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Brief paper

On weak topology for optimal control of switched nonlinear systems☆

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ABSTRACT

Optimal control of switched systems is challenging in general due to the discrete nature of the switching control input. One of the most well-known approaches is the embedding based method which addresses this challenge by solving a relaxed optimization problem with only continuous inputs and then projecting the relaxed solution back to obtain a solution to the original problem. In this paper, we present a unified topology based framework for analyzing and designing various embedding based switched optimal control algorithms. The proposed framework views the embedding based approaches from a novel topological perspective as a change of topology over the optimization space. A general procedure of constructing different switched optimal control algorithms with guaranteed convergence to a stationary point is described. Numerical examples are also provided to illustrate the effectiveness of the proposed framework.

1. Introduction

Switched systems consist of a family of subsystems and a switching signal which determines the active subsystem (mode) at each time instant. Optimal control of switched systems involves finding both the continuous inputs and switching signals to jointly optimize certain performance index. This problem has attracted considerable research attention due to its diverse engineering applications in power electronics (Oettmeier, Neely, Pekarek, DeCarlo, & Uthaichana, 2009), automotive systems (Hedlund & Rantzer, 1999; Rinehart, Dahleh, Reed, & Kolmanovsky, 2008; Uthaichana, DeCarlo, Bengea, Pekarek, & Žefran, 2011), robotics (Wei, Uthaichana, Žefran, & DeCarlo, 2013), and manufacturing (Cassandras, Pepyne, & Wardi, 2001).

Optimal control of switched systems is in general challenging due to the discrete nature of the switching signal, which prevents us from directly applying the classical optimal control and optimization techniques. To address this issue, numerous approaches have been investigated in the literature. The Maximum Principle was extended to characterize optimal hybrid control solutions (Piccoli, 1998; Shaikh & Caines, 2003; Sussmann, 1999, 2000). However, it is difficult to numerically compute the optimal solutions based on these abstract necessary conditions (Xu & Antsaklis, 2003).

The bilevel optimization method is a well-known approach proposed in Xu and Antsaklis (2003, 2004), which divides the switched optimal control problem into two sub-problems and solves them iteratively. At the lower level, switching instants are optimized with respect to a fixed switching mode sequence via classical variational methods. Then, the switching mode sequence is updated at the upper level once the optimal switching instants are obtained. Several more recent papers (Axelsson, Wardi, Egerstedt, & Verriest, 2008; Egerstedt, Wardi, & Axelsson, 2006; Gonzalez, Vasudevan, Kamgarpour, Sastry, Bajcsy, & Tomlin, 2010a, b) fall into this category, mainly dealing with the lower level optimization problem. Although various heuristic schemes have been proposed for the upper level update as well in these papers, solutions obtained by this method may still be unsatisfactory due to the restriction on mode sequences.

More recently, an alternative approach based on the so-called *embedding principle* has been investigated (Bengea & DeCarlo, 2005; Vasudevan, Gonzalez, Bajcsy, & Sastry, 2013a, b). This approach is closely related to the relaxed optimal control problems which optimize over the convex closure of the original control set (Berkovitz, 1974; Ge, Kohn, Nerode, & Remmel, 1996; Warga, 2014). The embedding based approach solves the relaxed optimal control problem first and applies a projection operator which maps the relaxed optimal control back to the original input space to generate a desired solution. In Bengea and DeCarlo (2005), the proposed approach solves the relaxed optimal control problem via the Maximum Principle and uses the Chattering Lemma (Berkovitz, 1974) as the projection operator. In Vasudevan et al. (2013a, b), the authors developed a comprehensive algorithm which uses a first order gradient-based approach (Polak, 1997) to solve the relaxed

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optimal control problem and provides a constructive method to generate the projection operator based on the Haar wavelet approximation.

This paper extends the particular embedding based approach (Bengea & DeCarlo, 2005; Vasudevan et al., 2013a, b) to a general topology based framework, which views the embedding based approach as a change of topology over the optimization space. Specifically, most existing embedding based algorithms adopt the weak topology induced by the state trajectory. The proposed framework allows for alternative choices of the topologies according to particular underlying problems. Our framework constitutes a weak topology over the optimization space dictating the embedding procedure, an algorithm solving the relaxed optimization problem, and a projection operator generating the desired solution. Different selections of these components will result in different embedding based switched optimal control algorithms. We also derive a set of conditions for these components so that the resulting algorithm converges to a stationary point of the original problem under the selected weak topology.

The main contributions of this paper lie in three aspects. First, the proposed framework offers a unified weak topology formulation of switched optimal control problems which includes most existing embedding based approaches as special cases. Second, the proposed framework provides more freedom to choose weak topologies, optimization algorithms, and projection operators, which expands the applicability of the embedding based approach. Last the set of convergence conditions derived in this paper establishes general criteria for effectiveness of the resulting embedding based algorithms. Two numerical examples are presented to illustrate the importance of weak topologies and the usage of the proposed framework in design and analysis of switched optimal control algorithms.

The rest of this paper is organized as follows: Section 2 formulates the switched optimal control problem of interest. Section 3 first reviews some important concepts in topology and then develops the proposed framework, along with its convergence analysis. Two numerical examples are presented in Section 4. Concluding remarks are given in Section 5.

2. Problem formulation and preliminaries

Consider the following switched nonlinear system

$$\dot{x}(t) = f_{\sigma(t)}(t, x(t), u(t)), \text{ for a.e. } t \in [0, t_f], \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$ is the system state, $u(t) \in U \subset \mathbb{R}^{n_u}$ is the continuous input constrained in a compact and convex set U, $\sigma(t) \in \Sigma \triangleq \{1, 2, \dots, n_\sigma\}$ is the switching signal which determines the active subsystem (mode) among a finite number n_σ of subsystems at time t, and t_f is the time horizon which is considered to be finite in this paper.

