$P^{(i)} = (A')^{i-2} P^{(2)} A^{i-2}, \quad i = 3, \ldots, d+1, \quad (4)$$

$$\sum_{i=1}^{d+1} P^{(i)} > 0.$$

Note that once $P^{(2)}$ is given, $P^{(i)}$, $i = 3, \ldots, d+1$ are determined by (4). Thus, there are only two independent terminal weighting matrices, $P^{(1)}$ and $P^{(2)}$. We shall show that the two matrices $P^{(1)}$ and $P^{(2)}$ play a key role in designing the RHC to guarantee mean square stability.

Remark 2. The cost function (3) is nonnegative. Because

$$\begin{aligned} &E_{k-1}\Big[(x_{k+N+1})' P^{(1)} x_{k+N+1} + (x_{k+N+1})' \sum_{i=2}^{d+1} P^{(i)} \\ &\quad \times \hat{x}_{k+N+1|k+N+i-d-1}\Big] \geq E_{k-1}\Big[(x_{k+N+1})' \sum_{i=1}^{d+1} P^{(i)} x_{k+N+1}\Big] \geq 0, \end{aligned}$$

$Q \geq 0$ and $R > 0$, we have that the cost function (3) is nonnegative.

Remark 3. Considering the input delay in the stochastic system (1), the terminal terms of the cost function herein are given by

$$(x_{k+N+1})' P^{(1)} x_{k+N+1} + (x_{k+N+1})' \sum_{i=2}^{d+1} P^{(i)} \hat{x}_{k+N+1|k+N+i-d-1}$$