Following similar notations used in Bengea and DeCarlo (2005), Vasudevan et al. (2013a), we rewrite the system as follows

$$\begin{aligned}\dot{x} &= \sum_{i=1}^{n_\sigma} d_i(t) f_i(t, x(t), u(t)) \\ &\triangleq f(t, x(t), u(t), d(t)), \text{ for a.e. } t \in [0, t_f],\end{aligned} \tag{2}$$

where $d(t) = [d_1(t), \dots, d_{n_\sigma}(t)] \in D$ for a.e. $t \in [0, t_f]$, and $D \triangleq \left\{(d_1, \dots, d_{n_\sigma}) \in \{0, 1\}^{n_\sigma} \middle| \sum_{i=1}^{n_\sigma} d_i = 1\right\}$ is the set of corners of the n_σ-simplex. The continuous input u and discrete input d can be viewed as mappings from $[0, t_f]$ to U and D, respectively. In this paper, we assume these mappings belong to the $\mathcal{L}^2$ space, defined as follows.

Definition 1. We say a function $g : [0, t_f] \to G \subseteq \mathbb{R}^n$ belongs to $\mathcal{L}^2([0, t_f], G)$, if

$$\|g\|_{\mathcal{L}^2} \triangleq \left(\int_0^{t_f} \|g(t)\|_2^2 \, dt\right)^{\frac{1}{2}} < \infty, \tag{3}$$

where the integration is taken with respect to the Lebesgue measure.

Let $\mathcal{U} = \mathcal{L}^2([0, t_f], U)$ be the space of continuous input signals and let $\mathcal{D} = \mathcal{L}^2([0, t_f], D)$ be the space of discrete input signals. We denote by $\mathcal{X}_p = \mathcal{U} \times \mathcal{D}$ the overall original input space and call $\xi \in \mathcal{X}_p$ an *original* input signal. Suppose the initial state $x(0) = x_0 \in \mathbb{R}^{n_x}$ is given and fixed, we denote by $\phi_t(\xi) \in \mathbb{R}^{n_x}$ the system state at time t driven by input signal ξ and $\phi(\xi) \in \mathcal{L}_2([0, t_f], \mathbb{R}^{n_x})$ the corresponding state trajectory.

With the above notations, the cost function considered in this problem is given by $h(\phi_{t_f}(\xi))$, which only penalizes terminal state. Optimal control problems with nontrivial running cost can be transformed into this form by augmenting the state space (Polak, 1997). We consider the following state and control constraints

$$\begin{aligned}&h^x_{j_1}(\phi(\xi)) \le 0, \quad \forall j_1 \in \mathcal{J}_1 \triangleq \{1, 2, \dots, n^x_c\}, \\ &h^u_{j_2}(\xi) \le 0, \quad \forall j_2 \in \mathcal{J}_2 \triangleq \{1, 2, \dots, n^u_c\}.\end{aligned}$$

Note that, the state and control constraints above are imposed on the state trajectory and control trajectory. Standard constraints imposed on state and input at each time instant can be easily incorporated.

The following assumptions are adopted to ensure the existence and uniqueness of the state trajectory of system (1) and the well-posedness of the optimal control problem.

Assumption 1.

1. f_i is Lipschitz continuous with respect to all arguments for all $i \in \Sigma$ with a common Lipschitz constant L,
2. h, $h^x_{j_1}$ and $h^u_{j_2}$ are Lipschitz continuous with respect to all arguments for all $j_1 \in \mathcal{J}_1$ and for all $j_2 \in \mathcal{J}_2$ with a common Lipschitz constant L.

Remark 1. We assume a common Lipschitz constant L to simplify notation. All the results in this paper extend immediately to the case where all these functions have different Lipschitz constants.

We further define $\Psi(\xi) \triangleq \max_{j_1 \in \mathcal{J}_1, j_2 \in \mathcal{J}_2} \left\{h^x_{j_1}(\xi), h^u_{j_2}(\xi)\right\}$. The constraints in (2) can then be rewritten compactly as $\Psi(\xi) \le 0$, since $\Psi(\xi) \le 0$ if and only if $h^x_{j_1}(\xi) \le 0$ for all $j_1 \in \mathcal{J}_1$ and $h^u_{j_2}(\xi) \le 0$ for all $j_2 \in \mathcal{J}_2$.

With the above notations, the switched optimal control problem is formulated as the following optimization problem:

$$\mathcal{P}_{\mathcal{X}_p} : \begin{cases} \inf\limits_{\xi \in \mathcal{X}_p} & J(\xi), \\ \text{subject to} & \Psi(\xi) \le 0. \end{cases} \tag{4}$$

The problem $\mathcal{P}_{\mathcal{X}_p}$ is a constrained optimization problem over function space $\mathcal{X}_p$. The classical optimization techniques cannot be directly applied to solve this problem due to the discrete nature of $\mathcal{X}_p$. The embedding based approach addresses this issue by first embedding the switched system into a larger class of continuous nonlinear systems with only continuous inputs. Then, a relaxed optimization problem associated with the continuous system is solved via the classical numerical optimization algorithms. Finally, the relaxed optimal control is projected back to the original input space to obtain a solution to the original problem.

In this paper, we propose to view the embedding based approach as a change of topology over the optimization space, resulting in a general procedure for developing embedding based switched optimal control algorithms. In the next section, we first briefly review some concepts in weak topology and then establish the topology based framework.